7093

Register Number

Part III — BUSINESS MATHEMATICS

(English Version)

Time Allowed : 3 Hours] [Maximum Marks : 200

SECTION – A

N. B. : i) Answer *all* the 40 questions.

ii) Choose and write the correct answer from the four choices given. 40 × 1 = 40

1. If the minor of a_{23} equal to the cofactor of a_{23} in $|a_{ij}|$, then the minor of a_{23} is

   a) 1   b) 2

   c) 0   d) 3.

2. If $|A| = 0$ then $|\text{Adj } A|$ is

   a) 0   b) 1

   c) -1 d) ± 1.

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3. If $A = \begin{pmatrix} 0 \cdot 8 & 0 \cdot 6 \\ -0 \cdot 6 & 0 \cdot 8 \end{pmatrix}$ then A^{-1} is

a) $\begin{pmatrix} -0 \cdot 8 & 0 \cdot 6 \\ -0 \cdot 6 & 0 \cdot 8 \end{pmatrix}$

b) $\begin{pmatrix} 0 \cdot 8 & -0 \cdot 6 \\ 0 \cdot 6 & 0 \cdot 8 \end{pmatrix}$

c) $\begin{pmatrix} 0 \cdot 8 & 0 \cdot 6 \\ 0 \cdot 6 & 0 \cdot 8 \end{pmatrix}$

d) $\begin{pmatrix} 0 \cdot 2 & 0 \cdot 4 \\ -0 \cdot 4 & 0 \cdot 2 \end{pmatrix}$.

4. A system of linear homogeneous equations has at least

a) one solution

b) two solutions

c) three solutions

d) four solutions.

5. If $T = \begin{matrix} & A & B \\ A & \begin{pmatrix} 0 \cdot 7 & 0 \cdot 3 \\ B & x & 0 \cdot 8 \end{pmatrix} \end{matrix}$ is a transition probability matrix, then the value of x is

a) 0·3

b) 0·2

c) 0·4

d) 0·7.

6. Focus of $y^2 = -4ax$ is

a) $(a, 0)$

b) $(0, a)$

c) $(0, -a)$

d) $(-a, 0)$.

7. Latus rectum of an ellipse $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$, $(a > b)$, is

a) $\dfrac{2a^2}{b}$

b) $\dfrac{a^2}{2b}$

c) $\dfrac{2b^2}{a}$

d) $\dfrac{b^2}{2a}$.

8. The difference between the focal distances of any point on the hyperbola is equal to length of its

    a)   transverse axis

    b)   semi-transverse axis

    c)   conjugate axis

    d)   semi-conjugate axis.

9. Eccentricity of the rectangular hyperbola is

    a)   2

    b) $\dfrac{1}{2}$

    c) $\sqrt{2}$

    d) $\dfrac{1}{\sqrt{2}}$.

10. If 20 units of some product cost Rs. 2500 and 50 units cost Rs. 3400 to produce, the linear cost function is

    a) $y = 30x + 1900$

    b) $y = 20x + 5900$

    c) $y = 50x + 3400$

    d) $y = 10x + 900$.

11. For the cost function $C = \dfrac{1}{10}\,e^{2x}$, the marginal cost is

    a) $\dfrac{1}{10}$

    b) $\dfrac{1}{5}\,e^{2x}$

    c) $\dfrac{1}{10}\,e^{2x}$

    d) $\dfrac{1}{10}\,e^{x}$.

12. For the function $y = 3x + 2$, the average rate of change of y when x increases from 1·5 to 1·6, is

    a)   1

    b)   0·5

    c)   0·6

    d)   3.

13. The slope of the tangent at (2, 8) on the curve $y = x^3$ is

   a) 3

   b) 12

   c) 6

   d) 8.

14. The slope of the curve $x = y^2 - 6y$ at the point where it crosses the y-axis is

   a) 5

   b) -5

   c) $\dfrac{1}{6}$

   d) $\dfrac{-1}{6}$.

