FACULTY OF SCIENCE

B.A. / B.Sc. I Year (Practical) Examination

Subject : MATHEMATICS

Paper – I
(Differential Equations and Solid Geometry)

QUESTION BANK

W.E.F. Annual 2009

Time : 3 Hours} **{Max. Marks: 50**

Unit – I:

Differential equations of first order and first degree :

1. Solve $xy - \frac{dy}{dx} = y^3 e^{-x}$

2. Solve $\frac{dy}{dx} + \frac{y}{x} = y^2 \sin x$.

3. Solve $(1 + e^{x/y})\, dx + e^{x/y} (1 - x/y)\, dy = 0$.

4. Solve $(xy \sin xy + \cos xy)\, y\, dx + (xy \sin xy - \cos xy)x\, dy = 0$.

5. Solve $(x^2 + y^2 + 2x)dx + 2y dy = 0$.

6. Solve $3e^x \tan y + (1 - e^x) \sec^2 y \frac{dy}{dx} = 0$.

7. Solve $(xy^3 + y)\, dx + 2(x^2 y^2 + x + y^4)\, dy = 0$.

8. Solve $x^2 y dx - (x^3 + y^3)\, dy = 0$.

Equations of the first order but not of the first degree :

9. Solve $p^3(x + 2y) + 3p^2(x + y) + (y + 2x)p = 0$.

10. Solve $x^2p^2 - 2xyp + (2y^2 - x^2) = 0$.

11. Solve $xp^2 - 2yp + ax = 0$.

12. Solve $y = 2px + \tan^{-1}(xp^2)$.

13. Solve $x^2 = a^2(1 + p^2)$.

14. Solve $y = 2p + 3p^2$.

15. Solve $(x - a)\, p^2 + (x - y)\, p - y = 0$.

16. Solve, $\sin px \cos y = \cos px \sin y + p$.

Applications of first order differential equations :

17. Find the orthogonal trajectories of $x^2 + y^2 = cx$.

18. Find the orthogonal trajectories of $r = c_1(1 - \sin\theta)$.

19. Find the orthogonal trajectories of $y = c_1 e^{-x}$.

20. Find the orthogonal trajectories of $x^{1/3} + y^{1/2} = c_1$.

Unit – II

Higher order differential equations :

21. Solve $(D^2 + a^2)y = \tan ax$.

22. Solve $(D^2 + 1)y = e^{-x} + \cos x + x^3 + e^x \cos x$.

23. Solve $(D^2 + 1)(D^2 + 4)y = \cos \frac{x}{2} \cos \frac{3x}{2}$

24. Solve $(D^2 + 1)y = \cos x + xe^{2x} + e^x \sin x$.

Solve the following differential equations by the method of undetermined coefficients :

25. $\frac{d^2y}{dx^2} + 3\frac{dy}{dx} + 2y = \sin x$.

26. $\frac{d^2y}{dx^2} + \frac{dy}{dx} + y = x^2$.

27. $(D^2 - 2D - 8)y = 9xe^x + 10e^{-x}$

28. $(D^2 - 3D)y = 2e^{2x} \sin x$.

Solve the following differential equations using the method of variation of parameters.

29. $y'' + 3y' + 2y = 12\, e^x$

30. $y'' + 2y' + y = x^2 e^{-x}$

31. $y'' + y = 4x \sin x$.

32. $y'' - 2y' + y = e^x \log x$.

Use the reduction of order method to find the solution of the following equation ; One solution of the homogeneous equation is given

33. $y'' - \frac{2}{x} y' + \frac{2}{x^2} y = 0 \quad y_1 = x$.

34. $(2x^2 + 1) y'' - 4xy' + 4y = 0 \quad y_1 = x$

35. $y'' - \frac{2}{x} y' + \frac{2}{x^2} y = x \log x, \ y_1 = x$.

36. $x^2y'' + xy' - y = x^2 e^{-x}$. $y_1 = x$.

