A

1521

Register Number

Part III — MATHEMATICS

(English Version)

Time Allowed : 3 Hours] [Maximum Marks : 200

SECTION – A

N. B. : i) *All* questions are compulsory.

ii) Each question carries *one* mark.

iii) Choose the most suitable answer from the given four alternatives. $40 \times 1 = 40$

1. The area of the parallelogram having a diagonal vector $3\vec{i} + \vec{j} - \vec{k}$ and a side vector $\vec{i} - 3\vec{j} + 4\vec{k}$ is

a) $10\sqrt{3}$ b) $6\sqrt{30}$

c) $\frac{3}{2}\sqrt{30}$ d) $3\sqrt{30}$.

2. If $\vec{a}, \vec{b}, \vec{c}$ are a right handed triad of mutually perpendicular vectors of magnitude *a*, *b*, *c* then the value of $\left[\vec{a}\ \vec{b}\ \vec{c}\right]$ is

a) $a^2 b^2 c^2$ b) 0

c) $\frac{abc}{2}$ d) *abc*.

[Turn over

3. $\vec{r} = s\vec{i} + t\vec{j}$ is the equation of

a) a straight line joining the points $\vec{i}$ and $\vec{j}$

b) xoy plane

c) yoz plane

d) zox plane.

4. The angle between the vectors $\vec{i} - \vec{j}$ and $\vec{j} - \vec{k}$ is

a) $\frac{\pi}{3}$

b) $-\frac{2\pi}{3}$

c) $-\frac{\pi}{3}$

d) $\frac{2\pi}{3}$.

5. The length of the perpendicular from the origin to the plane $\vec{r}.\left(3\vec{i} + 4\vec{j} + 12\vec{k}\right) = 26$ is

a) 2

b) $\frac{1}{2}$

c) 26

d) $\frac{26}{169}$.

6. The angle between the two tangents drawn from the point (– 4, 4) to $y^2 = 16x$ is

a) 45°

b) 30°

c) 60°

d) 90°.

7. The radius of the director circle of the conic $9x^2 + 16y^2 = 144$ is

a) $\sqrt{7}$

b) 4

c) 3

d) 5.

8. If the normal to the rectangular hyperbola $xy = c^2$ at t_1 meets the curve again at t_2 then $t_1^3 t_2 =$

a) 1 b) 0

c) − 1 d) − 2.

9. The slope of the tangent to the curve $y = 3x^2 + 3 \sin x$ at $x = 0$ is

a) 3 b) 2

c) 1 d) − 1.

10. For what values of x the rate of increase of $x^3 - 2x^2 + 3x + 8$ is twice the rate of increase of x ?

a) $\left(-\frac{1}{3}, -3\right)$

b) $\left(\frac{1}{3}, 3\right)$

c) $\left(-\frac{1}{3}, 3\right)$

d) $\left(\frac{1}{3}, 1\right)$.

11. The volume generated when the region bounded by $y = x$, $y = 1$, $x = 0$ is rotated about y-axis is

a) $\frac{\pi}{4}$ b) $\frac{\pi}{2}$

c) $\frac{\pi}{3}$ d) $\frac{2\pi}{3}$.

12. The length of arc of the curve $x^{\frac{2}{3}} + y^{\frac{2}{3}} = 4$ is

a) 48 b) 24

c) 12 d) 96.

13. $\int_0^{\infty} x^5 e^{-4x} \, dx =$

a) $\frac{\underline{|6}}{4^6}$ b) $\frac{\underline{|6}}{4^5}$

c) $\frac{\underline{|5}}{4^6}$ d) $\frac{\underline{|5}}{4^5}$.

14. The order and degree of $\left(\frac{dx}{dy}\right)^2 + 5y^{\frac{1}{3}} = x$ are

a) 2, 1 b) 1, 2

c) 1, 6 d) 1, 3.

15. The integrating factor of $\frac{dy}{dx} - y \tan x = \cos x$ is

a) $\sec x$

b) $\cos x$

c) $e^{\tan x}$

d) $\cot x$.

16. In the group $(G, \bullet)$, $G = \{1, -1, i, -i\}$, order of -1 is

a) -1 b) 1

c) 2 d) 0.