15. The stationary value of x for $f(x) = 3(x-1)(x-2)$ is

   a) 3

   b) $\dfrac{3}{2}$

   c) $\dfrac{2}{3}$

   d) $\dfrac{-3}{2}$.

16. If $u = x^y \ (x > 0)$ then $\dfrac{\partial u}{\partial y}$ is equal to

   a) $x^y \log x$

   b) $\log x$

   c) $y^x \log x$

   d) $\log y^x$.

17. If $z = x^3 + 3xy^2 + y^3$ then the marginal productivity of x is

   a) $x^2 + y^2$

   b) $6xy + 3y^2$

   c) $3(x^2 + y^2)$

   d) $(x^2 + y^2)^2$.

18. The cost function $y = 40 - 4x + x^2$ is minimum when

a) $x = 2$ b) $x = -2$

c) $x = 4$ d) $x = -4$.

19. $\int_{-2}^{2} x^4 \, dx$ is

a) $\dfrac{32}{5}$ b) $\dfrac{64}{5}$

c) $\dfrac{16}{5}$ d) $\dfrac{8}{5}$.

20. The area of the region bounded by $y = x + 1$, the x-axis and the lines $x = 0$ and

    $x = 1$ is

a) $\dfrac{1}{2}$ b) 2

c) $\dfrac{3}{2}$ d) 1.

21. If the marginal revenue $R'(x) = \dfrac{1}{x+1}$ then the revenue function is

a) $\log |x + 1| + k$ b) $-\dfrac{1}{x+1}$

c) $\dfrac{1}{(x+1)^2}$ d) $\log \dfrac{1}{x+1}$.

22. The degree and order of the differential equation $\dfrac{d^2 y}{dx^2} - 6\sqrt{\dfrac{dy}{dx}} = 0$ are

a)   2 and 1   b)   1 and 2

c)   2 and 2   d)   1 and 1.

23. The solution of the equation of the type $\dfrac{dy}{dx} + py = 0$ (p is a function of x) is given

. by

   a) $y\, e^{\int pdx} = c$

   b) $y \int pdx = c$

   c) $x\, e^{\int pdx} = y$

   d) $y = cx$.

24. The integrating factor of $x\dfrac{dy}{dx} - y = e^x$ is

   a) $\log x$

   b) $e^{-\frac{1}{x}}$

   c) $\dfrac{1}{x}$

   d) $-\dfrac{1}{x}$.

25. The complementary function of the differential equation $\left(D^2 - D\right)y = e^x$ is

   a) $A + Be^x$

   b) $(Ax + B)e^x$

   c) $A + Be^{-x}$

   d) $(A + Bx)e^{-x}$.

26. $\Delta\, f(x) =$

   a) $f(x + h)$

   b) $f(x) - f(x + h)$

   c) $f(x + h) - f(x)$

   d) $f(x) - f(x - h)$.

27. In a line of the best fit $y = 5 \cdot 8 \, (\, x - 1994 \,) + 41 \cdot 6$, the value of y when $x = 1997$ is

    a)　50

    b)　54

    c)　59

    d)　60.

28. If a discrete random variable has the probability mass function as

x	0	1	2	3
$P(x)$	k	$2k$	$3k$	$5k$

    then the value of k is

    a)　$\dfrac{1}{11}$

    b)　$\dfrac{2}{11}$

    c)　$\dfrac{3}{11}$

    d)　$\dfrac{4}{11}$

29. The standard deviation of a Poisson variate is 2. The mean of the Poisson variate is

    a)　2

    b)　4

    c)　$\sqrt{2}$

    d)　$\dfrac{1}{\sqrt{2}}$.

30. If $X \sim N\left(\mu, \sigma^2\right)$, the points of inflexion of normal distribution curve are

    a)　$\pm\,\mu$

    b)　$\mu \pm \sigma$

    c)　$\sigma \pm \mu$

    d)　$\mu \pm 2\sigma$.

