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Part III — MATHEMATICS

(English Version)

Time Allowed : 3 Hours]

[Maximum Marks : 200

SECTION - A

N. B. : i) All questions are compulsory.

ii) Each question carries *one* mark.

iii) Choose the most suitable answer from the given four alternatives.

40 × 1 = 40

1. The degree of the differential equation $\sqrt{1 + \left(\frac{dy}{dx}\right)^{1/3}} = \frac{d^2y}{dx^2}$ is

a) 1

b) 2

c) 3

d) 6.

2. The Particular Integral (P.I.) of $(3D^2 + D - 14) y = 13e^{2x}$ isa) $26xe^{2x}$ b) $13xe^{2x}$ c) xe^{2x} d) $\frac{x^2}{2} e^{2x}$.

[Turn over

3. $p \leftrightarrow q$ is equivalent to

a) $p \rightarrow q$

b) $q \rightarrow p$

c) $(p \rightarrow q) \vee (q \rightarrow p)$

d) $(p \rightarrow q) \wedge (q \rightarrow p)$

4. The order of $[7]$ in $[Z_9, +_9]$ is

a) 9

b) 6

c) 3

d) 1.

5. In the set of integers under the operation $*$ defined by $a * b = a + b - 1$, the identity element is

a) 0

b) 1

c) a

d) b .

6. $\lim_{x \rightarrow 0} \frac{a^x - b^x}{c^x - d^x} =$

a) ∞

b) 0

c) $\log \frac{ab}{cd}$

d) $\frac{\log \left(\frac{a}{b} \right)}{\log \left(\frac{c}{d} \right)}$

7. The stationary point of $f(x) = x^{3/5}(4-x)$ occurs at $x =$

a) $\frac{3}{2}$

b) $\frac{2}{3}$

c) 0

d) 4.

8. If $u = \log\left(\frac{x^2 + y^2}{xy}\right)$, then $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y}$ is

a) 0

b) u

c) $2u$

d) u^{-1} .

9. The curve $a^2 y^2 = x^2(a^2 - x^2)$, $a > 0$ is symmetrical about

a) x -axis onlyb) y -axis onlyc) $y = x$

d) both axes and about the origin.

10. The value of $\int_0^{\pi/4} \cos^3 2x \, dx$ is

a) $\frac{2}{3}$

b) $\frac{1}{3}$

c) 0

d) $\frac{2\pi}{3}$.

11. If $(n-5) + i(n+4)$ is the complex conjugate of $(2m+3) + i(3n-2)$, then (n, m) are

a) $\left(-\frac{1}{2}, -8\right)$

b) $\left(-\frac{1}{2}, 8\right)$

c) $\left(\frac{1}{2}, -8\right)$

d) $\left(\frac{1}{2}, 8\right)$.

12. The polar form of the complex number $(i^{25})^3$ is

a) $\cos \frac{\pi}{2} + i \sin \frac{\pi}{2}$

b) $\cos \pi + i \sin \pi$

c) $\cos \pi - i \sin \pi$

d) $\cos \frac{\pi}{2} - i \sin \frac{\pi}{2}$

13. The equation having $4 - 3i$ and $4 + 3i$ as roots is

a) $x^2 + 8x + 25 = 0$

b) $x^2 + 8x - 25 = 0$

c) $x^2 - 8x + 25 = 0$

d) $x^2 - 8x - 25 = 0$

14. The value of $e^{i\theta} - e^{-i\theta}$ is

a) $\sin \theta$

b) $2 \sin \theta$

c) $i \sin \theta$

d) $2i \sin \theta$

15. The length of the latus rectum of the parabola $y^2 - 4x + 4y + 8 = 0$ is

a) 8

b) 6

c) 4

d) 2.

16. If $A = \begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix}$, then $(\text{adj } A) A =$

a) $\begin{bmatrix} \frac{1}{5} & 0 \\ 0 & \frac{1}{5} \end{bmatrix}$

b) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

c) $\begin{bmatrix} 5 & 0 \\ 0 & -5 \end{bmatrix}$

d) $\begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix}$

17. If A and B are any two matrices such that $AB = 0$ and A is non-singular, then

a) $B = 0$

b) B is singular

c) B is non-singular

d) $B = A$.

