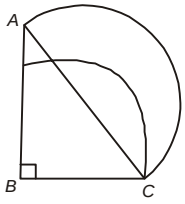
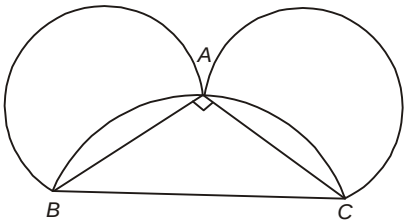
ELEMENTARY MATHEMATICS

1. What is the number which has to be added to each term of the ratio 49 : 68 so that it becomes 3 : 4?
   (a) 3 (b) 5
   (c) 8 (d) 9
2. What number must be added to each of 4, 10, 12, 24 so that the resulting numbers are in proportion?
   (a) 6 (b) 8
   (c) 10 (d) 4
3. The marked price of a machine is Rs. 18000. By selling it at a discount of 20%, the loss is 4%. What is the cost price of the machine?
   (a) Rs. 10000 (b) Rs. 12000
   (c) Rs. 14000 (d) Rs. 15000
4. At what rate percent per annum simple interest, will a sum of money triple itself in 25 yr?
   (a) 8% (b) 9%
   (c) 10% (d) 12%
5. If m men can do a job in p days, then $(m + r)$ men can do the job in how many days?
   (a) $(p + r)$ days (b) $\frac{mp}{m+r}$ days
   (c) $\frac{p}{m+r}$ days (d) $\frac{m+r}{p}$ days
6. If one man or two women or three boys can do piece of work in 55 days, then one man, one woman and one boy will do it in how many days?
   (a) 20 days (b) 30 days
   (c) 40 days (d) 50 days
7. A train crosses a telegraph post in 8 s and a bridge 200 m long in 24 s. What is the length of the train?
   (a) 100 m (b) 120 m
   (c) 140 m (d) 160 m
8. A train 110 m long is running with a speed of 60 km / h. What is the time in which it will pass a man who starts from the engine running at the speed of 6 km / h in the direction opposite to that of the train?
   (a) 5 s (b) 6 s
   (c) 10 s (d) 15 s
9. What is the value of
   $\left[\frac{(2.3)^3 - 0.027}{(2.3)^2 + 0.69 + 0.09}\right]$?
   (a) 2.6 (b) 2
   (c) 1.3 (d) 1
10. If $\sqrt{1+\frac{93}{196}} = 1+\frac{x}{14}$, then what does x equal to?
    (a) 1 (b) 2
    (c) 3 (d) 4
11. What is the simplifed form of $9\sqrt{2} - \sqrt{8} - 4\sqrt{2}$?
    (a) $4\sqrt{2}$ (b) $3\sqrt{2}$
    (c) $2\sqrt{2}$ (d) $\sqrt{2}$
12. A number, when divided by 987, gives a remainder 59. When the same number is divided by 21, what is the remainder?
    (a) 21 (b) 19
    (c) 17 (d) 15
13. Which one of the following statements is always correct?
    (a) The square of a prime number is prime
    (b) The sum of two square numbers is a square nmber
    (c) The number of digits in a square number is even
    (d) The product of two square numbers is a square number
14. 

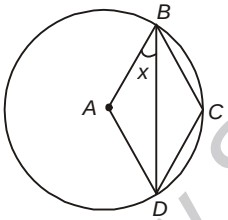
In the figure given above, ABC is a right angled triangle, right angled at B. $BC = 21$ cm and $AB = 28$ cm. With AC as diameter of a semicircle and with BC as radius a quarter circle are drawn. What is the area of the shaded portion?

(a) 425 cm^2 (b) 425.75 cm^2
(c) 428 cm^2 (d) 428.75 cm^2

15. 

In the figure given above, ΔABC is a right angled triangle, right angled at A. Semicircles are drawn on the sides AB, BC and AC. Then the area of the shaded portion is equal to which one of the following?

(a) Area of ΔABC
(b) 2 times the area of ΔABC
(c) Area of semicircle ABC
(d) None of the above

16. 

In the figure given above, A is the centre of the circle and $AB = BC = CD$. What is the value of x?

(a) 20° (b) $22\frac{1}{2}°$
(c) 25°
(d) None of these

17. Triangle ABC is right angled at A. $AB = 3$ unit, $AC = 4$ unit and AD is perpendicular to BC. What is the area of the triangle ABD?

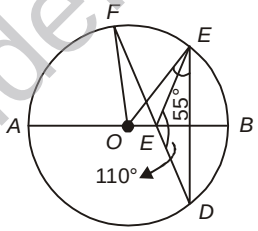
(a) $\frac{9}{25}$ sq unit (b) $\frac{54}{25}$ sq unit
(c) $\frac{72}{25}$ sq unit (d) $\frac{96}{25}$ sq unit

18. If A is the area of a right angled triangle and b is one of the sides containing the right angle, then what is the length of the altitude on the hypotenuse?