17. If $f(x) = \begin{cases} kx^2 & , \quad 0 < x < 3 \\ 0 & , \quad \text{elsewhere} \end{cases}$

is a probability density function, then the value of k is

a) $\frac{1}{3}$ b) $\frac{1}{6}$

c) $\frac{1}{9}$ d) $\frac{1}{12}$.

18. $Var(4X + 3)$ is

a) 7 b) 16 $Var(X)$

c) 19 d) 0.

19. If a random variable X follows Poisson distribution such that $E\left(X^2\right) = 30$, then the variance of the distribution is

a) 6 b) 5

c) 30 d) 25.

20. Which of the following is / are correct regarding normal distribution curve ?

I. Symmetrical about the line $X = \mu$ (mean)

II. Unimodal

III. Mean = Median = Mode

IV. Point of inflexion is at $X = \mu \pm \sigma$.

a) I, II only b) II, IV only

c) I, II, III only d) all of these.

21. If A is a scalar matrix with scalar $k \neq 0$ of order 3, then A^{-1} is

a) $\frac{1}{k^2} I$

b) $\frac{1}{k^3} I$

c) $\frac{1}{k} I$

d) kI.

22. If $ae^x + be^y = c$, $pe^x + qe^y = d$ and $\Delta_1 = \begin{vmatrix} a & b \\ p & q \end{vmatrix}$, $\Delta_2 = \begin{vmatrix} c & b \\ d & q \end{vmatrix}$,

$\Delta_3 = \begin{vmatrix} a & c \\ p & d \end{vmatrix}$, then the value of (x, y) is

a) $\left(\frac{\Delta_2}{\Delta_1}, \frac{\Delta_3}{\Delta_1} \right)$

b) $\left(\log \frac{\Delta_2}{\Delta_1}, \log \frac{\Delta_3}{\Delta_1} \right)$

c) $\left(\log \frac{\Delta_1}{\Delta_3}, \log \frac{\Delta_1}{\Delta_2} \right)$

d) $\left(\log \frac{\Delta_1}{\Delta_2}, \log \frac{\Delta_1}{\Delta_3} \right)$.

23. If the rank of $\begin{bmatrix} \lambda & -1 & 0 \\ 0 & \lambda & -1 \\ -1 & 0 & \lambda \end{bmatrix}$ is 2, then λ is

a) 1

b) 2

c) 3

d) any real number.

24. Every homogeneous system (linear)

a) is always consistent

b) has only trivial solution

c) has infinitely many solutions

d) need not be consistent.

25. If $\vec{u} = \vec{a} \times \left(\vec{b} \times \vec{c} \right) + \vec{b} \times \left(\vec{c} \times \vec{a} \right) + \vec{c} \times \left(\vec{a} \times \vec{b} \right)$, then

a) $\vec{u}$ is a unit vector

b) $\vec{u} = \vec{0}$

c) $\vec{u} \neq \vec{0}$

d) $\vec{u} = \vec{a} + \vec{b} + \vec{c}$.

26. The modulus and amplitude of $\left(e^{3 - \frac{i\pi}{4}} \right)^3$ are

a) $e^9, \frac{\pi}{2}$

b) $e^9, -\frac{\pi}{2}$

c) $e^6, -\frac{3\pi}{4}$

d) $e^9, -\frac{3\pi}{4}$

27. If $a = 3 + i$, $z = 2 - 3i$ then the points on the Argand diagram representing az, $3az$ and $-az$ are

a) vertices of a right angled triangle

b) vertices of an equilateral triangle

c) vertices of an isosceles triangle

d) collinear.

28. If $Z_n = \cos \frac{n\pi}{3} + i \sin \frac{n\pi}{3}$, then $Z_1 . Z_2 ... Z_6$ is

a) 1 b) -1

c) i d) $-i$.

29. If $| Z - Z_1 | = | Z - Z_2 |$ then the locus of Z is

a) a circle with centre at the origin

b) a circle with centre at Z_1

c) a straight line passing through the origin

d) a perpendicular bisector of the line joining Z_1 and Z_2.

30. $16x^2 - 3y^2 - 32x - 12y - 44 = 0$ represents

a) an ellipse

b) a circle

c) a parabola

d) a hyperbola.

31. If $f(a) = 2$, $f'(a) = 1$, $g(a) = -1$, $g'(a) = 2$, then the value of

$\lim_{x \to a} \frac{g(x) f(a) - g(a) f(x)}{x - a}$ is

a) 5 b) -5

c) 3 d) -3.

