MATHEMATICAL SCIENCES

Paper – II

1. Machine language is a computer language which
   - (A) needs to be translated
   - (B) is understood directly by the machine
   - (C) is machine independent
   - (D) is easy to learn.

2. Check the odd term out :
   - (A) Internet
   - (B) Linux
   - (C) Unix
   - (D) Windows.

3. Which of the following is not a computer hardware ?
   - (A) MS-DOS
   - (B) Intel 8088
   - (C) Channels
   - (D) Floppy disk.

4. Which of the following languages is frequently used to design Web pages ?
   - (A) COBOL
   - (B) C
   - (C) Java
   - (D) Pascal.

5. Which of the following is not a unit of computer memory ?
   - (A) Bit
   - (B) Byte
   - (C) Character
   - (D) Blotin.

6. A non-zero column vector X is an eigenvector of a square matrix A if there exists a number λ such that
   - (A) $A = \lambda X$
   - (B) $A = \frac{1}{\lambda} X$
   - (C) $AX = \lambda X$
   - (D) none of these.

7. The diag (1, 1, ...) is
   - (A) rectangular matrix
   - (B) non-symmetric matrix
   - (C) nilpotent matrix
   - (D) involutory matrix.

8. The number of function evaluations required to evaluate $\int_0^1 \frac{dx}{1+x}$ with an accuracy of 10^{-6} using Simpson's $\frac{1}{3}$rd rule is

(A) 8 (B) 9

(C) 10 (D) 11

9. The radius of convergence of the power series

$$\frac{1}{3} - x + \frac{x^2}{3^2} - x^3 + \frac{x^4}{3^4} - x^5 + \ldots\ldots \text{ is}$$

(A) $\frac{1}{2}$ (B) $\frac{1}{\sqrt{2}}$

(C) $\frac{1}{4}$ (D) 1

10. The variables to be determined in linear programming problems are always

(A) positive only (B) non-negative

(C) negative only (D) non-positive.

11. At any iteration of the usual simplex method if there is at least one basic variable in the basis at zero level and all $(Z_j - C_j) \geq 0$, the current solution is

(A) infeasible (B) unbounded

(C) non-degenerate (D) degenerate.

12. A probability density function f is said to be symmetric with respect to $x = a$ if for every real x,

(A) $f(a + x) = f(2a - x)$ (B) $f(a + 2x) = f(a - x)$

(C) $f(a + x) = f(a - x)$ (D) none of these.

13. If in a matrix A, two rows are interchanged and we obtain matrix B, then

(A) $|A| = |B|$ (B) $|A|\,|B| = 1$

(C) $|A| = -|B|$ (D) none of these.

14. The value of the determinant $\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^2 & b^2 & c^2 \end{vmatrix}$ is

(A) abc

(B) $a + b + c$

(C) $(a - b) (b - c) (c - a)$

(D) $abc (a - b) (b - c) (c - a)$.

15. If a natural number m is chosen at random from the first 100 natural numbers then the probability that $m + \frac{100}{m} > 50$ is given by

(A) $\frac{1}{20}$ (B) $\frac{3}{20}$

(C) $\frac{7}{20}$ (D) $\frac{11}{20}$.

16. The differential equation, derived from

$y = Ae^{2x} + Be^{-2x}$ has the order, where A, B are constant,

(A) 3 (B) 2

(C) 1 (D) none of these.

17. Which of the following sets of vectors $V = (v_1, v_2, \dots, v_n)$ in $I\!R^n$ are subspaces of $I\!R^n$ ($n \geq 3$), for all V such that (i) $v_1 \geq 0$, (ii) $v_1 + 3v_2 = v_3$?

(A) Both (i) and (ii) are subspaces of $I\!R^n$

(B) (i) is a subspace and (ii) is not a subspace of $I\!R^n$

(C) (i) is not a subspace $I\!R^n$ and (ii) is a subspace of $I\!R^n$

(D) Neither (i) nor (ii) are subspaces of $I\!R^n$.

18. If the vectors (0, 1, c), (1, c, 1), (c, 1, 0) of the vector space $I\!R^3$ (R) be linearly dependent, then the values of c are

(A) $-2, 0, 2$ (B) $-\sqrt{2}, 0, \sqrt{2}$

(C) $-4, 0, 4$ (D) none of these.

19. The series $1 + \frac{1}{1.3} + \frac{1}{1.3.5} + \ldots$ is

(A) divergent
(B) convergent
(C) unbounded
(D) none of these.

20. The series $\sum_{n=1}^{\infty} (-1)^n$ is

(A) convergent
(B) oscillatory
(C) unbounded
(D) none of these.