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31. If $X \sim N(8, 64)$, the standard normal variate Z will be

    a) $\dfrac{X-64}{8}$

    b) $\dfrac{X-8}{64}$

    c) $\dfrac{X-8}{8}$

    d) $\dfrac{X-8}{\sqrt{8}}$.

32. The standard error of the sample mean is

    a) Type I error

    b) Type II error

    c) Standard deviation of the sampling distribution of the mean

    d) Variance of the sampling distribution of the mean.

33. The central limit theorem states that the sampling distribution of the mean will approach normal distribution

    a) as the size of the population increases

    b) as the sample size increases and becomes larger

    c) as the number of samples gets larger

    d) as the sample size decreases.

34. Probability of rejecting the null hypothesis when it is true, is

    a) Type I error

    b) Type II error

    c) Sampling error

    d) Standard error.

35. The number of ways in which one can select 2 customers out of 10 customers is

    a) 90

    b) 60

    c) 45

    d) 50.

36. A time series consists of

    a) two components
    b) three components

    c) four components
    d) none of these.

37. Index number is a

    a) measure of relative changes
    b) a special type of an average

    c) a percentage relative
    d) all of these.

38. Laspeyre's index formula uses weights of the

    a) base year quantities

    b) current year prices

    c) average of the weights of number of years

    d) none of these.

39. Control charts in statistical quality consist of

    a) three control lines
    b) upper and lower control limits

    c) the level of process
    d) all of these.

40. The term 'regression' was introduced by

    a) R. A. Fisher
    b) Sir Francis Galton

    c) Karl Pearson
    d) none of them.

SECTION – B

N. B. : Answer any *ten* questions. $10 \times 6 = 60$

41. Find the inverse of $A = \begin{pmatrix} 2 & 3 & 4 \\ 3 & 2 & 1 \\ 1 & 1 & -2 \end{pmatrix}$, if it exists.

42. Find the rank of the matrix $\begin{pmatrix} 1 & 1 & 1 & 3 \\ 2 & -1 & 3 & 4 \\ 5 & -1 & 7 & 11 \end{pmatrix}$.

43. Find the equation of the parabola which has focus (1, 2) and directrix $x + y - 2 = 0$.

44. The demand curve for a monopolist is given by $x = 100 - 4p$.

    i) Find the total revenue, average revenue and marginal revenue.

    ii) At what value of x, the marginal revenue is equal to zero ?

45. If the total cost C of making x units is $C = 50 + 10x + 5x^2$, find the average cost and marginal cost when $x = 13$.

46. If $u = x^3 + y^3 + z^3 - 3xyz$, prove that $x\dfrac{\partial u}{\partial x} + y\dfrac{\partial u}{\partial y} + z\dfrac{\partial u}{\partial z} = 3u$.

47. Find the area enclosed by the parabola $y^2 = 4x$, $x = 1$, $x = 4$ and the x-axis.

48. Solve : $\left(1 - x^2\right)\dfrac{dy}{dx} - xy = 1$

49. Solve : $\left(4D^2 - 12D + 9\right)y = 0$

50. From the following data find $f(3)$:

X :	1	2	3	4	5
$f(x)$	2	5	—	14	32

51. Fit a straight line for the following data :

X	0	1	2	3	4
Y	1	1	3	4	6

52. A continuous random variable has the following p.d.f. :

$$f(x)=\begin{cases} kx^2 & ;0\le x\le 10 \\ 0 & ;\text{otherwise} \end{cases}$$

Determine k and evaluate

i) $P(0\cdot2 \le X \le 0\cdot5)$

ii) $P(X \le 3)$

53. A random sample of marks in mathematics secured by 50 students out of 200 students show a mean of 75 and a standard deviation of 10. Find the 95% confidence limits for the estimate of their mean marks.

54. From the following data compute the correlation co-efficient :

$N = 11$, $\Sigma X = 117$, $\Sigma Y = 260$, $\Sigma X^2 = 1313$, $\Sigma Y^2 = 6580$, $\Sigma XY = 2827$.

