MATHS

Q. 1. The mean of the numbers a, b, 8, 5, 10 is 6 and the variance is 6.80 . Then which one of the following gives possible values a and b?

i. a = 1, b = 6
ii. a = 3, b = 4
iii. a = 0, b = 7
iv. a = 5, b = 2

Sol.

$$Mean = \frac{\sum x}{n} = 6$$

$$Variance = \frac{\sum x^2}{n} - \left(\frac{\sum x}{n}\right)^2 = 6.8$$

$$- \frac{a^2 + b^2 + 64 + 25 + 100}{5} - 36 - 6.8$$

$$\Rightarrow a^2 + b^2 + 189 - 180 = 34$$

$$\Rightarrow a^2 + b^2 = 25$$

Possible values of a and b is given by (2)

Q. 2. The vector $\vec{a} = a\hat{i} + 2\hat{j} + \beta\hat{K}$ lies in the plane of the vectors $\vec{b} = \hat{i} + \hat{j} \ and + \vec{c} = \hat{j} + \hat{k}$ and bisects the angle between $\vec{b} \ and \ \vec{c}$. Then which one of the following gives possible values of $\alpha \ and \ \beta$?

i. $\alpha = 2, \beta = 1$
ii. $\alpha = 1, \beta = 1$
iii. $\alpha = 2, \beta = 2$
iv. $\alpha = 1, \beta = 2$

Sol.

$As\ \vec{a}, \vec{b}\ and\ \vec{c}\ are\ coplanar$

$\therefore \left[\vec{a}\ \vec{b}\ \vec{c}\right] = 0$

$Or, \alpha + \beta = 2 \qquad (i)$

$Also\ \vec{a}\ bisects\ the\ angle\ between\ \vec{b}\ and\ \vec{c}$

$\therefore \vec{a} = \lambda\left(\vec{b} + \vec{c}\right)$

$or, \vec{a} = \lambda\left(\frac{\hat{i} + 2\vec{j} + \vec{k}}{\sqrt{2}}\right) \qquad (ii)$

$But\ \ \vec{a} = \alpha\,\vec{2} + 2\vec{j} + \beta\vec{k}$

$Hence\ \lambda = \sqrt{2}\ and\ \alpha = 1,\ \beta = 1$

$Which\ also\ satisfy \quad (i)$

$\therefore Correct\ answer\ is\ (2)$

Q. 3.

$The\ non-zero\ vectors\ \vec{a}, \vec{b}\ and\ \vec{c}\ are\ related\ by\ \vec{a} = 8\vec{b}\ and\ \vec{c} = -7\vec{b}$
$Then\ the\ angle\ between\ \vec{a}\ and\ \vec{c}\ is$

i. $\frac{\pi}{2}$
ii. π
iii. 0
iv. $\frac{\pi}{4}$

Sol. The sign of $\vec{a}\ and\ \vec{c}$ are opposite. Hence they are parallel but directions are opposite. Therefore angle between $\vec{a}\ and\ \vec{c}\ is\ \pi$

$\therefore correct\ answer\ is\ (2)$

Q. 4. The line passing through the points (5, 1, a) and (3, b, 1) crosses the yz-plane at the point $\left(0, \frac{17}{2}, \frac{-13}{2}\right). Then$

i. a = 6, b = 4
ii. a = 8, b = 2
iii. a = 2, b = 8
iv. a = 4, b = 6

Sol. Equation of line through (5, 1, a) and (3, b, 1) is

$$\frac{x-5}{-2}=\frac{y-1}{b-1}=\frac{z-a}{1-a}=\lambda$$

any point on (i) is

$$\{5-2\lambda, 1+(b-1)\lambda, a+(1-a)\lambda\} \qquad (ii)$$

$$As\left(0,\frac{17}{2},-\frac{13}{2}\right) \text{ lies on } (i)$$

$$5-2\lambda=0 \Rightarrow \alpha=\frac{5}{2} \qquad (iii)$$

$$1+(b-1)\times\frac{5}{2}=\frac{17}{2}$$

$$or,\ 2+5b-5=17$$

$$or,\ b=4$$

$$and\ a+(1-a)\times\frac{5}{2}=-\frac{13}{2}$$

$$or,\ 2a+5-5a=-13$$

$$or,\ a=6$$

$\therefore$ *Correct answer is* (1)

