

**Standard - XII**

**Sample Question Paper - I**

**MATHEMATICS**

**(Science Group)**

## **Weightage to score of content**

### **MATHEMATICS (Science Group)**

**Std: XII**

| Sl. No. | CO's     | Unit                             | No. of Questions | Score     | %          |
|---------|----------|----------------------------------|------------------|-----------|------------|
| 1.      | 1 - 2    | Matrices and Determinants        | 2                | 8         | 10         |
| 2.      | 3 - 5    | Boolean Algebra                  | 1                | 5         | 6          |
| 3.      | 6 - 7    | Probability                      | 1                | 4         | 5          |
| 4.      | 8 - 13   | Functions, limits and continuity | 1                | 3         | 4          |
| 5.      | 14 - 17  | Differentiation                  | 1                | 10        | 13         |
| 6.      | 18 - 23  | Application of differentiation   | 1                | 7         | 9          |
| 7.      | 24 - 28  | Indefinite Integral              | 2                | 9         | 11         |
| 8.      | 29 - 31* | Definite Integral                | 1                | 7         | 9          |
| 9.      | 32*      | Differential Equation            | 1                | 5         | 6          |
| 10.     | 33 - 35  | Vector I                         | 1                | 4         | 5          |
| 11.     | 36       | Vector II                        | 1                | 7         | 9          |
| 12.     | 37       | 3D I                             | 1                | 5         | 6          |
| 13.     | 38 - 41  | 3D II                            | 1                | 6         | 7          |
|         |          |                                  | <b>15</b>        | <b>80</b> | <b>100</b> |

\* Internal choices were given to this questions

**Weightage to type of questions****MATHAMATICS****Std: XII**

| Type of Questions | Scores    | %          |
|-------------------|-----------|------------|
| Objective         | 18        | 22.5       |
| Short answer      | 36        | 45         |
| Essay             | 26        | 32.5       |
| <b>Total</b>      | <b>80</b> | <b>100</b> |

**Weightage to level of questions****MATHEMATICS****Std: XII**

| Type of Questions | Scores    | %          |
|-------------------|-----------|------------|
| Essay             | 15        | 19         |
| Average           | 42        | 52         |
| Difficulty        | 23        | 29         |
| <b>Total</b>      | <b>80</b> | <b>100</b> |

# Blue Print

## MATHEMATICS - Paper I (Science Group)

**Std: XII**

| Sl.<br>No. | CO's    | Unit                             | Types of Questions |              |           | Total<br>Score |
|------------|---------|----------------------------------|--------------------|--------------|-----------|----------------|
|            |         |                                  | Objective          | Short answer | Essay     |                |
| 1.         | 1 - 2   | Matrices and Determinants        | 2                  | 2            | 4         | 8              |
| 2.         | 3 - 5   | Boolean Algebra                  | 1                  | 1            | 3         | 5              |
| 3.         | 6 - 7   | Probability                      | 2                  | 2            | -         | 4              |
| 4.         | 8 - 13  | Functions, limits and continuity | 3                  | -            | -         | 3              |
| 5.         | 14 - 17 | Differentiation                  | -                  | 8            | 2         | 10             |
| 6.         | 18 - 23 | Applications of differentiation  | 2                  | 2            | 3         | 7              |
| 7.         | 24 - 28 | Indefinite Integral              | 1                  | -            | 8         | 9              |
| 8.         | 29 - 31 | Definite Integral                | -                  | 5            | 2         | 7              |
| 9.         | 32      | Differential Equation            | 1                  | 2            | 2         | 5              |
| 10.        | 33 - 35 | Vector I                         | 2                  | 2            | -         | 4              |
| 11.        | 36      | Vector II                        | 1                  | 4            | 2         | 7              |
| 12.        | 37      | 3D I                             | 1                  | 4            | -         | 5              |
| 13.        | 38 - 41 | 3D II                            | 2                  | 4            | -         | 6              |
|            |         | Total                            | <b>18</b>          | <b>36</b>    | <b>26</b> | <b>80</b>      |

## Part III - MATHEMATICS

### (Science Group)

Std - XII

Maximum Score: 80

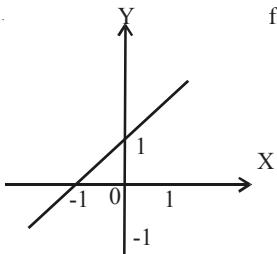
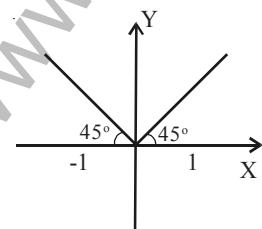
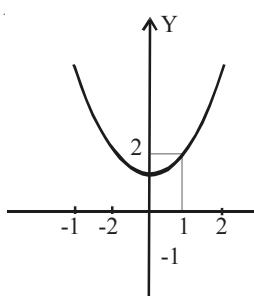
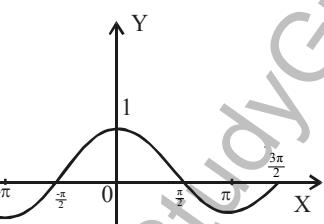
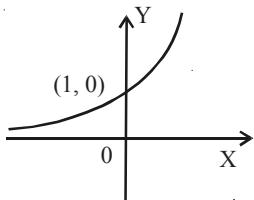
Time:  $2\frac{1}{2}$  hrs.

Cool off time: 15 mts.

#### **Instructions**

- Read the question carefully before answering.
- Maximum time allowed is 2 hours 45 minutes including cool off time.
- First 15 minutes is cool-off time, during which the candidate should neither write answers nor have discussion with others.
- All questions are compulsory and only internal choices are allowed.
- In the case of question having internal choice only the sub-questions of the same questions should be answered.
- Calculations, figures and graphs should be shown in the answer sheet itself.

1. (i) Find the graph of the function  $f(x) = x + 1$  from the graphs given below. [1]

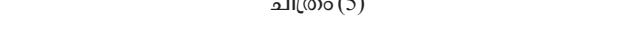
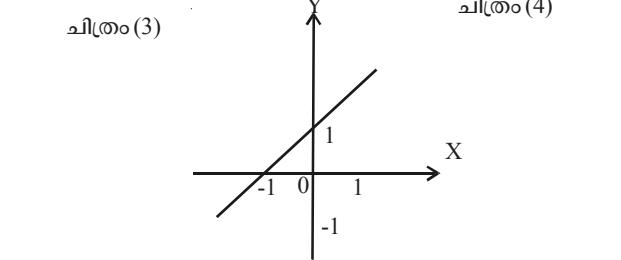
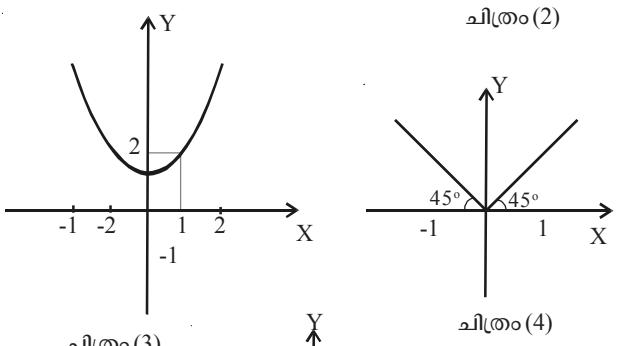
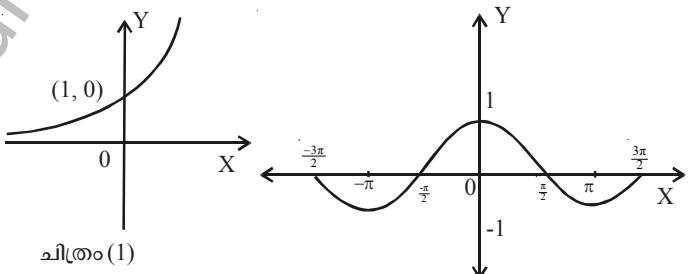


#### **നിർദ്ദേശങ്ങൾ:**

- \$ ഉത്തരം ഏഴുതി തുടങ്ങുന്നതിനു മുമ്പ് ശ്രദ്ധാപൂർവ്വം ചോദ്യങ്ങൾ വയിക്കുക.
- \$ ‘കുൾ ഓഫ്’ സമയം ഉൾപ്പെടെ ചോദ്യങ്ങൾക്ക് അനുവദിക്കപ്പെട്ട സമയം 2 മണിക്കൂറ് 45 മിനിറ്റാണ്.
- \$ അദ്യത്തെ 15 മിനിട്ട് ‘കുൾ ഓഫ്’ സമയമാണ്. ഈ സമയത്ത് മറ്റൊരുവരുമായി ചർച്ച ചെയ്യുകയോ ഉത്തരമെഴുതാനോ പാടില്ല.
- \$ ചോയ്സ് നമ്പ്കാരത്തെ ചോദ്യങ്ങൾക്കല്ലാം ഉത്തരം ഏഴു തേണ്ടതാണ്.
- \$ ചോയിസ്യൂളും ചോദ്യങ്ങൾക്ക് ഉത്തരമെഴുതുന്നോൾ ആ ചോദ്യത്തിൽ ഉപചോദ്യങ്ങൾക്ക് മാത്രം ഉത്തരം ഏഴുതുക.
- \$ ക്രിയകൾ ചിത്രങ്ങൾ ശാമ്പൂകൾ ഉത്തരക്കടലാസിൽ തന്നെ ഏഴുതേണ്ടതാണ്.