18. The system of equations $ax + y + z = 0$, $x + by + z = 0$, $x + y + cz = 0$ has a non-trivial solution. Then $\frac{1}{1-a} + \frac{1}{1-b} + \frac{1}{1-c} =$

a) 1

b) 2

c) -1

d) 0.

19. If $\rho(A) = \rho(A, B)$, then the system is

a) consistent and has infinitely many solutions

b) consistent and has a unique solution

c) consistent

d) inconsistent.

20. If \vec{a} is a non-zero vector and m is a non-zero scalar, then $m\vec{a}$ is a unit vector if
- $m = \pm 1$
 - $a = |m|$
 - $a = \frac{1}{|m|}$
 - $a = 1$.
21. Let p be 'Kamala is going to school' and q be 'there are twenty students in the class'. 'Kamala is not going to school or there are twenty students in the class' stands for
- $p \vee q$
 - $p \wedge q$
 - $\sim p$
 - $\sim p \vee q$.
22. If $f(x) = \frac{A}{\pi} \frac{1}{16+x^2}$, $-\infty < X < \infty$ is a p.d.f. of a continuous random variable X , then the value of A is
- 16
 - 8
 - 4
 - 1.
23. In a Poisson distribution if $P(X=0) = k$, then the variance is
- $\log \frac{1}{k}$
 - $\log k$
 - e^k
 - $\frac{1}{k}$.
24. In a binomial distribution if $n = 5$, $P(X=3) = 2P(X=2)$, then
- $p = 2q$
 - $2p = q$
 - $p = q$
 - $3p = 2q$.

25. If $f(x)$ is a *p.d.f.* of a normal distribution with mean μ then $\int_{-\infty}^{\infty} f(x) dx$ is
- a) 1
 - b) 0.5
 - c) 0
 - d) 0.25.
26. The area between the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ and its auxiliary circle is
- a) $\pi b(a-b)$
 - b) $2\pi a(a-b)$
 - c) $\pi a(a-b)$
 - d) $2\pi b(a-b)$.
27. The volume of the solid that results when the region enclosed by $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is revolved about the minor axis ($a > b > 0$) is
- a) $\frac{4}{8} \pi ab^2$
 - b) $\frac{4}{3} \pi a^2 b$
 - c) $\frac{4}{3} \pi ab^2$
 - d) $\frac{3}{4} \pi a^2 b$.
28. The surface area of the solid of revolution of the region bounded by $y = 2x$, $x = 0$ and $x = 2$ about x -axis is
- a) $8\sqrt{5} \pi$
 - b) $2\sqrt{5} \pi$
 - c) $\sqrt{5} \pi$
 - d) $4\sqrt{5} \pi$.

29. Integrating factor of $\frac{dy}{dx} + \frac{1}{x \log x} y = \frac{2}{x^2}$ is

a) e^x

b) $\log x$

c) $\frac{1}{x}$

d) e^{-x}

30. In finding the differential equation corresponding to $y = e^{mx}$ where m is the arbitrary constant, then m is

a) $\frac{y}{y'}$

b) $\frac{y'}{y}$

c) y'

d) y

31. The eccentricity of the conic $9x^2 + 5y^2 - 54x - 40y + 116 = 0$ is

a) $\frac{1}{3}$

b) $\frac{2}{3}$

c) $\frac{4}{9}$

d) $\frac{2}{\sqrt{5}}$

32. The coordinates of the vertices of the rectangular hyperbola $xy = 16$ are

a) $(4, 4); (-4, -4)$

b) $(2, 8); (-2, -8)$

c) $(4, 0); (-4, 0)$

d) $(8, 0); (-8, 0)$

33. The difference between the focal distances of any point on the hyperbola

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \text{ is } 24 \text{ and the eccentricity is } 2. \text{ Then the equation of the}$$

hyperbola is

a) $\frac{x^2}{144} - \frac{y^2}{432} = 1$

b) $\frac{x^2}{432} - \frac{y^2}{144} = 1$

c) $\frac{x^2}{12} - \frac{y^2}{12\sqrt{3}} = 1$

d) $\frac{x^2}{12\sqrt{3}} - \frac{y^2}{12} = 1$

34. The slope of the tangent to the curve $y = 3x^2 + 3 \sin x$ at $x = 0$ is

a) 3

b) 2

c) 1

d) -1

35. If $S = t^3 - 4t^2 + 7$, the velocity, when the acceleration is zero, is

a) $\frac{32}{3}$ m/sec

b) $-\frac{16}{3}$ m/sec

c) $\frac{16}{3}$ m/sec

d) $-\frac{32}{3}$ m/sec

36. If $\vec{u} = \vec{a} \times (\vec{b} \times \vec{c}) + \vec{b} \times (\vec{c} \times \vec{a}) + \vec{c} \times (\vec{a} \times \vec{b})$, then