Q. 5. If the straight lines $\frac{x-1}{k}=\frac{y-2}{2}=\frac{z-3}{3}$ *and* $\frac{x-2}{3}=\frac{y-3}{k}=\frac{z-1}{2}$ intersect at a point, then the integer k is equal to

i. 2
ii. 2
iii. 5
iv. 5

Sol. As the given lines intersect

$$\therefore \begin{vmatrix} 2-1 & 3-2 & 1-3 \\ k & 2 & 3 \\ 3 & k & 2 \end{vmatrix}=0$$

$$Or, \begin{vmatrix} 1 & 1 & 2 \\ k & 2 & 3 \\ 3 & k & 2 \end{vmatrix}=0$$

$$or,\ k=-5, \frac{5}{2}$$

Integer is -5 *only*

$\therefore$ *Correct answer is* (3)

Q. 6. The differential of the family of circles with fixed radius 5 units and centre on the line y = 2 is

i. $(y-2)^2\ y'^2 = 25-(y-2)^2$
ii. $(x \quad 2)^2\ y'^2 = 25 \quad (y \quad 2)^2$
iii. $(x-2)\ y'^2 = 25-(y-2)^2$
iv. $(y-2)\ y'^2 - 25-(y-2)^2$

Sol. The required equation of circle is

$$(x-a)^2+(y-2)^2=25 \qquad (i)$$

differentiating we get

$$2(x-a)+2(y-2)\,y'=0$$

$$or,\ a=x+(y-2)\,y' \qquad (ii)$$

putting a in (i)

$$\{x-x-(y-2)\,y'\}^2+(y-2)^2=25$$

$$or,\ (y-2)^2\ y'^2=25-(y-2)^2$$

$\therefore$ *The correct answer is* (1)

Q. 7. Let a, b, c be any real numbers. Suppose that there are real numbers x, y, z not all zero such that $x=cy+bz,\ y=az+cx \text{ and } z=bx+ay.$ *Then* $a^2+b^2+c^2+2abc$ *is equal to*

i. 0
ii. 1
iii. 2
iv. - 1

Sol.

$$x=cy+bz \Rightarrow x-cy-bz=0 \qquad (i)$$

$$y=az+bx \Rightarrow bx-y+az=0 \qquad (ii)$$

$$z=bx+ay \Rightarrow bx+ay-z=0 \qquad (iii)$$

Eliminating x, y, z from (i), (ii) *and* (iii) *we get*

$$\begin{vmatrix} 1 & -c & -b \\ c & -1 & a \\ b & a & -1 \end{vmatrix}=0$$

$$or,\ a^2+b^2+c^2+2abc=1.$$

$\therefore$ *The correct answer is* (2)

Q. 8. Let A be a square matrix all of whose entries are integers. Then which one of the following is true?

i. $If\ \det A = \pm 1,\ then\ A^{-1}\ exists\ and\ all\ its\ entries\ are\ integers$

ii. $If\ \det A = \pm 1,\ then\ A^{-1}\ need\ not\ exist$

iii. $If\ \det A = \pm 1,\ then\ A^{-1}\ exist\ but\ all\ its\ entries\ are\ not\ necessarily\ integers$

iv. $If\ \det A = \pm 1,\ then\ A^{-1}\ exist\ and\ all\ its\ entries\ are\ non-integers$

Sol. The obvious answer is (1).

Q. 9. The quadratic equations $x^2 - 6x\ a = 0\ and\ x^2 - cx + 6 = 0$ and have one root in common. The other roots of the first and second equations are integers in the ratio 4 : 3. Then the common root is

i. 3
ii. 2
iii. 1
iv. 4

Sol.

$Let\ the\ roots\ of\ x^2 - 6x + a = 0$

$be\ \alpha\ and\ 4\beta\ and\ that\ of\ x^2 - cx + 6 = 0\ be\ \alpha\ and\ 3\beta$

$\therefore\ \alpha + 4\beta = 6 \qquad (i)$

$4\alpha\beta = a \qquad (ii)$

$\alpha + 3\beta = c \qquad (iii)$

$3\alpha\beta = 6 \qquad (iv)$

Using $(ii)\ \&\ (iv)$

$$\frac{4}{3} = \frac{a}{6} \Rightarrow a = 8$$

$Then \qquad x^2 - 6x + a = 0$

$reduces\ to$

$$x^2 - 6x + 8 = 0$$

$$x = \frac{6 \pm \sqrt{36 - 32}}{2}$$

$$= \frac{6 \pm 2}{2} = 4,\ 2$$

$$\therefore\ \alpha = 2,\ \beta = 1$$

$$\therefore\ Correct\ answer\ is\ (2)$$

Q. 10. How many different words can be formed by jumbling the letters in the word MISSISSIPPI in which no two S are adjacent?