32. $x = x_0$ is a root of even order for the equation $f'(x) = 0$ then $x = x_0$ is a/an

a) maximum point

b) minimum point

c) inflexion point

d) critical point.

33. If $u = y \sin x$ then $\frac{\partial^2 u}{\partial x \partial y}$ is

a) $\cos x$
b) $\cos y$

c) $\sin x$
d) 0.

34. An asymptote to the curve $y^2(a + 2x) = x^2(3a - x)$ is

a) $x = 3a$

b) $x = -\frac{a}{2}$

c) $x = \frac{a}{2}$

d) $x = 0$.

35. The value of $\int_0^\pi \sin^2 x \cos^3 x \, dx$ is

a) π
b) $\frac{\pi}{2}$

c) $\frac{\pi}{4}$
d) 0.

36. The particular integral of $\left(D^2 - 4D + 4\right) y = e^{2x}$ is

a) $\frac{x^2}{2} e^{2x}$

b) xe^{2x}

c) xe^{-2x}

d) $\frac{x}{2} e^{-2x}$.

37. In finding the differential equation corresponding to $y = e^{mx}$ where m is the arbitrary constant, m is

a) $\frac{y}{y'}$

b) $\frac{y'}{y}$

c) y'

d) y.

38. The number of rows in the truth table of $\sim [\, p \wedge (\sim q)\,]$ is

a) 2

b) 4

c) 6

d) 8.

39. Which of the following is not a binary operation on R ?

a) $a * b = ab$

b) $a * b = a - b$

c) $a * b = \sqrt{ab}$

d) $a * b = \sqrt{a^2 + b^2}$.

40. The order of [7] in $\left(Z_9, +_9\right)$ is

a) 9

b) 6

c) 3

d) 1.

SECTION - B

N. B. : i) Answer any *ten* questions.

ii) Question No. **55** is compulsory and choose any *nine* questions from the remaining.

iii) Each question carries *six* marks. $10 \times 6 = 60$

41. Solve by matrix inversion method :

$$x + y = 3, \quad 2x + 3y = 8.$$

42. Find the rank of $\begin{bmatrix} 3 & 1 & 2 & 0 \\ 1 & 0 & -1 & 0 \\ 2 & 1 & 3 & 0 \end{bmatrix}$.

43. Prove that $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$ by vector method.

44. a) The work done by the force $\vec{F} = a\vec{i} + \vec{j} + \vec{k}$ in moving the point of application from (1, 1, 1) to (2, 2, 2) along a straight line is given to be 5 units. Find the value of a.

b) If $\vec{x} \cdot \vec{a} = 0$, $\vec{x} \cdot \vec{b} = 0$, $\vec{x} \cdot \vec{c} = 0$ and $\vec{x} \neq \vec{0}$, show that $\vec{a}, \vec{b}, \vec{c}$ are coplanar.

45. For any two complex numbers Z_1 and Z_2, prove that

a) $| Z_1 . Z_2 | = | Z_1 | \, | Z_2 |$

b) $arg (Z_1 . Z_2) = arg \, Z_1 + arg \, Z_2$.

46. If $n \in N$, prove that $(1 + i\sqrt{3})^n + (1 - i\sqrt{3})^n = 2^{n+1} \cos \dfrac{n\pi}{3}$.

47. Find the equation of the hyperbola whose directrix is $2x + y - 1 = 0$, focus (1, 2) and eccentricity $\sqrt{3}$.

48. Prove that the curves $2x^2 + 4y^2 = 1$ and $6x^2 - 12y^2 = 1$, cut each other at right angles.

49. Find the volume of the solid that results when the region enclosed by the curve $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is revolved about major axis ($a > b > 0$).

50. a) Form the differential equation for the equation $y = e^{2x} (A + Bx)$.

b) Solve : $\left(D^2 + D + 1 \right) y = 0$.

51. Use the truth table to establish that the following statement is tautology or contradiction :

$(p \wedge (\sim p)) \wedge ((\sim q) \wedge p)$.

52. Find the order of all the elements of the group $\left(Z_6, +_6 \right)$.

53. The probability distribution of a random variable X is given below :

X :	0	1	2	3
$P(X = x)$:	0·1	0·3	0·5	0·1

If $Y = X^2 + 2X$, find the mean and variance of Y.