21. If $x = \cos\theta + i\sin\theta$, then $x^n + x^{-n}$, where n is a positive integer, is equal to

(A) $2 \sin n\theta$
(B) $2 \tan n\theta$
(C) $2 \cot n\theta$
(D) $2 \cos n\theta$.

22. The dimension of the vector space of all 5×5 real symmetric matrices is

(A) 5
(B) 15
(C) 12
(D) 10.

23. The variables to be determined in linear programming problems are

(A) non-linear only
(B) linear and non-linear both
(C) linear only
(D) none of these.

24. The function $f : I\!R \to I\!R$ defined by $f(x) = 6^x + 6^{|x|}$ is

(A) one-one and onto
(B) many-one and onto
(C) one-one and into
(D) many-one and into.

25. Which of the following statements is true for the function defined by

$$f(x) = \begin{cases} \frac{1}{2^p}, & \text{when } \frac{1}{2^{p+1}} < x < \frac{1}{2^p}, \; p = 0, 1, 2, \ldots\ldots \\ 0, & \text{when } x = 0 \end{cases} \quad ?$$

(A) It has infinite number of points of discontinuity at $x = \frac{1}{2^p}$, but is not of bounded variation in [0, 1]

(B) It has infinite number of points of discontinuity at $x = \frac{1}{2^p}$, but is of bounded variation in [0, 1]

(C) It has finite number of points of discontinuity at $x = \frac{1}{2^p}$ and is of bounded variation in [0, 1]

(D) None of these.

26. The function $f(z) = \frac{z}{(z+2)(z-1)^3}$ has a pole of

(A) first order at $z = 1$ (B) second order at $z = 1$

(C) third order at $z = 1$ (D) none of these.

27. The angular domain in the complex plane is defined by $0 < amp(z) < \frac{\pi}{4}$. The mapping which maps this region onto the left half plane is

(A) $w = z^4$ (B) $w = iz^4$

(C) $w = -z^4$ (D) $w = -iz^4$.

28. The value of the integral $\int_C \frac{dz}{z^2 - 8}$ in the counter-clockwise sense where C is the unit circle is

(A) $\frac{\pi}{\sqrt{2}} i$ (B) $-\frac{\pi}{\sqrt{2}} i$

(C) 0 (D) $2\pi i$.

29. If a point z_0 is the pole of the function $f(z)$, then as $z \to z_0$,

(A) $|f(z)|$ increases without bound

(B) $|f(z)|$ decreases without bound

(C) $|f(z)|$ attains a bound

(D) none of these.

30. Let $\mathbb{Z}$ be the set of integers and Q the set of all rational numbers. Then

(A) there exists a unique one-to-one-map from $\mathbb{Z}$ to Q

(B) there cannot exist a bijective map from $\mathbb{Z}$ to Q

(C) there exists a unique onto map from Q to $\mathbb{Z}$

(D) there exists infinitely many bijective maps from $\mathbb{Z}$ to Q.

31. What is the probability that 3 out of 10 students will pass the examination given that anyone of that can pass it with probability 0·80 ?

(A) 0·20 (B) 0·10

(C) 0·30 (D) 0·40.

32. The second moment about the origin for a random variable X with probability density function

$$f(x) = \begin{cases} \dfrac{4}{\pi(1+x^2)} , & 0 < x \leq 1 \\ 0 , & \text{otherwise} \end{cases}$$

(A) $\frac{4}{\pi}$ (B) $\frac{4}{\pi} - 1$

(C) 4π (D) $\frac{\pi}{4} - 1$.

33. Let f be a real-valued function defined on the interval [0, 1]. Then we have

(A) if f is not Riemann integrable, f must be unbounded

(B) if f has infinitely many points of discontinuity on [0, 1], it cannot be Riemann integrable

(C) if f assumes only the values 0 and 1 on [0, 1], it cannot be Riemann integrable

(D) if f is not Riemann integrable, it cannot be continuous on [0, 1].

34. If $f(x)$ is a probability density function of a continuous random variable then

(A) $\int_{-\infty}^{\infty} f(x)\, dx < 0$ (B) $\int_{-\infty}^{\infty} f(x)\, dx = 0$

(C) $\int_{-\infty}^{\infty} f(x)\, dx > 1$ (D) $\int_{-\infty}^{\infty} f(x)\, dx = 1$.

35. Which of the following is NOT a criterion for a good estimator ?

(A) Consistency (B) Invariance

(C) Efficiency (D) Sufficiency.