1. (i) ചുവവെട കൊടുത്തതിൽക്കുന്ന ശ്രാമ്പുകളിൽ നിന്ന്  $f(x) = x + 1$  എന്ന ഫലം മംഗൾപ്പര്ഗി ശ്രാമ്പ കണ്ണെത്തുക.

[1]



- (ii) Match the following using the graph given above. [2]

| A - functions    | B - Graph  |
|------------------|------------|
| $f(x) = \cos x$  | figure (1) |
| $f(x) =  x $     | figure (2) |
| $f(x) = e^x$     | figure (3) |
| $f(x) = x^2 + 1$ | figure (4) |
|                  | figure (5) |

- (iii) One of the above graphs represents a function which is not differentiable at  $x = 0$ . Identify that graph. Justify your answer. [2]

2. (i) Match the following

| A   | B                                    |
|---|--------------------------------------|
| $\frac{d}{dx}(\sin x)$                                | $-\cos x$                            |
| $\frac{d}{dx}(e^{\cos x})$                            | $\frac{2}{1+x^2}$                    |
| $\frac{d}{dx} \sin^{-1}\left(\frac{2x}{1+x^2}\right)$ | $-\sin x \cdot e^{\cos x}$           |
| $\frac{d}{dx} \sin^{-1}(x)$                           | $\cos x$<br>$\frac{1}{\sqrt{1-x^2}}$ |

- ii) Differentiate  $\sin x$  with respect to  $x$  using first principle. [2]

- iii) If  $y = \sin^{-1} x$ , prove that

$$(1 - x^2) y_2 - xy_1 = 0. \quad [2]$$

- (ii) മുകളിൽ പറയ്തിരിക്കുന്ന ശാമ്പുകൾ ഉപയോഗിച്ച് ചേരുപ്പടി ചേർക്കുക. [2]

| A - ഘണ്ടശമ്പൻ    | B - ശാമ്പ് |
|------------------|------------|
| $f(x) = \cos x$  | ചിത്രം (1) |
| $f(x) =  x $     | ചിത്രം (2) |
| $f(x) = e^x$     | ചിത്രം (3) |
| $f(x) = x^2 + 1$ | ചിത്രം (4) |
|                  | ചിത്രം (5) |

- (iii) മുകളിൽ കൊടുത്തിരിക്കുന്ന ഘണ്ടശമ്പന്കൾ ഒരു ശാമ്പുകളിൽ ഒന്ന്  $x = 0$  തെ ഡിഫീൻഷിഡ് അല്ല. ആ ശാമ്പ് എത്ര? എന്തു കൊണ്ട്? [2]

2. (i) ചേരുപ്പടി ചേർക്കുക.

| A   | B                                    |
|---|--------------------------------------|
| $\frac{d}{dx}(\sin x)$                                | $-\cos x$                            |
| $\frac{d}{dx}(e^{\cos x})$                            | $\frac{2}{1+x^2}$                    |
| $\frac{d}{dx} \sin^{-1}\left(\frac{2x}{1+x^2}\right)$ | $-\sin x \cdot e^{\cos x}$           |
| $\frac{d}{dx} \sin^{-1}(x)$                           | $\cos x$<br>$\frac{1}{\sqrt{1-x^2}}$ |

- ii) ഫലം പ്രിംസിപ്പിൾ ഉപയോഗിച്ച്  $\sin x$  എന്ന അസ്പദമാക്കി ഡിഫീൻഷ്യർ ചെയ്യുക. [2]

- iii)  $y = \sin^{-1} x$  ആയാൽ

$$(1 - x^2) y_2 - xy_1 = 0 \text{ എന്നു തെളിയിക്കുക.} \quad [2]$$

3. An open box of maximum volume is to be made from a square piece of tin sheet 24 cm on a side by cutting equal squares from the corners and turning of the sides?

i) Complete the following table

| Height of the box (x cm) | Width of the box  | Volume of the box ( $v$ cm $^3$ )    |
|--------------------------|-------------------|--------------------------------------|
| 1                        | $24 - 2 \times 1$ | $1 \times (24 - 2 \times 1)^2 = 484$ |
| 2                        | $24 - 2 \times 2$ | $2 \times (24 - 2 \times 2)^2 = 800$ |
| 3                        | -----             | -----                                |
| 4                        | -----             | -----                                |
| 5                        | -----             | -----                                |
| 6                        | -----             | -----                                |

[2]

- ii) Using the above table, express  $v$  as a function of  $x$  and determine its domain. [2]
- iii) Find height (x cm) of the box when volume (v) is maximum by using differentiation. [3]

4. i) Choose the correct answer from the bracket.

$$\int e^x dx = \dots \dots \dots$$

$(e^{2x} + c, e^x + c, e^{-x} + c, e^{-2x} + c)$  [1]

ii) Evaluate:  $\int e^x \sin x dx$  [3]

iii) Evaluate  $\int_0^2 e^x dx$  as the limit of sum. [2]

3. 24 സെ.മീ വശമുള്ള സമചതുരകൃതിയിലുള്ള ഒരു ടിന്റിറിൻരെ മുലകളിൽ നിന്ന് സമചതുരകൃതിയിൽ ഒരു ഭാഗം മുറിച്ചുമാറ്റി ശേഷം വശങ്ങൾ മുകളിലേക്ക് മടക്കി പരമാവധി വ്യാപ്തമുള്ള ഒരു തുറന്ന പെട്ടി നിർമ്മിച്ചിരിക്കുന്നു.

i) ചുവവെട കൊടുത്തിരിക്കുന്ന പട്ടിക പൂരിപ്പിക്കുക.

| പെട്ടിയുടെ ഉയരം (x cm) | പെട്ടിയുടെ വീതി   | പെട്ടിയുടെ വ്യാപ്തം (v cm $^3$ )     |
|------------------------|-------------------|--------------------------------------|
| 1                      | $24 - 2 \times 1$ | $1 \times (24 - 2 \times 1)^2 = 484$ |
| 2                      | $24 - 2 \times 2$ | $2 \times (24 - 2 \times 2)^2 = 800$ |
| 3                      | _____             | _____                                |
| 4                      | _____             | _____                                |
| 5                      | _____             | _____                                |
| 6                      | _____             | _____                                |

[2]

- ii) മുകളിൽ കൊടുത്തിരിക്കുന്ന പട്ടിക ഉപയോഗിച്ച് പെട്ടിയുടെ വ്യാപ്തം  $v$  യെ  $x$  ലെ ഉള്ള ഒരു ഫംഗ്ഷൻ ആയി എഴുതുക. [2]
- iii) പെട്ടിയുടെ വ്യാപ്തം പരമാവധി ആകുമ്പോൾ അതിന്റെ ഉയരം എത്രയെന്ന് ഡിഫറൻസിയേഷൻ ഉപയോഗിച്ച് കണ്ടെത്തുക. [3]

4. i) ബ്രാക്കറ്റിൽ നിന്ന് ശരിയായ ഉത്തരം തെരഞ്ഞെടുത്തെത്തഫുത്തുക.

$$\int e^x dx = \dots \dots \dots$$

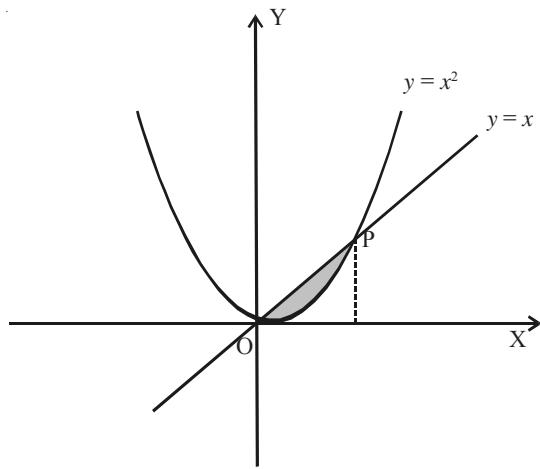
$(e^{2x} + c, e^x + c, e^{-x} + c, e^{-2x} + c)$  [1]

ii)  $\int e^x \sin x dx$  റെറ്റ് വില കാണുക. [3]

iii)  $\int_0^2 e^x dx$  റെറ്റ് വില തുകയുടെ ലിമിറ്റ് ഉപയോഗിച്ച് കാണുക. [2]

5. Direction: Answer any one 5 or 6.

Consider the following figure



- Find the point of intersection (P) of the parabola and the line. [2]
- Find the area of the shaded region. [3]

OR

- Evaluate  $\int_0^r \sqrt{r^2 - x^2} dx$ , where  $r$  is a fixed positive number. Hence prove the area of the circle of radius  $r$  is  $\pi r^2$ . [3]

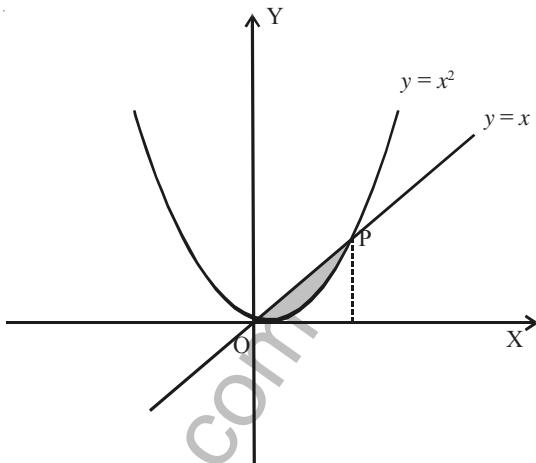
- Find the area of the circle:  $x^2 + y^2 = 16$ , which is exterior to parabola  $y^2 = 6x$ . [2]

- Let  $(B, +, .)$  be a Boolean algebra. State the following statements are true or false. Justify your answer.