a) \vec{u} is a unit vector

b) $\vec{u} = \vec{a} + \vec{b} + \vec{c}$

c) $\vec{u} = \vec{0}$

d) $\vec{u} \neq \vec{0}$

37. If $|\vec{a} + \vec{b}| = |\vec{a} - \vec{b}|$ then

- a) \vec{a} is parallel to \vec{b}
- b) \vec{a} is perpendicular to \vec{b}
- c) $|\vec{a}| = |\vec{b}|$
- d) \vec{a} and \vec{b} are unit vectors.

38. The non-parametric vector equation of a plane passing through three points, whose position vectors are $\vec{a}, \vec{b}, \vec{c}$ is

- a) $[\vec{r} - \vec{a}, \vec{b} - \vec{a}, \vec{c} - \vec{a}] = 0$
- b) $[\vec{r}, \vec{a}, \vec{b}] = 0$
- c) $[\vec{r}, \vec{b}, \vec{c}] = 0$
- d) $[\vec{a}, \vec{b}, \vec{c}] = 0$.

39. The shortest distance of the point $(2, 10, 1)$ from the plane

$$\vec{r} \cdot (3\vec{i} - \vec{j} + 4\vec{k}) = 2\sqrt{26} \text{ is}$$

- a) $2\sqrt{26}$
- b) $\sqrt{26}$
- c) 2
- d) $\frac{1}{\sqrt{26}}$

40. The point of intersection of the line $\vec{r} = (\vec{i} - \vec{k}) + t(3\vec{i} + 2\vec{j} + 7\vec{k})$ and the plane $\vec{r} \cdot (\vec{i} + \vec{j} - \vec{k}) = 8$ is

- a) $(8, 6, 22)$
- b) $(-8, -6, -22)$
- c) $(4, 3, 11)$
- d) $(-4, -3, -11)$.

SECTION - B

N. B. : i) Answer any ten questions.

ii) Question No. 55 is compulsory and choose any nine questions from the remaining.

iii) Each question carries six marks.

10 × 6 = 60

41. Find the adjoint of the matrix $A = \begin{bmatrix} 1 & 2 \\ 3 & -5 \end{bmatrix}$ and verify the result

$$A (\text{adj } A) = (\text{adj } A) A = |A| \cdot I.$$

42. Find the rank of the matrix $\begin{bmatrix} 3 & 1 & -5 & -1 \\ 1 & -2 & 1 & -5 \\ 1 & 5 & -7 & 2 \end{bmatrix}$.

43. a) Show that the vectors $2\vec{i} - \vec{j} + \vec{k}$, $\vec{i} - 3\vec{j} - 5\vec{k}$ and $-3\vec{i} + 4\vec{j} + 4\vec{k}$ form the sides of a right angled triangle.

b) A force given by $3\vec{i} + 2\vec{j} - 4\vec{k}$ is applied at the point $(1, -1, 2)$. Find the moment of the force about the point $(2, -1, 3)$.

44. Find the meeting point of the line $\vec{r} = (2\vec{i} + \vec{j} - 3\vec{k}) + t(2\vec{i} - \vec{j} - \vec{k})$ and the plane $x - 2y + 3z + 7 = 0$.

45. If $x = \cos \alpha + i \sin \alpha$ and $y = \cos \beta + i \sin \beta$, prove that

$$x^m y^n + \frac{1}{x^m y^n} = 2 \cos (m\alpha + n\beta).$$

46. Find the square root of $(-7 + 24i)$.

47. Find the equation of the hyperbola if centre is $(0, 0)$, length of semi-transverse axis is 6, $e = 3$ and the transverse axis is parallel to y -axis.