Solve the following differential equations:

37. $x^4 \frac{d^3y}{dx^3} + 2x^3 \frac{d^2y}{dx^2} - x^2 \frac{dy}{dx} + xy = 1.$

38. $x^2 \frac{d^2y}{dx^2} - x \frac{dy}{dx} + 2y = x \log x.$

39. $(2x+3)^2 \frac{d^2y}{dx^2} -2(2x+3) \frac{dy}{dx} - 12y = 6x.$

40. $(x+3)^2 \frac{d^2y}{dx^2} -4(x+3) \frac{dy}{dx} + 6y = \log (x+3).$

<u>Unit – III</u>

Planes :

41. A variable plane is at a constant distance 3p from the origin and meets the axes in A, B and C. Show that the locus of the centroid of the triangle ABC is $x^{-2} + y^{-2} + z^{-2} = p^{-2}$.

42. A variable plane passes through a fixed point (a, b, c) and meets the co-ordinate axes in A, B, C. Show that the locus of the point common to the planes through A, B, C parallel to the co-ordinate planes is $a/x + b/y + c/z = 1$.

43. Show that the equation $12x^2 – 2y^2 – 6z^2 – 2xy + 7yz + 6zx = 0$ represents a pair of planes and also find the angle between them.

Right Line :

44. Find the coordinates of the foot of the perpendicular drawn from the origin to the plane $2x + 3y – 4z + 1 = 0$; also find the coordinates of the point which is the image of the origin in the plane.

45. Find the equation of the plane through the point (1, 1, 1) and perpendicular to the line $x – 2y + z = 2$, $4x + 3y – z + 1 = 0$.

46. A square ABCD of diagonal 2a is folded along the diagonal AC, so that plane DAC, BAC are at right angles. Show that the shortest distance between DC and A is then $2a / \sqrt{3}$.

47. Find the magnitude and the equations of the line of shortest distance between the two lines :

$$\frac{x-3}{2} = \frac{y+15}{-7} \quad \frac{z-9}{5} \quad \frac{x+1}{2} \quad \frac{y-1}{1} = \frac{z-9}{-3}$$

48. Find the length and the equations of the shortest distance line between

$5x – y – z = 0$, $x – 2y + z + 3 = 0$;

$7x – 4y – 2z = 0$, $x – y + z – 3 = 0$;

49. Find the magnitude and the equations of the line of shortest distance between the lines.

$$\frac{x}{4} = \frac{y+1}{3} = \frac{z-2}{2} \text{ ; } 5x - 2y - 3z + 6 = 0; \; x - 3y + 2z - 3 = 0.$$

50. Obtain the co-ordinates of the points where the shortest distance line between the lines.

$$\frac{x-23}{-6} = \frac{y-19}{-4} = \frac{z-25}{3} \quad \frac{x-12}{-9} \quad \frac{y-1}{4} \quad \frac{z-5}{2}$$

Spheres:

51. A variable plane through a fixed point (a, b, c) cuts the co-ordinate axes in the point A,B, C show that the locus of the centres of the spheres O ABC is

$$\frac{a}{x} + \frac{b}{y} + \frac{c}{z} = 2$$

52. Find the equation of the sphere through the four the four points (0, 0, 0), (-a, b, c), (a, -b, c) , (a, b, -c) and determine its radius.

53. Obtain in the equation of the sphere which passes through the three points (1, 0, 0), (0, 1, 0), (0, 0,1), and has its radius as small as possible.

54. Find the center and the radius of the circle $x^2 + y^2 + z^2 - 2y - 4z = 11$, $x + 2y + 2z = 15$.

55. Obtain the equation of the sphere having the circle.

$$x^2 + y^2 + z^2 + 10y - 4z - 8 = 0, \quad x + y + z = 3 \text{ as the great circle.}$$

56. Show that the plane $2x - 2y + z + 12 = 0$ touches the sphere

$$x^2 + y^2 + z^2 - 2x - 4y + 2z = 3 \text{ and find the point of contact.}$$

57. Obtain the equations of the sphere which pass through the circle

$x^2 + y^2 + z^2 - 2x - 2y + 4z - 3 = 0$, $2x + y + z = 4$ and touches the plane $3x + 4y = 14$.

58. Show that the polar line of $(x + 1)/2 = \frac{y-2}{3} = (z+3)$, with respect to the

sphere $x^2 + y^2 + z^2 = 1$ is the line $\frac{7x+3}{11} = \frac{y-2}{5} = \frac{z}{-1}$

59. Find the equation of the sphere that passes through the circle. $x^2 + y^2 + z^2 - 2x + 3y - 4z + 6 = 0$, $3x - 4y + 5z - 15 = 0$ and cuts the sphere $x^2 + y^2 + z^2 + 2x + 4y - 6z + 11 = 0$ orthogonally.