- $x + y = y + x$  [1]
- $x + 1 = 1$ , where 1 is the unit element in  $(B, +, .)$  [1]
- $(x + y) + (x' \cdot y') = 1$ , where  $x'$  and  $y'$  are complements of  $x$  and  $y$  respectively. [3]

5. നിർദ്ദേശം: ചോദ്യം 5, 6 ഇവയിൽ ഒന്നിനു മാത്രം ഉത്തരം എഴുതുക.

ചുവടെ കൊടുത്തിരിക്കുന്ന ചിത്രം പരിഗണിക്കുക.



- പരബോളയും റേഖയും സംഗമിക്കുന്ന വിന്ദു വിന്ദു “കോ-ഓർഡിനേറ്റ്സ്” കാണുക? [2]
- ഷൈലിയും ചെയ്തിരിക്കുന്ന ഭാഗത്തിന്റെ വിസ്തീർണ്ണം കാണുക. [3]

അലേപ്പകിൽ

- $r$  ഒരു സ്ഥിരമായ പോസിറ്റീവ് സംവ്യൂദ്ധായാൽ  $\int_0^r \sqrt{r^2 - x^2} dx$  കാണുക? ഇതിനെ അടിസ്ഥാനമാക്കി ആരം  $r$  യൂണിറ്റായ ഒരു വൃത്തത്തിന്റെ വിസ്തീർണ്ണം  $\pi r^2$  ആയി രിക്കും എന്ന് തെളിയിക്കുക. [3]
- $y^2 = 6x$  എന്ന പരബോളയുടെ പുറത്ത്  $x^2 + y^2 = 16$  എന്ന വൃത്തഭാഗത്തിന്റെ വിസ്തീർണ്ണം എത്ര ആയിരിക്കും? [2]

- $(B, +, .)$  ഒരു ബൂളിയൻ ആർബിബൈ ആധാർ ചുവടെ കൊടുത്തിരിക്കുന്നവ ശരിയോ തെറ്റോ എന്നു പറയുക? എന്തുകൊണ്ട്?

- $x + y = y + x$  [1]
- $x + 1 = 1$ ,  $(B, +, .)$  ലെ യൂണിറ്റ് എല്ലാമാർഗ്ഗം 1 ആണ്. [1]
- $x', y'$  ഇവ തമാക്രമം  $x, y$  ഇവയുടെ കോൺജിമെറ്റ് ആധാർ  $(x + y) + (x' \cdot y') = 1$ . [3]

8. Consider a random experiment - two dice are thrown simultaneously.
- Write the sample space of the experiment.  
[1]
  - What is the probability of 'getting a sum 12' from the above experiment.  
[1]
  - Write any two events which are mutually exclusive and exhaustive from the above experiment.  
[2]
9. Given that  $A + B = \begin{bmatrix} 2 & 5 \\ 7 & 8 \end{bmatrix}$  and  $A - B = \begin{bmatrix} 6 & 8 \\ 4 & 3 \end{bmatrix}$ .
- Find  $2A$   
[1]
  - Find  $A^2 - B^2$ . Is it equal to  $(A + B)(A - B)$ . Given reason.  
[2]
10. Consider a square matrix of order 3  $A = [a_{ij}]$ , where  $a_{ij} = 2_i + j$ .
- $a_{21} = \dots$   
[1]
  - Construct A and thus find  $A + AT$ .  
[2]
  - Express A as sum of a symmetric and a skew-symmetric matrices.  
[2]
11. Consider the point  $(-1, -2, -3)$ .
- In which octant, the above point lies.  
[1]
  - Find the direction cosines of the line joining  $(-1, -2, -3)$  and  $(3, 4, 5)$ .  
[2]
  - If P is any point such that  $OP = \sqrt{50}$  and direction cosine of OP are  $\frac{3}{\sqrt{50}}, \frac{4}{\sqrt{50}}$  and  $\frac{5}{\sqrt{50}}$ , then find the co-ordinate of P.  
[2]
8. രണ്ടു 'ഡിസ്' കൾ ഓൺച്ച് എറിയുന്നു എന്ന 'രാഖിയം എക്സ്‌പിരിമെന്റ്' പരിഗണിക്കുക.
- 'രാഖിയം എക്സ്‌പിരിമെന്റ്' സാമ്പിൾ സ്വേച്ഛ ആത്?  
[1]
  - ഈ 'രാഖിയം എക്സ്‌പിരിമെന്റ്' തുക 12 വരുന്നതിനുള്ള 'പ്രോബബിലിറ്റ്' എത്ര?  
[1]
  - മുകളിൽ കൊടുത്തിരിക്കുന്ന ചോദ്യം ഉപയോഗിച്ച് 'മൃച്ഛലി എക്സ്‌സ്കൂസി' വും 'എക്സോസ്റ്റോവൈം' ആയ രണ്ടു ഇവർഗ്ഗകൾ എഴുതുക.  
[2]
9.  $A + B = \begin{bmatrix} 2 & 5 \\ 7 & 8 \end{bmatrix}, A - B = \begin{bmatrix} 6 & 8 \\ 4 & 3 \end{bmatrix}$  ആയാൽ
- $2A$  കാണുക.  
[1]
  - $A^2 - B^2$  എത്ര? ഈത്  $(A + B)(A - B)$  കുതുല്യമാണോ? കാരണം എന്ത്?  
[2]
10.  $a_{ij} = 2_i + j$  ആകത്തക്കവിധം  $A = [a_{ij}]$  എന്ന ഓർഡർ 3 ആയ മെട്രിക്സ് പരിഗണിക്കുക.
- $a_{21} = \dots$   
[1]
  - മെട്രിക്സ് A നിർമ്മിച്ച്  $A + AT$  കാണുക.  
[2]
  - A എന്ന മെട്രിക്സിനെ ഒരു സിമിട്രിക് മെട്രിക്സിന്റെയും ഒരു സ്ക്യൂസിമിട്രിക്സ് മെട്രിക്സിന്റെയും തുകയായി എഴുതുക.  
[2]
11.  $(-1, -2, -3)$  എന്ന ബിന്ദു പരിഗണിക്കുക.
- മുകളിൽ പറഞ്ഞിരിക്കുന്ന ബിന്ദു ഏത് ഒക്റ്ററ്റിൽ സ്ഥിതി ചെയ്യുന്നു.  
[1]
  - $(-1, -2, -3), (3, 4, 5)$  എന്നീ ബിന്ദുകൾ യോജിപ്പിച്ചുണ്ടാകുന്ന രേഖയുടെ 'ധയരക്ഷണ കോൺസൻസ്' കാണുക.  
[2]
  - $OP = \sqrt{50}$  ആകത്തക്കവിധമുള്ള ഒരു ബിന്ദു വാൻ P. OP യുടെ 'ധയരക്ഷണ രേഖയാണ്'  $\frac{3}{\sqrt{50}}, \frac{4}{\sqrt{50}}, \frac{5}{\sqrt{50}}$ , എന്നിവ ആയാൽ P എന്ന ബിന്ദുവിന്റെ കോ-ഓർഡിനേറ്റേംസ് കാണുക.  
[2]

**Direction:** Answer 12 or 13.

12. Consider the following spheres

$$S_1 = x^2 + y^2 + z^2 - 2x - 2y - 6z + 7 = 0$$

$$S_2 = x^2 + y^2 + z^2 + 2x - 2z - 7 = 0$$

- i) Find the centre and radius of circles  $S_1$  and  $S_2$ . [2]

- ii) Find the distance between the centres of  $S_1$  and  $S_2$ .

If  $C_1$  and  $C_2$  are centres of  $S_1$  and  $S_2$  and  $A$  is

the point which  $S_1$  cuts the line  $C_1 C_2$ . Find the co-ordinate of  $A$ . [2]

- iii) Find the equation of the smallest sphere which contains spheres  $S_1$  and  $S_2$ . [2]

13. Consider the sphere

$$S : x^2 + y^2 + z^2 - 2x - 4y + 2z - 3 = 0.$$

- i) Find ccentre and radius of  $S$ . [1]

- ii) Find the perpendicular distance from the centre of the sphere  $S$  to the plane  $2x - 2y + z + 12 = 0$ .

Show that this plane touches the sphere  $S$ .