48. Evaluate $\lim_{x \rightarrow 0} \left(\operatorname{cosec} x - \frac{1}{x} \right)$.

49. If $u = \log (\tan x + \tan y + \tan z)$, prove that $\sum \sin 2x \frac{\partial u}{\partial x} = 2$.

50. Evaluate $\int_0^3 \frac{\sqrt{x} dx}{\sqrt{x} + \sqrt{3-x}}$ using properties of integration.

51. Solve : $xdy = (y + 4x^5 e^{x^4}) dx$.

52. G is a group, $a, b \in G$. Prove that $(a * b)^{-1} = b^{-1} * a^{-1}$.

53. Show that $p \leftrightarrow q = ((\sim p) \vee q) \wedge ((\sim q) \vee p)$.

54. Find c , μ and σ of the normal distribution whose probability distribution function is given by $f(x) = ke^{-2x^2 - 4x + 2}$.

55. a) Verify Lagrange's law of mean for the following function :

$$f(x) = x^3 - 5x^2 - 3x \quad [1, 3].$$

OR

b) A discrete random variable X has the following probability distribution :

x	0	1	2	3	4	5	6	7	8
$P(x)$	a	$3a$	$5a$	$7a$	$9a$	$11a$	$13a$	$15a$	$17a$

i) Find the value of a

ii) Find $P(X < 3)$

iii) Find $P(3 < x < 7)$.

SECTION - C

N. B. : i) Answer any ten questions.

ii) Question No. 70 is compulsory and choose any nine questions from the remaining.

iii) Each question carries ten marks. $10 \times 10 = 100$

56. Discuss the solutions of the system of equations for all values of λ .

$$x + y + z = 2, \quad 2x + y - 2z = 2, \quad \lambda x + y + 4z = 2.$$

57. Find the vector and Cartesian equations of the plane passing through the points $(-1, 1, 1)$ and $(1, -1, 1)$ and perpendicular to the plane $x + 2y + 2z = 5$.

58. If α and β are the roots of the equation $x^2 - 2px + (p^2 + q^2) = 0$ and

$$\tan \theta = \frac{q}{y+p}, \text{ show that } \frac{(y+\alpha)^n - (y+\beta)^n}{\alpha - \beta} = q^{n-1} \frac{\sin n\theta}{\sin^n \theta}.$$

59. Find the axis, vertex, focus, equation of directrix and latus rectum, length of the latus rectum for the following parabola and hence sketch the graph.

$$y^2 + 8x - 6y + 1 = 0.$$

60. The ceiling in a hallway 20 ft wide is in the shape of a semi-ellipse and 18 ft high at the centre. Find the height of the ceiling 4 ft from either wall if the height of the side walls is 12 ft.

61. Find the equation of the rectangular hyperbola which has for one of its asymptotes the line $x + 2y - 5 = 0$ and passes through the points $(5, 0)$ and $(-3, 0)$.

62. Find the angle between the curves $y = x^2$ and $y = (x - 2)^2$ at the point of intersection.

63. Find the intervals of concavity and the points of inflexion of the function

$$y = 12x^2 - 2x^3 - x^4.$$

64. Using Euler's theorem, prove that

$$x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = \frac{1}{2} \tan u \text{ if } u = \sin^{-1} \left(\frac{x - y}{\sqrt{x} + \sqrt{y}} \right).$$

65. Find the common area enclosed by the parabolas $y^2 = x$ and $x^2 = y$.

66. Find the length of the curve $x = a(t - \sin t)$, $y = a(1 - \cos t)$ between $t = 0$ and $t = \pi$.

67. Solve: $(1 - x^2) \frac{dy}{dx} + 2xy = x\sqrt{(1 - x^2)}$.

68. Show that the set $\{[1], [3], [4], [5], [9]\}$ forms an Abelian group under multiplication modulo 11.

69. An urn contains 4 white and 3 red balls. Find the probability distribution of the number of red balls in three draws one by one from the urn (i) with replacement, (ii) without replacement.

70. a) Solve : $(D^2 - 1)y = \cos 2x - 2 \sin 2x$

OR

- b) If $\vec{a} = 2\vec{i} + 3\vec{j} - \vec{k}$, $\vec{b} = -2\vec{i} + 5\vec{k}$, $\vec{c} = \vec{j} - 3\vec{k}$, verify that

$$\vec{a} \times (\vec{b} \times \vec{c}) = (\vec{a} \cdot \vec{c})\vec{b} - (\vec{a} \cdot \vec{b})\vec{c}.$$