60. Find the limiting points of the co-axial system of spheres. $x^2 + y^2 + z^2 - 20x + 30y - 40z + 29 + \lambda (2x - 3y - 4z) = 0$.

Unit – IV:

CONES AND CYLINDERS :

61. Find the equation of the cone whose vertex is the point (1, 1,0) and whose guiding curve is $y = 0, x^2 + z^2 = 4$.

62. Find the equation of the cone with vertex at (1, 2, 3) and guiding curve

$$x^2 + y^2 + z^2 = 4, \quad x + y + z = 1.$$

63. Find enveloping cone of the sphere $x^2 + y^2 + z^2 – 2x + 4z = 1$ with its vertex at (1, 1, 1).

64. Find the equation of the cone whose vertex is at the origin and the direction cosines of whose generators satisfy the relation $3\ell^2 – 4m^2 + 5n^2 = 0$.

65. Find the equation to the cone which passes through the three cc-ordinate axes as well as the two lines

$$\frac{x}{1} = \frac{y}{-2} = \frac{z}{3} \quad \frac{x}{3} = \frac{y}{-1} = \frac{z}{1}$$

66. Show that the equation $x^2 - 2y^2 + 3z^2 – 4xy + 5yz – 6zx + 8x – 19y – 2z – 20 = 0$ represents a cone with vertex (1, -2, 3).

67. Find the angle between the lines of intersections of $x + y + z = 0$ and $x^2 + yz + xy – 3z^2 = 0$.

68. If the planes $2x – y + cz = 0$ cuts the cone $yz + zx + xy = 0$ in perpendicular lines, find the values of c.

69. Find the equation of the lines in which the plane $2x + y – z = 0$ cuts the cone $4x^2 - y^2 + 3z^2 = 0$.

70. Show that the locus of mid-points of chords of the cone

$ax^2 + by^2 + cz^2 + 2fyz + 2gx + 2hxy = 0$.

drawn parellel to the line $x / \ell = y / m = z/n$ is the plane

$$x(a\ell + hm + gn) + y(h\ell + bm + fn) + z(gV + fm + cn) = 0.$$

71. Find the plane which touches the cone $x^2 + 2y^2 – 2yz – 5zx + 3xy = 0$ along the generator whose direction rations are 1, 1,1.

72. Prove that the cones $fyz + gzx + hxy = 0$; $\sqrt{fx} + \sqrt{gy} + \sqrt{hz} = 0$ are resiprocal. .

73. Find the equation of the right circular cone whose vertex is (1, -2, -1), axis the line

$$\frac{x-1}{3} = \frac{y+2}{4} = \frac{z+1}{5}$$ and semi-vertical angle 60^o.

74. Find the equation to the cylinder whose generators are parallel to $\frac{x}{1} = \frac{y}{2} = \frac{z}{3}$ and guiding curve is $x^2 + y^2 = 16, z = 0$

75. Find the equation of the enveloping cylinder of the conicoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$ generator are parallel to the line $\frac{x}{\ell} = \frac{y}{m} = \frac{z}{n}$

76. Obtain the equation of a cylinder whose generators touch the sphere $x^2 + y^2 + z^2 + 2uv + 2vy + 2wz + d = 0$. whose generators are parallel to the line $\frac{x}{\ell} = \frac{y}{m} = \frac{z}{n}$

77. Obtain the equation of the right circular cylinder whose guiding curve is the circle through the points (1, 0, 0), (0, 1, 0), (0, 0, 1).

78. Find the equation of the right circular cylinder of radius 2 whose axis is the line $(x - 1) / 2 = (y – 2) / 2 = (z – 2) / 2$.

79. Find the equation of the right circular whose axis is $\frac{x-2}{2} = \frac{y-1}{1} = \frac{z}{3}$ and passes through (0, 0, 3).

80. Prove that the right circular cylinder whose one section is the circle.

$$x^2 + y^2 + z^2 - x - y – z = 0, \quad x + y + z = 1, \text{ is}$$

$$x^2 + y^2 + z^2 - yz – zx – xy = 1.$$

*****