[Turn over

55. Calculate Fisher's Ideal Index from the following data :

Commodity	Price		Quantity	
	1985	1986	1985	1986
A	8	20	50	60
B	2	6	15	10
C	1	2	20	25
D	2	5	10	8
E	1	5	40	30

SECTION – C

N. B. : Answer any *ten* questions. 10 × 10 = 100

56. Solve the following equations by determinant method :

$x + 2y + 5z = 23$, $3x + y + 4z = 26$, $6x + y + 7z = 47$.

57. The newspapers A and B are published in a city. Their present market shares are 15% for A and 85% for B. Of those who bought A the previous year 65% continue to buy it again while 35% switch over to B. Of those who bought B the previous year 55% buy it again and 45% switch over to A. Find their market shares after two years.

58. Find the equation to the hyperbola which has the lines $x + 4y - 5 = 0$ and $2x - 3y + 1 = 0$ for its asymptotes and which passes through the point (1, 2).

59. Find the equation of the tangent and normal to the curve $y(x-2)(x-3)-x+7=0$ at the point where it cuts the x-axis.

60. $R=21x-x^2$ and $C=\dfrac{x^3}{3}-3x^2+9x+16$ are respectively the sales revenue and cost function of x units sold. Find :

   i)   At what output the revenue is maximum ? What is the total revenue at this point ?

   ii)   What is the marginal cost at a minimum ?

   iii) What output will maximise the profit ?

61. The demand for a commodity A is $q_1=240-p_1^2+6p_2-p_1p_2$. Find the partial elasticities $\dfrac{Eq_1}{Ep_1}$ and $\dfrac{Eq_1}{Ep_2}$ when $p_1=5$ and $p_2=4$.

62. Evaluate : $\displaystyle\int_{\frac{\pi}{6}}^{\frac{\pi}{3}}\dfrac{dx}{1+\sqrt{\tan x}}$

63. The elasticity of demand (x) with respect to price p is $\dfrac{3-x}{x}, x<3$. Find the demand function and the revenue function when the price is 2 and the demand is 1.

64. Suppose that the quantity demanded $Q_d=42-4p-4\dfrac{dp}{dt}+\dfrac{d^2p}{dt^2}$ and quantity supplied $Q_s=-6+8p$ where p, is the price. Find the equilibrium price for market clearance.

65. Using Gregory-Newton's formula, find $y(22{\cdot}4)$:

X	19	20	21	22	23
Y	91	100	110	120	131

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66. Find the mean and variance for the following probability distribution :

$$f(x) = \begin{cases} 2e^{-2x} & ; \quad x \geq 0 \\ 0 & ; \quad x < 0 \end{cases}$$

67. The number of accidents in a year attributed to taxi drivers in a city follows Poisson distribution with mean 3. Out of 500 taxi drivers, find the approximate number of drivers with

i) no accident in a year

ii) more than 2 accidents in a year. $\left(e^{-3} = 0 \cdot 04979\right)$

68. To test the conjecture of the management that 60 per cent employees favour a new bonus scheme, a sample of 150 employees was drawn and their opinion was taken whether they favoured it or not. Only 55 employees out of 150 favoured the new bonus scheme. Test the conjecture at 1% level of significance.

69. Calculate the seasonal Indices for the following data using average method :

Year	Quarters			
	I	II	III	IV
1982	72	68	80	70
1983	76	70	82	74
1984	74	66	84	80
1985	76	74	84	78
1986	78	74	86	82

70. The following data shows the value of sample mean $\overline{X}$ and the range R for ten samples of size 5 each. Calculate the values for central line and control limits for mean chart and range chart and determine whether the process is in control.

Sample no.	1	2	3	4	5	6	7	8	9	10
Mean $\overline{X}$	11·2	11·8	10·8	11·6	11·0	9·6	10·4	9·6	10·6	10·0
Range R	7	4	8	5	7	4	8	4	7	9

(Given for $n = 5$, $A_2 = 0·577$, $D_2 = 0$, $D_4 = 2·115$)

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