54. Let X be a binomially distributed variable with mean 2 and standard deviation $\frac{2}{\sqrt{3}}$. Find the corresponding probability function.

55. a) Find the equation of the tangent and normal to the curve $y = x^3$ at the point (1, 1).

OR

b) If $u = \log (\tan x + \tan y + \tan z)$, prove that $\sum \sin 2x \frac{\partial u}{\partial x} = 2$.

SECTION – C

N. B. : i) Answer any *ten* questions.

ii) Question No. **70** is compulsory and choose any *nine* questions from the remaining.

iii) Each question carries *ten* marks. $10 \times 10 = 100$

56. Solve by Cramer's rule :

$$\frac{1}{x} + \frac{2}{y} - \frac{1}{z} = 1, \quad \frac{2}{x} + \frac{4}{y} + \frac{1}{z} = 5, \quad \frac{3}{x} - \frac{2}{y} - \frac{2}{z} = 0.$$

57. Prove by vector method that altitudes of a triangle are concurrent.

58. Find the vector and cartesian equation of the plane passing through the points (1, 2, 3), (2, 3, 1) and perpendicular to the plane $3x - 2y + 4z - 5 = 0$.

59. Solve : $x^4 - x^3 + x^2 - x + 1 = 0$.

60. Find the axis, vertex, focus, directrix, equation of the latus rectum and the length of the latus rectum of the parabola $y^2 - 8x + 6y + 9 = 0$.

61. Find the equation of the hyperbola if its asymptotes are parallel to $x + 2y - 12 = 0$ and $x - 2y + 8 = 0$. (2, 4) is the centre of the hyperbola and it passes through the point (2, 0).

62. A water tank has the shape of an inverted circular cone with base radius 2 metres and height 4 metres. If water is being pumped into the tank at a rate of 2 m^3/min, find the rate at which the water level is rising when the water is 3 m deep.

63. Obtain the Maclaurin's series expansion for $\tan x, \; -\frac{\pi}{2} < x < \frac{\pi}{2}$.

64. Trace the curve $y = x^3$.

65. Find the common area enclosed by the parabolas $y^2 = x$ and $x^2 = y$.

66. Find the length of the curve $\left(\frac{x}{a}\right)^{\frac{2}{3}} + \left(\frac{y}{a}\right)^{\frac{2}{3}} = 1$.

67. The sum of Rs. 1,000 is compounded continuously, the nominal rate of interest being 4% per annum. In how many years will the amount be twice the original principal ?

$[\log_e 2 = 0{\cdot}6931]$.

68. Let G be the set of all rational numbers except 1 and $*$ be defined on G by $a * b = a + b - ab$ for all $a, b \in G$. Show that $(G, *)$ is an infinite Abelian group.

69. If X is normally distributed with mean 6 and standard deviation 5, find

i) $P(0 \le X \le 8)$

ii) $P(|X - 6| < 10)$.

[Given that $P(0 < z < 1\cdot2) = 0\cdot3849$, $P(0 < z < 0\cdot4) = 0\cdot1554$,

$P(0 < z < 1) = 0\cdot3413$, $P(0 < z < 2) = 0\cdot4772$].

70. a) A satellite is travelling around the earth in an elliptical orbit having the earth at a focus and of eccentricity $\frac{1}{2}$. The shortest distance that the satellite gets to the earth is 400 kms. Find the longest distance that the satellite gets from the earth.

OR

b) Solve : $\left(D^2 - 6D + 9\right) y = x + e^{2x}$.

69. (i) X is normally distributed with mean 6 and standard deviation 5, find

a) $P(0 \le X \le 8)$

b) $P(|X - 6| < 10)$.

[Given that $P(0 < z < 1 \cdot 2) = 0 \cdot 3849$, $P(0 < z < 0 \cdot 4) = 0 \cdot 1554$,

$P(0 < z < 1) = 0 \cdot 3413$, $P(0 < z < 2) = 0 \cdot 4772$].

70. a) A satellite is travelling around the earth in an elliptical orbit having the earth at a focus and of eccentricity $\frac{1}{2}$. The shortest distance that the satellite gets to the earth is 400 kms. Find the longest distance that the satellite gets from the earth.

OR

b) Solve : $(D^2 - 6D + 9)\, y = x + e^{2x}$.