[2]

- iii) Find the point of contact of sphere  $S$  and the plane. [3]

14. Consider  $\bar{a} = i + 2\bar{j} - 3\bar{k}$

$$\bar{b} = 3i - \bar{j} + 2\bar{k}, \bar{c} = 11i + \bar{j}$$

- i) Find  $\bar{a} + \bar{b}$  and  $\bar{a} \cdot \bar{b}$ . [1]

- ii) Find the unit vector in the direction of  $\bar{a} + \bar{b}$ . [1]

- iii) Show that  $\bar{a} + \bar{b}$  and  $\bar{a} - \bar{b}$  are orthogonal. [2]

- iv) Find the value of  $\lambda$  and  $\mu$  such that

$$\bar{c} = \lambda \bar{a} + \mu \bar{b}. [2]$$

**നിർദ്ദേശം:** ചോദ്യം 12, 13 റൂവയിൽ നന്നിനു മാത്രം ഉത്തരം എഴുതുക.

താഴെ കൊടുത്തിരിക്കുന്ന ഗോളങ്ങൾ പരിഗണിക്കുക.

$$S_1 = x^2 + y^2 + z^2 - 2x - 2y - 6z + 7 = 0$$

$$S_2 = x^2 + y^2 + z^2 + 2x - 2z - 7 = 0$$

- i)  $S_1, S_2$  എന്നീ ഗോളങ്ങളുടെ കേന്ദ്രം, ആരം എന്നിവ കാണുക. [2]

- ii)  $S_1, S_2$  എന്നീ ഗോളങ്ങളുടെ കേന്ദ്രങ്ങൾ തമ്മിലുള്ള അകലം കാണുക?

$$S_1, S_2 \text{ എന്നീ }$$

ഗോളങ്ങളുടെ കേന്ദ്രങ്ങൾ തമ്മിലുള്ള അകലം  $C_1, C_2$  എന്നിവ

യാമാക്കം  $S_1, S_2$  എന്നീ ഗോളങ്ങളുടെ കേന്ദ്രങ്ങൾ  $A$  എന്ന ബിന്ദു  $C_1, C_2$  എന്ന രേഖയെ  $S_1$  വണിക്കുന്ന ബിന്ദുവുമായാൽ  $A$  എന്ന ബിന്ദുവിന്റെ കോ-ഓർഡിനേറ്റ് കാണുക. [2]

- iii)  $S_1, S_2$  എന്നീ ഗോളങ്ങളെ ഉൾക്കൊള്ളാൻ കഴിയുന്ന ഏറ്റവും ചെറിയ ഗോളത്തിന്റെ സമാക്ക്യം എഴുതുക. [2]

13.  $S : x^2 + y^2 + z^2 - 2x - 4y + 2z - 3 = 0.$

എന്ന ഗോളം പരിഗണിക്കുക.

- i)  $S$  ലൈൻ കേന്ദ്രവും ആരവും കാണുക. [1]

- ii) ഗോളത്തിന്റെ കേന്ദ്രവും  $2x - 2y + z + 12 = 0$  എന്ന ‘പ്ലാനിനു’ തമ്മിലുള്ള ലംബ അകലം എത്ര? ഈ പ്ലാനിൽ ഗോളവുമായി സ്പർശിക്കുന്നവെന്ന് തെളിയിക്കുക. [2]

- iii) പ്ലാനിൽ ഗോളവുമായി സ്പർശിക്കുന്ന സ്പർശബിന്ദു കാണുക. [3]

4.  $\bar{a} = i + 2\bar{j} - 3\bar{k}, \bar{b} = 3i - \bar{j} + 2\bar{k},$

$\bar{c} = 11i + \bar{j}$  എന്നീ വെക്ടറുകൾ പരിഗണിക്കുക.

- i)  $\bar{a} + \bar{b}, \bar{a} \cdot \bar{b}$  റൂവ കാണുക. [1]

- ii)  $\bar{a} + \bar{b}$  യൂടെ ദിശയിലുള്ള ആണിറ്റ് വെക്ടർ കാണുക. [1]

- iii)  $\bar{a} + \bar{b}, \bar{a} - \bar{b}$  റൂവ ഓർത്തേശണൽ എന്നു തെളിയിക്കുക. [2]

- iv)  $\bar{c} = \lambda \bar{a} + \mu \bar{b}$  ആയാൽ  $\lambda, \mu$  റൂവയുടെ വിലകൾ കാണുക.. [2]

15. Let  $\bar{a} = 2\bar{i} + 3\bar{j} - 5\bar{k}$   
 $\bar{b} = 6\bar{i} - 4\bar{j} + 2\bar{k}$  and  
 $\bar{c} = 8\bar{i} + 2\bar{j} + 3\bar{k}$

Consider the product  $(\bar{a} \cdot \bar{b}) \times \bar{c}$ ,  
 $\bar{a} \cdot (\bar{b} \times \bar{c})$  and  $\bar{a} \times (\bar{b} \times \bar{c})$

- i) Out of the above three products, which is not possible to find out. [1]
- ii) Find the volume of the parallelopiped whose co-terminal edges are  $\bar{a}$ ,  $\bar{b}$  and  $\bar{c}$ . [2]
- iii) Show that

$$\bar{a} \times (\bar{b} \times \bar{c}) = (\bar{a} \cdot \bar{c}) \bar{b} - (\bar{a} \cdot \bar{b}) \bar{c}$$

by using above vectors. [2]

**Direction:** Answer any one of 16 or 17.

16. A horizontal beam of length 21 m carrying a uniform load of  $w \text{kg/m}$  of length, is freely supported at the both ends satisfying the differential equation.  $E.I = \frac{d^2y}{dx^2} - \frac{1}{2}wx^2 - wlx$ ,  $y$  being the deflection at the distance  $x$  from one end.

- i) What is the order and degree of the above differential equation. [1]

ii) Find  $\frac{dy}{dx}$ . [2]

- iii) If  $y = 0, x = 0$  and  $\frac{dy}{dx} = 0$  at  $x = 1$ . Find the deflection at any point. [2]

15.  $\bar{a} = 2\bar{i} + 3\bar{j} - 5\bar{k}$   
 $\bar{b} = 6\bar{i} - 4\bar{j} + 2\bar{k}$   
 $\bar{c} = 8\bar{i} + 2\bar{j} + 3\bar{k}$  എന്നീ വെക്ടറുകൾ പരിഗണിക്കുക.

കുടാതെ  $(\bar{a} \cdot \bar{b}) \times \bar{c}$ ,  $\bar{a} \cdot (\bar{b} \times \bar{c})$ ,  $\bar{a} \times (\bar{b} \times \bar{c})$  എന്നീ പ്രോഡക്ടുകളും പരിഗണിക്കുക.

- i) മുകളിൽ പറഞ്ഞ പ്രോഡക്ടുകളിൽ കണ്ണ താൻ കഴിയാത്തതെ. [1]
- ii)  $\bar{a}, \bar{b}, \bar{c}$  ഇവ ഒരു “പാരലലോപിപ്പിഡ്” എന്നു മുലയിലുള്ള വശങ്ങളായാൽ വ്യാപ്തമായതെ? [2]
- iii)  $\bar{a} \times (\bar{b} \times \bar{c}) = (\bar{a} \cdot \bar{c}) \bar{b} - (\bar{a} \cdot \bar{b}) \bar{c}$  എന്ന് മുകളിൽ പറഞ്ഞ വെക്ടറുകളുടെ സഹായത്താൽ തെളിയിക്കുക. [2]

**നിർദ്ദേശം:** ചോദ്യം 16, 17 ഇവയിൽ ഒന്നിനു മാത്രം ഉത്തരം എഴുതുക.

16. 21 മീറ്റർ നീളമുള്ളതും തിരശ്ചീനവുമായ ഒരു ബീം  $w \text{kg/m}$  ലോഡു താങ്ങുന്നതും ഇതിന്റെ ഒരഗത്തിൽ നിന്നും  $x$  യൂണിറ്റ് അകലെയുള്ള ബിന്ദുവിലെ വളവ്  $y$  യും ആയാൽ ഇത്  $E.I = \frac{d^2y}{dx^2} = \frac{1}{2}wx^2 - wlx$  എന്ന ഡിഫറൻഷ്യൽ ഇക്കോഷ്ട് അനുസ്യതമാണ്.

- i) മുകളിൽ പറഞ്ഞ ഡിഫറൻഷ്യൽ ഇക്കോഷ്ട് ഓർഡർ, ഡിഗ്രി എന്നിവ കാണുക. [1]

ii)  $\frac{dy}{dx}$  കാണുക. [2]

- iii) If  $y = 0$  എന്നും  $\frac{dy}{dx} = 0$  എന്നും  $x = 1$  എന്നും  $\frac{dy}{dx} = 0$  എന്നും ആയാൽ ഏതൊരു ബിന്ദുവിന്റെയും ഡിഫറൻഷ്യൽ കാണുക. [2]

17. Consider the differential equation.

$$(x^2 - 1) \frac{dy}{dx} + 2(x+2)y = 2(x+1)$$

- i) Find  $\frac{dy}{dx}$  degree and order of the differential equation. [2]
- ii) Find the integrating factor of the above differential equaiton. [1]
- iii) Solve the differential equation. [2]

18. Find  $\int \sqrt{\tan x} dx$ .

[5]

17.  $(x^2 - 1) \frac{dy}{dx} + 2(x+2)y = 2(x+1)$  എന്ന ഡിഫീരിൻഷ്യൽ റൂക്കോഷൻ പരിഗണിക്കുക.