36. If X is a *r.v.* with mean μ and variance σ^2, then for any positive number k, we have

(A) $P\{ | X - \mu | > k\sigma \} \leq \frac{1}{k^2}$

(B) $P\{ | X - \mu | > k\sigma \} > \frac{1}{k^2}$

(C) $P\{ | X - \mu | < k\sigma \} \leq \frac{1}{k^2}$

(D) $P\{ | X - \mu | < k\sigma \} > \frac{1}{k^2}$.

37. Limits for multiple correlation coefficient are

(A) $(-\infty, \infty)$ (B) $(-1, 1)$

(C) $(0, \infty)$ (D) $(0, 1)$.

38. $\frac{\partial^2 z}{\partial x^2} - \frac{\partial^2 z}{\partial y^2} + \frac{\partial z}{\partial x} - \frac{\partial z}{\partial y} = 0$ has the solution

(A) $f_1(y - x) + e^x f_2(y - x)$

(B) $f_1(y + x) + f_2(y - x)$

(C) $e^{-x} f(y - x)$

(D) $f_1(y + x) + e^{-x} f_2(y - x)$.

OR

Suppose the random variable X has normal distribution with zero mean and unit variance. Then the random variable $Y = e^X$ has

(A) normal distribution

(B) Pareto distribution

(C) exponential distribution

(D) lognormal distribution.

39. The solution of the integral equation $f(x) = x + \int_0^1 x u^2 g(u)\, du$ is

(A) $\frac{3x}{4}$

(B) $\frac{4x}{3}$

(C) $\frac{2x}{3}$

(D) $\frac{3x}{2}$.

OR

Suppose $R = \begin{pmatrix} 1 & 0 \cdot 3 & 0 \cdot 2 \\ & 1 & 0 \cdot 5 \\ & & 1 \end{pmatrix}$ is the correlation matrix corresponding to the three component-random variable $X = (X_1, X_2, X_3)$. Then the multiple correlation coefficient $\rho_{1 \cdot 23}$ between X_1 and (X_2, X_3) is approximately

(A) 0·03

(B) 0·33

(C) 0·90

(D) 0·75.

40. Volterra Integral Equation can be reduced to

(A) Boundary value problem

(B) Initial value problem

(C) Initial boundary value problem

(D) none of these.

OR

Let y be the smallest order statistic for a random sample of size 10 drawn from the uniform distribution (0, 1). Then the distribution function of Y is given by

(A) $(1-y)^{10}, \ 0 \leq y \leq 1$

(B) $1-(1-y)^{10}, \ 0 \leq y \leq 1$

(C) $y^{10}, \ 0 \leq y \leq 1$

(D) $1-y^{10}, \ 0 \leq y \leq 1$

41. The linear congruence $ax = b \pmod{m}$, where $(a, m) = 1$ has

(A) an infinite number of solutions

(B) more than one but finite number of solutions

(C) only one solution

(D) no solution.

OR

Let X be a positive-valued random variable with expectation 1. Then $P(X > 2)$ has

(A) the least upper bound 0·5

(B) a lower bound 0·5

(C) an upper bound 0·5

(D) the greatest lower bound 0·5.

42. The function $f(z) = x^3 + ax^2 y + bxy^2 + cy^3$, where a, b, c are complex constants, is analytic in $\mathbb{C}$ if

(A) $a = 3i, \ b = -3, \ c = -i$

(B) $a = i, \ b = -3, \ c = -3i$

(C) $a = 3, \ b = -3i, \ c = i$

(D) none of these.

OR

Suppose the random variable X has the probability mass function

$p(x) = p(1-p)^x, \ x = 0, 1, 2, \ldots, 0 < p < 1$. Then

(A) $E(X) < Var(X)$

(B) $E(X) > Var(X)$

(C) $E(X) = Var(X)$

(D) there is no definite relation between $E(X)$ and $Var(X)$.

43. The principal value of $(1+i)^i$ is

(A) $e^{-\pi/4}\left[\cos\left(\frac{1}{2}\log 2\right)+i\sin\left(\frac{1}{2}\log 2\right)\right]$

(B) $e^{\pi/4}\left[\cos\left(\frac{1}{2}\log 2\right)+i\sin\left(\frac{1}{2}\log 2\right)\right]$

(C) $e^{-\pi/4}\left[\cos\left(\frac{1}{2}\log 2\right)-i\sin\left(\frac{1}{2}\log 2\right)\right]$

(D) none of these.