i. $6.8.{}^{7}C_4$

ii. $7.{}^{6}C_4.\ {}^{8}C_4$

iii. $8.{}^{6}C_4.\ {}^{7}C_4$

iv. $6.7.{}^{8}C_4$

Sol. M = 1, I = 4, P = 2

These letters can be arranged by

$$\frac{(1+4+2)!}{1!\,4!\,2!} = 7\ {}^{6}C_4 \ ways$$

The remaining 8 gaps can be filled by 4 S by ${}^{8}C_4 \ ways$

$$\therefore \ Total\ no.\ of\ ways = 7\ {}^{6}C_4 \quad {}^{8}C_4$$

$$\therefore \ Correct\ answer\ is\ (2)$$

Q. 11.

$$Let\ I = \int_0^1 \frac{\cos x}{\sqrt{x}}\,dx.\ Then\ which\ one\ of\ the\ following\ is\ true?$$

i. $I < \frac{2}{3}\ and\ J > 2$

ii. $I < \frac{2}{3}\ and\ J < 2$

iii. $I > \frac{2}{3}\ and\ J > 2$

iv. $I < \frac{2}{3}\ and\ J > 2$

Sol.

$We\ Know\ \frac{\sin x}{x} < 1,\ when\ x \in (0, 1)$

$\therefore \frac{\sin x}{\sqrt{x}} < \sqrt{x}$

$\Rightarrow \int_0^1 \frac{\sin x}{\sqrt{x}}\,dx < \int_0^1 \sqrt{x}\,dx$

$\Rightarrow \int_0^1 \frac{\sin x}{\sqrt{x}}\,dx < \frac{2}{3}$

$Also,\ \cos x < 1,\ when\ x \in (0,1)$

$\therefore \frac{\cos x}{\sqrt{x}} < \frac{1}{\sqrt{x}}$

$\Rightarrow \int_0^1 \frac{\cos x}{\sqrt{x}}\,dx < \int \frac{1}{\sqrt{x}}\,dx$

$\int_0^1 \frac{\cos x}{\sqrt{x}}\,dx < 2$

$\therefore I < \frac{2}{3}\ and\ J < 2$

$\therefore$ *Correct answer is* (4)

Q. 12. The area of the plane region bounded by the curve $x + 2y^2 = 0\ and\ 3y^2 = 1\ is\ equal\ to$

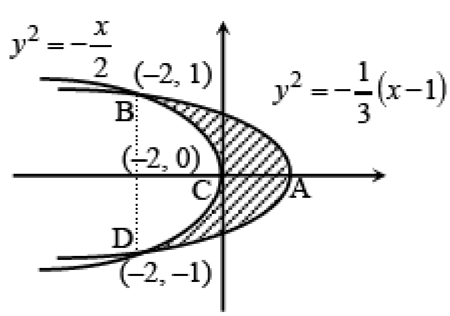
i. $\frac{2}{3}$

ii. $\frac{4}{3}$

iii. $\frac{5}{3}$

iv. $\frac{1}{3}$

Sol.



$x + 2y^2 = 0 \Rightarrow y^2 = -\frac{x}{2}$

$x + 3y^2 = 1 \Rightarrow y^2 = -\frac{1}{3}(x-1)$

$\therefore -\frac{x}{2} = -\frac{1}{3}(x-1)$

$or, \quad -\frac{x}{2} = -\frac{x}{3} + \frac{1}{3}$

$or, \quad \frac{x}{3} - \frac{x}{2} = \frac{1}{3}$

$or, \quad -\frac{x}{6} = \frac{1}{3}$

$or, \quad x = -2$

$\therefore y^2 = 1 \Rightarrow y = \pm 1$

Area of the region BCA

$$= \left| \int_0^1 \left\{ \left(-2y^2\right) - \left(1 - 3y^2\right) \right\} dy \right|$$

$$= \left| \int_0^1 \left(y^2 - 1\right) dy \right|$$

$$= \left| \left[\frac{y^3}{3}\ y \right]_0^1 \right|$$

$$= \left| \frac{1}{3} - 1 \right| = \frac{2}{3}$$

Hence area of the region bounded by the curve is equal to $2 \times \frac{2}{3} = \frac{4}{3}$

$\therefore$ *Correct answer is* (2)