- i)  $\frac{dy}{dx}$ , ഡിഗ്രി, ഓർഡർ എന്നിവ കാണുക. [2]
- ii) മുകളിൽ പറയ്ത ഡിഫീരിൻഷ്യൽ റൂക്കോഷൻ ഇൻഗ്രേറ്റീറ് ഫാക്ടർ കാണുക. [1]
- iii) മുകളിൽ പറയ്ത ഡിഫീരിൻഷ്യൽ റൂക്കോഷൻ ഇൻഗ്രേറ്റീറ് മൂല്യം കാണുക. [2]

18.  $\int \sqrt{\tan x} dx$  കണ്ടുപിടിക്കുക.

[5]

## Questionwise Analysis

| Q.     | CO No. | M.P            | Content                        | Type  | Level | Score | Time in Min |
|--------|--------|----------------|--------------------------------|-------|-------|-------|-------------|
| 1.i)   | 8      | 4, 5           | Limit & Continuity             | O     | E     | 1     | 2           |
| ii)    | 8      | 4, 5, 7        | Limit & Continuity             | O     | A     | 2     | 4           |
| iii)   | 14     | 2, 4, 7, 10    | Differentiation                | SA    | D     | 2     | 4           |
| 2. i)  | 14, 16 | 2, 5, 7        | Differentiation                | SA    | A     | 4     | 7           |
| ii)    | 14     | 3, 5, 7        | Differentiation                | SA    | A     | 2     | 4           |
| iii)   | 17     | 2, 3, 5, 7, 10 | Differentiation                | Essay | D     | 2     | 4           |
| 3. i)  | 20     | 2, 5           | Application of differentiation | O     | A     | 2     | 4           |
| ii)    | 20     | 2, 5, 7        | Application of differentiation | SA    | A     | 2     | 4           |
| iii)   | 20     | 2, 5, 7, 10    | Application of differentiation | Essay | D     | 3     | 5           |
| 4. i)  | 24, 25 | 2, 5           | Indefinite integral            | O     | E     | 1     | 2           |
| ii)    | 24, 25 | 2, 5, 7        | Indefinite integral            | Essay | A     | 3     | 5           |
| iii)   | 24, 25 | 2, 5, 7, 10    | Definite integral              | Essay | D     | 2     | 4           |
| 5. i)  | 31     | 2, 5, 7        | Definite integral              | SA    | A     | 2     | 4           |
| ii)    | 31     | 2, 5, 7, 10    | Definite integral              | SA    | A     | 3     | 5           |
| 6* i)  | 31     | 2, 5, 7, 10    | Definite integral              | Essay | A     | 3     | 5           |
| ii)    | 31     | 2, 5, 7, 10    | Definite integral              | Essay | D     | 2     | 4           |
| 7. i)  | 3      | 2, 5           | Boolean Algebra                | O     | E     | 1     | 2           |
| ii)    | 3      | 2, 5, 7        | Boolean Algebra                | SA    | A     | 1     | 2           |
| iii)   | 3      | 2, 5, 7, 10    | Boolean Algebra                | Essay | D     | 3     | 5           |
| 8. i)  | 6      | 2, 5           | Probability                    | O     | E     | 1     | 2           |
| ii)    | 6      | 2, 5, 7        | Probability                    | O     | A     | 1     | 2           |
| iii)   | 6      | 2, 5, 7        | Probability                    | SA    | A     | 2     | 4           |
| 9. i)  | 1      | 2, 5           | Matrices and determinants      | O     | E     | 1     | 2           |
| ii)    | 1      | 2, 5, 7        | Matrices and determinants      | Essay | A     | 2     | 4           |
| 10. i) | 1      | 2              | Matrices and determinants      | O     | E     | 1     | 2           |
| ii)    | 1      | 2, 5           | Matrices and determinants      | SA    | A     | 2     | 4           |
| iii)   | 1      | 2, 5, 7        | Matrices and determinants      | Essay | A     | 2     | 4           |
| 11. i) | 37     | 2, 5           | 3D - I                         | O     | E     | 1     | 2           |
| ii)    | 37     | 2, 5           | 3D - I                         | SA    | A     | 2     | 4           |
| iii)   | 37     | 2, 5, 7        | 3D - I                         | SA    | A     | 2     | 4           |

| Q.      | CO No. | M.P            | Content               | Type  | Level | Score | Time in Min |
|---------|--------|----------------|-----------------------|-------|-------|-------|-------------|
| 12.i)   | 41     | 2, 5           | 3D - II               | O     | E     | 2     | 3           |
| ii)     | 41     | 2, 5, 7        | 3D - II               | SA    | D     | 2     | 4           |
| iii)    | 41     | 2, 5, 7        | 3D - II               | SA    | D     | 2     | 4           |
| 13* i)  | 41     | 2, 5, 6        | 3D - II               | O     | E     | 1     | 2           |
| ii)     | 41     | 2, 5, 6, 7     | 3D - II               | SA    | A     | 2     | 4           |
| iii)    | 41     | 2, 5, 6, 7, 10 | 3D - II               | Essay | D     | 3     | 5           |
| 14. i)  | 34     | 2, 5           | Vector I              | O     | E     | 1     | 2           |
| ii)     | 33     | 2, 5           | Vector I              | P     | E     | 1     | 2           |
| iii)    | 36     | 2, 5, 6        | Vector II             | SA    | A     | 2     | 4           |
| iv)     | 35     | 2, 5, 6, 7     | Vector I              | SA    | A     | 2     | 4           |
| 15. i)  | 36     | 2, 5, 6        | Vector II             | O     | E     | 1     | 2           |
| ii)     | 36     | 2, 5, 6, 7     | Vector II             | SA    | A     | 2     | 4           |
| iii)    | 36     | 2, 5, 6, 7, 10 | Vector II             | Essay | D     | 2     | 4           |
| 16. i)  | 32     | 2, 5           | Differential equaiton | O     | E     | 1     | 1           |
| ii)     | 32     | 2, 5, 6, 7     | Differential equaiton | SA    | A     | 2     | 4           |
| iii)    | 32     | 2, 5, 6, 7, 10 | Differential equaiton | Essay | D     | 2     | 4           |
| 17*. i) | 32     | 2, 5           | Differential equaiton | SA    | E     | 2     | 4           |
| ii)     | 32     | 2, 5, 6        | Differential equaiton | SA    | A     | 1     | 2           |
| iii)    | 32     | 2, 5, 6, 7, 10 | Differential equaiton | Essay | D     | 2     | 4           |
| 18.     | 25     | 2, 5, 6, 7, 10 | Indefinite integral   | Essay | D     | 5     | 8           |

O - Objective, SA - Short Answer, A - Average, D - Difficult, E - Easy

**Sample Question Paper**  
**MATHEMATICS**  
**(Science Group)**  
**Scoring Key**

**Std - XII**

**Total Score: 80**

Time :  $2 \frac{1}{2}$  hours

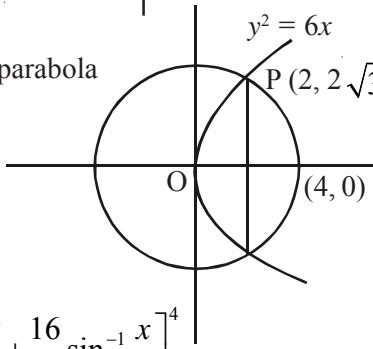
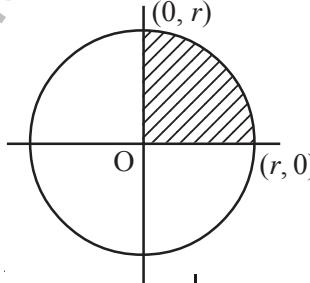
Cool off Time: 15 mts.

| Qn.No. | Scoring indicators  | Score       |       |
|--------|---|-------------|-------|
|        |   | Stage score | Total |
| 1. i)  | Figure 5  | 1           |       |
| ii)    | $f(x) = \cos x$ - figure 2  | 1           |       |
|        | $f(x) =  x $ - figure 4   |             |       |
|        | $f(x) = e^x$ - figure 1   |             |       |
|        | $f(x) = x^2 + 2$ - figure 3   | 1           |       |
| iii)   | Figure 4.<br>In the figure, there is a sharp edge at $x = 0$ . So it is not differentiable at $x = 0$ .   | 1           | 5     |
| 2. i)  | $\frac{d}{dx} (\sin x) \rightarrow \cos x.$   | 1           |       |
|        | $\frac{d}{dx} (e^{\cos x}) \rightarrow -\sin x e^{\cos x}$  | 1           |       |
|        | $\frac{d}{dx} \sin^{-1} \left( \frac{2x}{1+x^2} \right) \rightarrow \frac{2x}{1+x^2}$   | 1           |       |
|        | $\frac{d}{dx} \sin^{-1}(x) \rightarrow \frac{1}{\sqrt{1-x^2}}$  | 1           |       |
| ii)    | $\begin{aligned} f'(x) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \\ &= \lim_{h \rightarrow 0} \frac{\sin(x+h) - \sin(x)}{h} \\ &= \lim_{h \rightarrow 0} \frac{2 \cos(x + \frac{h}{2}) - \sin(\frac{h}{2})}{2} \\ &= \cos x. \end{aligned}$ | 1           |       |
| iii)   | $\begin{aligned} y_1 &= \frac{1}{\sqrt{1-x^2}} \\ \sqrt{1-x^2} y_1 &= 1 \\ \text{Differentiate again with respect to } x. \\ (1-x^2) y_2 - xy_1 &= 0 \end{aligned}$   | 1           | 8     |