OR

Suppose the random variable X has the exponential distribution with mean 2. Then the probability density function of X is given by

(A) $\frac{1}{2}e^{-x/2}$, $x > 0$

(B) $2e^{-2x}$, $x > 0$

(C) $2e^{-x/2}$, $x > 0$

(D) $\frac{1}{2}e^{-2x}$, $x > 0$.

44. The accuracy of Simpson's one-third integration formula for a step size h is

(A) $O(h^2)$ (B) $O(h^3)$

(C) $O(h^4)$ (D) $O(h^5)$.

OR

Suppose 100 independent Bernoulli trials with unknown but common success probability p ($0 < p < 1$) produce 20 successes. Then an estimate of the standard error of the maximum likelihood estimate of p is

(A) 0·04 (B) 0·016

(C) 0·004 (D) 0·4

45. If for feasible solution $\overline{x}_B$, the problem min $z = \overline{c}\ \overline{x}$, subject to $\overline{A}\ \overline{x} = \overline{b}$, $\overline{x} \geq \overline{0}$, $Z_j - C_j \leq 0$ for all j, then $\overline{x}_B$ is a/an

(A) optimal solution

(B) non-optimal solution

(C) only basic feasible solution

(D) none of these.

OR

Let 3·3, 4·7, 5·9, 5·6, 4·6 and 4·8 be a random sample from the uniform distribution $u(0, \theta)$, where θ is unknown. Then the maximum likelihood estimate of θ is

(A) 5·9 (B) 4·8 (approximately)

(C) 3·3 (D) 4·6.

46. A necessary and sufficient condition for the differential equation $\vec{X} \cdot d\vec{r} = 0$, where $\vec{X} = (P, Q, R)$ and $\vec{r} = (x, y, z)$, to be integrable is

(A) $(\vec{\nabla} \times \vec{X}) \cdot \vec{X} = 0$ (B) $(\vec{X} \cdot \vec{\nabla})\vec{X} = 0$

(C) $\vec{\nabla} \times \vec{X} = \vec{0}$ (D) $(\vec{\nabla} \times \vec{X}) \times \vec{X} = \vec{0}$.

OR

A population of N-units is divided into k-strata of $N_1, N_2, \ldots\ldots, N_k$ units. A SRSWOR of $n_1, n_2, \ldots, n_k$ are taken from these k-strata. The proportional allocation for the i^{th} stratum is

(A) $n_i = \frac{n}{N} N_i$ (B) $n_i = \frac{N}{n} N_i$

(C) $n_i = n N_i$ (D) $n_i = N N_i$.

47. The function $f(z) = \ln z$ is analytic in the region

(A) $|z - 1| \leq 1$ (B) $|z - 1| < 1$

(C) $|z| \leq 1$ (D) $|z| < 1$.

OR

The method in which auxiliary information is used in estimating the population mean is known as

(A) inverse sampling method

(B) ratio method

(C) double sampling method

(D) none of these.

48. The set of natural numbers has

(A) maximal element

(B) upper bound

(C) lower bound

(D) none of these.

OR

In testing of hypothesis, the type II error is defined as

(A) rejecting a true null hypothesis

(B) rejecting a false null hypothesis

(C) accepting a false null hypothesis

(D) accepting a true null hypothesis.

49. The series $\sum_{n=0}^{\infty} \frac{(100 + 75i)^n}{\lfloor n}$ is

(A) convergent

(B) divergent

(C) non-zero

(D) absolutely convergent.

OR

The *m.g.f.* of χ^2 distribution with n *d.f.* is

(A) $(1 - t)^{-n/2}$, $|t| < 1$

(B) $(1 - 2t)^{n/2}$, $|t| < 1$

(C) $(1 - 2t)^{n/2}$, $|2t| < 1$

(D) $(1 - 2t)^{-n/2}$, $|2t| < 1$.

50. The variation of a functional $v[y(x)]$ is

(A) $\frac{\partial}{\partial \alpha} v[y(x) + \alpha \delta y]_{\alpha=0}$

(B) $v[y(x) + \alpha \delta y]_{\alpha=0}$

(C) $\left\{\frac{\partial}{\partial \alpha} v[y(x)] + \delta y\right\}_{\alpha=0}$

(D) $\left\{v[y(x)] + \frac{\partial}{\partial \alpha}(\delta y)\right\}_{\alpha=0}$.

OR

In a 2^4 design with 4 blocks in a replicate, the elements of the control block are

| (1) *abd* *acd* *bc* | . Then sub-group confounded is

(A) none of the following

(B) [*I, BC, AB, AC*]

(C) [*I, A, B, AB*]

(D) [*I, ABC, BCD, AD*].