|    |  |                  |   |
|----|--|------------------|---|
|    |  |                  |   |
| 3. | i) $24 - 2 \times 3 = 3 \times 18^2 = 972$<br>$24 - 2 \times 4 = 4 \times 16^2 = 1024$<br>$24 - 2 \times 5 = 5 \times 14^2 = 980$<br>$24 - 2 \times 6 = 6 \times 12^2 = 864$   | 1                |   |
|    | ii) $V = x(24 - 2x)^2$<br>Domain $0 < x < 12$  | 1                |   |
|    | iii) $\frac{dv}{dx} = -4x(24 - 2x) + (24 - 2x)^2 = 0$<br>$\Rightarrow x = 12, 4$<br>$\frac{d^2v}{dx^2} = -4x \times -2 + -4(24 - 2x) + -2(24 - 2x)$<br>$\Rightarrow x = 4, \frac{d^2v}{dx^2} < 0.$   | 1<br>1<br>1      |   |
|    | Volume is maximum when $x = 4$ cm.   | 1                | 7 |
| 4. | i) $e^x + c$   | 1                |   |
|    | ii) $I = \int e^x \sin x dx$<br>$= \sin x \cdot e^x - \int \cos x \cdot e^x dx$<br>$= \sin x \cdot e^x - \left( \cos x e^x - \int -\sin x e^x dx \right)$<br>$2I = e^x (\sin x - \cos x)$<br>$I = \frac{1}{2} e^x (\sin x - \cos x)$   | 1<br>1<br>1<br>1 |   |
|    | iii) $\int_0^2 e^x dx = 2 \lim_{n \rightarrow \infty} \frac{1}{n} \left[ e^0 + e^{\frac{2}{n}} + e^{\frac{4}{n}} + \dots + e^{\frac{2(n-2)}{n}} \right]$<br>$= 2 \lim_{n \rightarrow \infty} \frac{1}{n} \left[ \frac{e^{\frac{2n}{n}} - 1}{e^{\frac{2}{n}} - 1} \right]$<br>$= \frac{2(e^2 - 1)}{2}$<br>$= e^2 - 1$ | 1<br>1<br>1      | 6 |
|    |  |                  |   |

|   |                     |
|---|---------------------|
| <p>5. i)</p> $x = x^2$ $x^2 - x = 0$ $x(x - 1) = 0$ $x = 0, 1$ <p>When <math>x = 0, y = 0</math></p> <p>When <math>x = 1, y = 1</math></p> <p>Points of intersection <math>(0, 0)</math> and <math>(1, 1)</math></p>  | 1<br>1              |
| <p>ii)</p> $\int_0^1 x \, dx - \int_0^1 x^2 \, dx = \left( \frac{x^2}{2} \right)_0^1 - \left( \frac{x^3}{3} \right)_0^1$ $= \frac{1}{2} - \frac{1}{3} = \frac{1}{6}$ <p>sq.units.</p>   | 1 + 1<br>1 5        |
| <p>6. i)</p> $\int_0^r (r^2 - x^2) \, dx = \left[ \frac{x}{2} \sqrt{r^2 - x^2} + \frac{r^2}{2} \sin^{-1} \frac{x}{r} \right]_0^r$ $= \frac{r^2}{2} \sin^{-1} 1$ $= \frac{\pi r^2}{4}$ $y = \sqrt{r^2 - x^2}$ <p>Area = <math>4 \int_0^r y \, dx</math></p> $= 4 \int_0^r \sqrt{r^2 - x^2} \, dx$ $= 4 \cdot \frac{\pi r^2}{4} = \pi r^2$  | 1<br>1<br>1         |
| <p>ii)</p> <p>Area = Area of the circle - Interior area of the parabola</p> $= 16\pi - 2 \int_0^2 y \, dx - 2 \int_2^4 y \, dx$ $= 16\pi - 2 \int_0^2 \sqrt{6} x^{1/2} \, dx - 2 \int_2^4 \sqrt{16-x^2} \, dx$ $= 16\pi - 2\sqrt{6} \left[ \frac{2}{3} x^{3/2} \right]_0^2 - 2 \left[ \frac{x}{2} \sqrt{16-x^2} + \frac{16}{2} \sin^{-1} \frac{x}{4} \right]_2^4$ $= 16\pi - \frac{16}{\sqrt{3}} - 16 \cdot \frac{\pi}{2} + 4\sqrt{3} + \frac{8\pi}{3}$ $= \frac{-4}{\sqrt{3}} + \frac{32\pi}{3}$ $= \frac{4}{3} (8\pi - \sqrt{3})$ | 1+1*<br>1<br>1<br>1 |

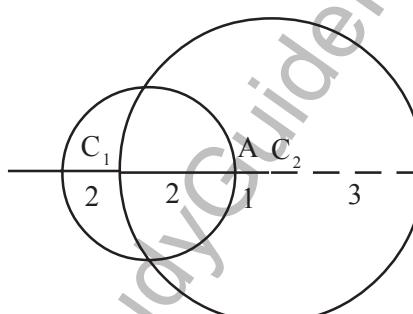
\*Picture



|    |   |   |   |
|----|---|---|---|
|    |   |   |   |
| 7. | i) True, operation + is commutative in Boolean Algebra.<br>ii) True<br>$\begin{aligned} 1 &= x + x' \\ &= x + (x' \cdot 1) \\ &= (x + x') \cdot (x + 1) \\ &= 1 \cdot (x + 1) \\ &= (x + 1) \cdot 1 = x + 1 \end{aligned}$ iii) True<br>$\begin{aligned} (x + y) + (x' \cdot y') &= (y + x) + (x' \cdot y') \\ &= y + [x + (x' \cdot y')] \\ &= y + [(x + x') \cdot (x' + y')] \\ &= y + [1 \cdot (x + y')] \\ &= y + [(x + y') \cdot 1] \\ &= y + (x + y') \\ &= (y + x) + y' \\ &= (x + y) + y' \\ &= x + (y + y') \end{aligned}$ | 1<br>1<br>1<br>1<br>1<br>1<br>1<br>1<br>1 |   |
| 8. | i) $S = \{(1, 1), (1, 2) \dots (1, 6)$<br>$(2, 1), (2, 2) \dots (2, 6)$<br>$(3, 1), (2, 2) \dots (3, 6)$<br>$(4, 1) \dots (4, 6)$<br>$(5, 1) \dots (5, 6)$<br>$(6, 1) \dots (6, 6)\}$<br><br>ii) $\frac{1}{36}$<br>iii) A = Getting of a number whose sum is less than 6.<br>B = Getting a number whose sum is greater than or equal to 6.<br>(Hint: Two events whose union is sample space and their intersection is null set)   | 1<br>1<br>1<br>1<br>1<br>1<br>1<br>1<br>1 | 5 |
| 9. | i) $2A = \begin{bmatrix} 8 & 13 \\ 11 & 11 \end{bmatrix}$   | 1   |   |

\*Picture

|     |   |             |   |
|-----|---|-------------|---|
|     | <p>ii) <math>A = \begin{bmatrix} 4 &amp; \frac{13}{2} \\ \frac{1}{2} &amp; \frac{1}{2} \end{bmatrix}</math>, <math>B = \begin{bmatrix} -2 &amp; \frac{-3}{2} \\ \frac{3}{2} &amp; \frac{5}{2} \end{bmatrix}</math></p> $A^2 - B^2 = \begin{bmatrix} \frac{207}{4} & \frac{247}{4} \\ \frac{209}{4} & \frac{264}{4} \end{bmatrix} - \begin{bmatrix} \frac{7}{4} & \frac{-9}{2} \\ \frac{3}{4} & 4 \end{bmatrix}$ $= \begin{bmatrix} \frac{200}{4} & \frac{265}{2} \\ \frac{206}{4} & \frac{248}{4} \end{bmatrix}$ $(A + B)(A - B) = \begin{bmatrix} 32 & 31 \\ 74 & 80 \end{bmatrix}$ $A^2 - B^2 \neq (A + B)(A - B)$ <p>This is because <math>AB \neq BA</math>.</p>                      | 1           | 3 |
| 10. | <p>i) <math>a_{21} = 5</math></p> <p>ii) <math>A = \begin{bmatrix} 3 &amp; 4 &amp; 5 \\ 5 &amp; 6 &amp; 7 \\ 7 &amp; 8 &amp; 9 \end{bmatrix}</math></p> $A + A^T = \begin{bmatrix} 3 & 4 & 5 \\ 5 & 6 & 7 \\ 7 & 8 & 9 \end{bmatrix} + \begin{bmatrix} 3 & 5 & 7 \\ 4 & 6 & 8 \\ 5 & 7 & 9 \end{bmatrix}$ $= \begin{bmatrix} 6 & 9 & 12 \\ 9 & 12 & 15 \\ 12 & 15 & 18 \end{bmatrix}$ <p>iii) <math>\begin{bmatrix} 3 &amp; \frac{1}{2} &amp; 6 \\ \frac{1}{2} &amp; 6 &amp; \frac{15}{2} \\ 6 &amp; \frac{15}{2} &amp; 9 \end{bmatrix} + \begin{bmatrix} 0 &amp; \frac{-1}{2} &amp; -1 \\ \frac{1}{2} &amp; 0 &amp; \frac{-1}{2} \\ 1 &amp; \frac{1}{2} &amp; 0 \end{bmatrix}</math></p> | 1           | 5 |
| 11  | <p>i) 7<sup>th</sup> octant</p> <p>ii) Direction ratio 2, 6, 8</p> <p>Direction cosines are <math>\frac{1}{\sqrt{26}}, \frac{3}{\sqrt{26}}, \frac{4}{\sqrt{26}}</math></p>  | 1<br>1<br>1 |   |

|     |   |             |   |
|-----|---|-------------|---|
|     | <p>iii) Let P <math>(x, y, z)</math> be the point<br/>         Direction ratios of OP = <math>x, y, z</math></p> <p>Direction cosines of OP are <math>\frac{x}{\sqrt{x^2 + y^2 + z^2}}, \frac{y}{\sqrt{x^2 + y^2 + z^2}}, \frac{z}{\sqrt{x^2 + y^2 + z^2}}</math></p> $= \frac{3}{\sqrt{50}}, \frac{4}{\sqrt{50}}, \frac{5}{\sqrt{50}}$ $\Rightarrow x = 3, y = 4, z = 5$ <p>Since <math>\sqrt{x^2 + y^2 + z^2} = \sqrt{50}</math></p> <p>Therefore the point is <math>(3, 4, 5)</math></p> | 1           | 5 |
| 12. | <p>i) <math>S_1</math>; centre <math>(1, 1, 3)</math>; Radius = 2<br/> <math>S_2</math>; centre <math>(-1, 0, 1)</math>; Radius = 3</p> <p>ii) <math>C_1 C_2 = \sqrt{4+1+4} = 3</math><br/>         Centre of the smallest sphere which contains the <math>S_1</math> and <math>S_2</math> is A. (From fig.)</p>  | 1<br>1<br>1 |   |
|     |   | 1           |   |
|     | <p>A divides <math>C_1 C_2</math> in the ratio <math>2 : 1</math></p> <p>Co-ordinate of A = <math>\left(\frac{-1}{3}, \frac{1}{3}, \frac{5}{3}\right)</math></p>  | 1           |   |
|     | <p>iii) Radius 4 units.<br/>         Equation of the sphere is</p> $\left(x + \frac{1}{3}\right)^2 + \left(y - \frac{1}{3}\right)^2 + \left(z - \frac{5}{3}\right)^2 = 0$   | 1           | 6 |

|     |   |                                    |
|-----|---|------------------------------------|
| 13* | i) Centre $(1, 2, -1)$ , Radius 3<br><br>ii) Perpendicular distance = $\frac{2 \times 1 - 2 \times 2 - 1 + 12}{\sqrt{4 + 4 + 1}} = 3$<br>Since radius and perpendicular distance are equal, the plane touches the sphere.<br>iii) Equation to the line from the centre $(1, 2, -1)$ perpendicular to plane is<br>$\frac{x-1}{2} = \frac{y-2}{-2} = \frac{z+1}{1} = k.$<br>Any point on the line is $(2k+1, -2k+2, k-1)$<br>Point of contact is $(-1, 4, -2)$  | 1<br>1<br>1<br>1<br>1<br>1<br>1    |
| 14. | i) $\bar{a} + \bar{b} = 4\mathbf{i} + \mathbf{j} - \mathbf{k}$<br>$\bar{a} \cdot \bar{b} = -5$<br><br>ii) $\bar{a} + \bar{b} = \frac{4\mathbf{i} + \mathbf{j} - \mathbf{k}}{3\sqrt{2}}$<br><br>iii) $\bar{a} - \bar{b} = -2\mathbf{j} + 3\mathbf{j} - 5\mathbf{k}$<br>$(\bar{a} + \bar{b}) \cdot (\bar{a} - \bar{b}) = -8 + 3 + 5$<br>$= 0$<br>$\bar{a} + \bar{b}$ and $\bar{a} - \bar{b}$ are orthogonal.<br>iv) $\lambda = 2, \mu = 3.$   | 1<br>1<br>1<br>1<br>1<br>4         |
| 15. | i) $(\bar{a} \cdot \bar{b}) \times \bar{c}$<br><br>ii) $\bar{a} \cdot (\bar{b} \times \bar{c}) = \begin{bmatrix} 2 & 3 & -5 \\ 6 & -4 & 2 \\ 8 & 2 & 3 \end{bmatrix} = 258$ cub units.<br><br>iii) $\bar{b} \times \bar{c} = \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 6 & -4 & 2 \\ 8 & 2 & 3 \end{bmatrix} = 16\mathbf{i} - 2\mathbf{j} - 44\mathbf{k}$<br><br>$\bar{a} \times (\bar{b} \times \bar{c}) = \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2 & 3 & -5 \\ -16 & -2 & -44 \end{bmatrix} = 122\mathbf{i} - 2\mathbf{j} - 44\mathbf{k}$<br><br>$(\bar{a} \cdot \bar{c})\bar{b} - (\bar{a} \cdot \bar{b})\bar{c} = 122\mathbf{i} - 2\mathbf{j} - 44\mathbf{k}$ .<br>Thus the results follows. | 1<br>1 + 1<br>1 + 1<br>1<br>1<br>7 |

|  |   |
|--|---|
| <p>16 i) order 2, degree 1</p> <p>ii) <math>\frac{d^2y}{dx^2} = \frac{1}{EI} \left( \frac{1}{2}wx^2 - wlx \right)</math></p> $\frac{dy}{dx} = \frac{w}{2EI} \cdot \frac{x^3}{3} - \frac{wl}{EI} \frac{x^2}{2} + c$ <p>iii) <math>\frac{dy}{dx} = 0</math>, when <math>x = 1</math><br/>         Then <math>c = \frac{wl^3}{3EI}</math><br/> <math>y = \frac{1}{EI} \left( \frac{1}{24}wx^4 - \frac{wlx^3}{6} \right) + \frac{wl^3}{3EI} + c</math><br/> <math>y = 0</math> where <math>x = 0 \Rightarrow c_1 = 0</math><br/>         Therefore deflection at any point is<br/> <math>y = \frac{w}{24EI} (x^4 - 41x^3 + 81^3x)</math></p> | <p>1</p> <p>1 + 1</p> <p>1</p>                        |
| <p>17. i) <math>\frac{dy}{dx} = \frac{-2(x+2)}{x^2-1} y + \frac{2(x+1)}{x^2-1}</math><br/>         degree 1, order 1.</p> <p>ii) <math display="block">\begin{aligned} \int pdx &amp;= \int \frac{-2(x+2)}{x^2-1} dx \\ &amp;= \int \left( \frac{-3}{x-1} + \frac{1}{x+1} \right) dx = \log \int \frac{x+1}{(x-1)^3} \end{aligned}</math></p> $\int e^{pdx} = \frac{x+1}{(x-1)^3}$ $\text{I.F} = \frac{x+1}{(x-1)^3}$ <p>iii) Solution is</p> $y \cdot \frac{x+1}{(x-1)^3} = c + \int \frac{2(x+1)}{x^2-1} \cdot \frac{x+1}{(x-1)^3} dx$   | <p>1</p> <p>1</p> <p>1</p> <p>1</p> <p>1</p> <p>1</p> |

|          |   |   |   |   |   |   |   |
|----------|---|---|---|---|---|---|---|
| 18    i) | $I = \int \sqrt{\tan x} dx$ <p>put <math>\tan x = t</math>, <math>\sec^2 x dx = 2t dt</math></p> $dx = \frac{2t dt}{1+t^4}$ $I = \int \frac{2t^2 dt}{1+t^4} = \int \frac{t^2 + 1 dt}{t^4 + 1} + \int \frac{t^2 - 1}{t^4 + 1} dt = I_1 + I_2$ $I_1 = \int \frac{t^2 + 1}{t^4 + 1} dt = \int \frac{(1 + \frac{1}{t^2})}{(t + \frac{1}{t})^2 + (\sqrt{2})^2}$ $= \frac{1}{\sqrt{2}} \tan^{-1} \left( \frac{u}{\sqrt{2}} \right) + c_1$ $= \frac{1}{\sqrt{2}} \tan^{-1} \left( \frac{\tan x - 1}{\sqrt{2} \tan x} \right) + c_1$ $I_2 = \int \frac{t^2 - 1}{t^4 + 1} dt = \int \frac{\left(1 - \frac{1}{t^2}\right) dt}{\left(t + \frac{1}{t}\right)^2 - (\sqrt{2})^2}$ $= \int \frac{du}{u^2 - (\sqrt{2})^2} \text{ where } u = t + \frac{1}{t}$ $= \frac{1}{2\sqrt{2}} \log \left  \frac{u - \sqrt{2}}{u + \sqrt{2}} \right  + c_2$ $= \frac{1}{2\sqrt{2}} \log \left  \frac{t^2 - \sqrt{2}t + 1}{t^2 + \sqrt{2}t + 1} \right  + c_2$ $\int \sqrt{\tan x} dx = I_1 + I_2$ | 1 | 1 | 1 | 1 | 1 | 5 |
|----------|---|---|---|---|---|---|---|

# APPENDIX

## Curriculum Objectives

### ***Unit - 1- Matrices and Determinants***

1. Understand the concept of matrices, familiarise different types of matrices, matrix operations and the algebra of matrices by discussion, assignment, project etc.
2. Develop the concept of determinant of a square matrix and its properties, inverse of a square matrix, consistency of linear equations and their solutions by discussion, assignment, seminar etc.

### ***Unit - 2 - Boolean Algebra***

3. Develop the concepts of Boolean algebra as an algebraic structure, understand the principle of duality and prove the related basic theorems through discussion, seminar, project etc.
4. Develop the concepts of Boolean function, basic gates, combinatorial circuits and their applications in switching circuits through discussion, lab work, project, assignment etc.
5. Develop the concepts of conditional statements, biconditional statements, arguments and validity of arguments through discussion, assignment etc.

### ***Unit - 3 - Probabilty***

6. Understand the concepts of random experiment, sample space, events, types of events, equally-likely outcomes, mutually exclusive events, exhaustive events, algebra of events, probability of an event, addition rule, conditional probability, independent events, independent experiments and multiplication rule through discussion, lab work, seminar, assignment etc.
7. Develop the concept of random variables and probability distribution through discussion, seminar etc.

### ***Unit - 4 - Functions, Limit and Continuity***

8. Develop the concepts of real functions, domain and range, composite functions, inverse of a function, familiarize the functions - modulus function, greatest integer function, signum function, trigonometric function, inverse trigonometric function and draw the graphs of above functions by discussion, assignment, seminar etc.
9. Develop the concept of limit of a function, left hand limit, right hand limit, familiarize the related notations and the fundamental theorems on limits by drawing graphs, assignment, discussion etc.
10. Derive the standard results i)  $\lim_{x \rightarrow a} \frac{x^n - a^n}{x - a} = na^{n-1}$ ,  $n$  is a positive integer.  
ii)  $\lim_{x \rightarrow 0} \frac{\sin x}{x} = 1$  and familiarize the results (i)  $\lim_{x \rightarrow 0} \frac{e^x - 1}{x} = 1$  and (ii)  $\lim_{x \rightarrow 0} \frac{\log(1+x)}{x} = 1$   
by seminar, assignment etc.
11. Develop the concepts of limit at infinity, infinite limits and familiarise theorems on limits by drawing graphs, assignment, discussion etc.
12. Develop the concept of continuity of a function (i) at a point, (ii) over an open/closed interval, familiarize the concept of continuity of sum, difference, product and quotient of continuous functions by drawing graphs, assignments, discussion etc.

13. Develop the concept of continuity of special functions - trigonometric functions, logarithmic functions, exponential functions, inverse trigonometric functions and polynomial functions by drawing graphs, discussion, assignment etc.

#### **Unit -5- Differentiation**

14. Develop the concept of derivative of a function, understand its physical and geometrical significance, derive the derivatives of algebraic, trigonometric, exponential and logarithmic functions using first principle through discussion, assignment, seminar etc.
15. Develop the ideas of derivative of sum, difference, product, quotient of functions and chain rule through discussion.
16. Familiarize the methods of logarithmic differentiation, derivative of a function expressed in parametric forms, implicit functions and differentiation by substitution through discussion, seminar etc.
17. Develop the concept of second order derivative by discussion, assignment etc.

#### **Unit - 6- Application of Derivatives**

18. Develop the concepts of rate of change of quantities, applying in different situations by discussion, assignment etc.
19. Deriving the equations of tangents and normals using derivatives by discussion, lab work etc.
20. Application of the derivatives in increasing and decreasing functions, maxima and minima, greatest and least values of functions by discussion, drawing graphs, assignments etc.
21. Familiarize and apply the ideas of Rolle's theorem and Mean Value theorem, by model preparations, seminars etc.
22. Applies differentiation to find approximate values of certain quantities by discussion, assignment etc.
23. Applies differentiation to sketch simple curves by lab work, assignment etc.

#### **Unit - 7 - Indefinite Integrals**

24. Develop the concept of indefinite integrals as antiderivatives and comprehend the properties of indefinite integrals through discussion, seminar etc.
25. Develop the idea of integration of functions involving algebraic, trigonometric, exponential and logarithmic functions using suitable substitutions and trigonometric identities through discussion, seminar, etc.
26. Derive the integrals of the form  $\int \frac{dx}{x^2 \pm a^2}$ ,  $\int \frac{dx}{a^2 - x^2}$ ,  $\int \frac{dx}{\sqrt{x^2 \pm a^2}}$ ,  $\int \frac{dx}{\sqrt{a^2 - x^2}}$  and apply them in integrating function such as  $\frac{1}{ax^2 + bx + c}$ ,  $\frac{1}{\sqrt{ax^2 + bx + c}}$ ,  $\frac{px+q}{ax^2 + bx + c}$ ,  $\frac{px+q}{\sqrt{ax^2 + bx + c}}$ ,  $\frac{1}{a+b \sin x}$ ,  $\frac{1}{a+b \cos x}$  through discussion, assignment etc.
27. Familiarize the methods of partial fractions and their use in integration of rational expressions through discussion, seminar etc.

28. Familiarize the method of integration by parts and apply it in the evaluation of integrals of the type  $\int \sqrt{x^2 \pm a^2} dx$ ,  $\int \sqrt{a^2 - x^2} dx$ ,  $\int \sqrt{ax^2 + bx + c} dx$ ,  $\int (px + q) \sqrt{ax^2 + bx + c} dx$  through discussion, assignment etc.

#### **Unit -8 - Definite Integral**

29. Develop the concept of definite integral as the limit of a sum and familiarise the fundamental theorems of integral calculus through discussion, project etc.
30. Familiarise the methods of evaluating definite integral by the method of substitution and using properties of definite integrals through discussion, seminar etc.
31. Develop the method to find the area bounded by a curve and the co-ordinate axes, by a curve, a straight line and between two curves by discussion, seminar, project etc.

#### **Unit - 9- Differential Equations**

32. Familiarize the concept of differential equations, its order, degree, the general and particular solutions and formation of a differential equation whose general solution is given and finding the solutions of different types of differential equations by discussion, assignment, seminar etc.

#### **Unit - 10- Vectors - I (Part - B)**

33. Develop the concept of vectors and differentiate different types of vectors such as equal vectors, unit vectors, zero vector, localised vector, collinear vectors, coplanar vectors, negative of a vector by group work, assignment etc.
34. Develop the idea of addition of vectors, multiplication of vectors by a scalar and its algebra through assignment, discussion etc.
35. Familiarize the concept of position vector and finding the position vector of a point dividing the line segment in the given ratio by discussion, assignment etc.

#### **Unit -11 - Vectors - II**

36. Develop and familiarize the concept of product of two vectors, dot product, cross product, area of triangle and parallelogram, solving problem in geometry and trigonometry using vector, scalar and vector triple product of three vectors by lab work, seminar, discussion, assignment, model preparation etc.

#### **Unit - I2 - Three Dimensional Geometry - I**

37. Develop the concept of co-ordinate planes in three dimensional space, co-ordinate of a point in a space, derive the formula of distance between two points and section formulae and also develop the concept of direction cosines and direction ratios of a line joining two points, projection of the join of two points on a given line, angle between two lines whose direction ratios are given through discussion, seminar, assignment, model preparation etc.

#### **Unit - I3 - Three Dimensional Geometry - II**

38. Familiarize the concepts of cartesian and vector equation of a line through (i) a point and parallel to a given vector (ii) two points and collinearity of three points by discussion, assignment, seminar etc.

39. Develop the idea of coplanar, skew lines, derive the method to find the shortest distance between two lines and find the condition for intersection of two lines through discussion, assignment etc.
40. Comprehend the vector and Cartesian equation of a plane and derives the formulae for the angle between (i) two lines (ii) two planes (iii) a line and a plane, the condition of coplanarity of two lines in vector and cartesian form, perpendicular distance of a point from a plane by both vectors and cartesian methods using discussion, seminar, assignment etc.
41. Derive the vector and Cartesian equation of a sphere using its centre and radius, diameter form of equation of the sphere using discussion, assignment, seminar etc.

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## **Mental Process**

- 1 Retrieves/ Recalls/ Retells information
- 2 Readily makes connection to new information based on past experience and formulate initial ideas.
- 3 Detects similarities and differences
- 4 Classifies/ Categorises/ Organises information appropriately
- 5 Translates/ Transfers knowledge or understanding and apply them in new situations
- 6 Establishes cause and effect relationship
- 7 Makes connection/ relates prior knowledge to new information. Apply reasoning and draw inferences
- 8 Communicates knowledge/ understands different media
- 9 Imagines/ Fantasises/ designs/ predicts based on received information
- 10 Judges/ appraises/ evaluates the merits or demerits of idea, develops own solution to problems.