(a) $2Ab/\sqrt{b^4+4A^2}$ (b) $2A^2b/\sqrt{b^4+4A^2}$
(c) $2Ab^2/\sqrt{b^4+4A^2}$ (d) $2A^2b^2/\sqrt{b^4+A^2}$

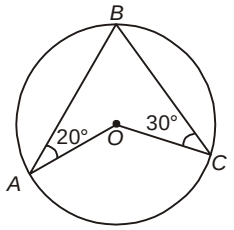
19. If ABC is a triangle, right angled at B and M, N are mid pionts of AB and BC respectively, then what is $4(AN^2 + CM^2)$ equal to?

(a) $3AC^2$ (b) $4AC^2$
(c) $5AC^2$ (d) $6AC^2$

20. 

In the figure given above AB is a diameter of the circle with centre O and $EC = ED$. What is $\angle EFO$?

(a) 15° (b) 20°
(c) 25° (d) 30°

21. 

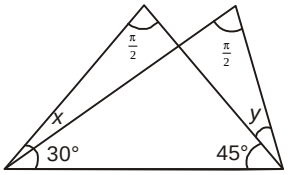
In the figure given above, O is the centre of the circle. What is $\angle AOC$?

(a) 160°
(b) 150°
(c) 120° (d) 100°

22. *ABCD* is a square. The diagonals *AC* and *BD* meet at *O*. Let *K, L* be the points on *AB* such that *AO* = *AK* , *BO* = *BL*. If θ = ∠*LOK*, then what is the value of tan θ?

(a) $1/\sqrt{3}$ (b) $\sqrt{3}$

(c) 1 (d) 1/2

23. 

In the figure given above, what is the value of (2*x* – *y*)?

(a) 10° (b) 15°

(c) 20° (d) 25°

24. The sides of a triangle are 50 m, 40 m and 30 m. What is the length of the altitude of the vertex opposite to the side 50 m long?

(a) 22 m (b) 24 m

(c) 25 m (d) 26 m

25. A ladder 34 m long is placed in a lane so as to reach a window 30 m high and on turning the ladder to the other side of the lane and keeping its foot at the same place, it reaches a point 16 m high. What is the breadth of the lane?

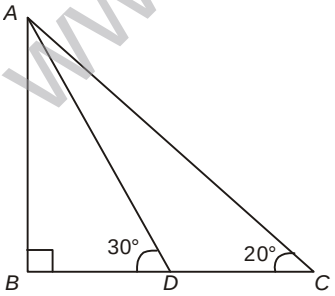
(a) 18 m (b) 40 m

(c) 46 m (d) 50 m

26. If the angles of a triangle are 90°, 60° and 30°, then what is the ratio of the sides opposite to these angles?

(a) $\sqrt{3} : \sqrt{2} : 1$ (b) $1 : \sqrt{2} : 2$

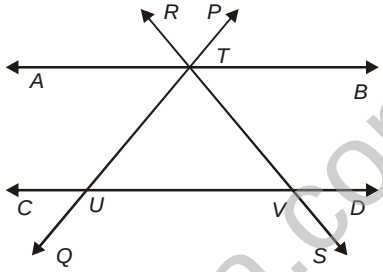
(c) $2 : \sqrt{3} : 1$ (d) 3 : 2 : 1

27. 

In the given figure, ∠*ABD* = 90°, ∠*BDA* = 30° and ∠*BCA* = 20°. What is ∠*CAD*?

(a) 10° (b) 20°

(c) 30° (d) 15°

28. 

In the given figure, if *AB*‖*CD*, ∠*PTB* = 55° and ∠*DVS* = 45°, then what is the sum of the measures of ∠*CUQ* and ∠*RTP*?

(a) 180° (b) 135°

(c) 110° (d) 100°

29. *ABC* is a right angled triangle, right angled at *C* and *p* is the length of the perpendicular from *C* on *AB*. If *a*, *b*, *c* are the sides of the triangle, then which one of the following is correct?

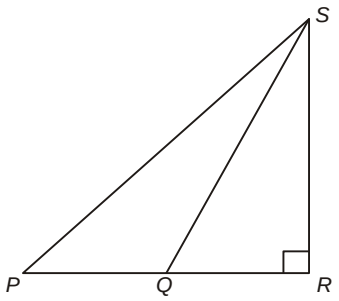
(a) $(a^2 + b^2)\, p^2 = a^2b^2$ (b) $a^2 + b^2 = a^2b^2p^2$

(c) $p^2 = a^2 + b^2$ (d) $p^2 = a^2 - b^2$

30. If a clock is started at noon, then what is the angle turned by hour hand at 3 : 45 pm?

(a) 67.5° (b) 97.5°

(c) 112.5° (d) 142.5°

31. 

In the figure given above, *PQ* = *QS* and *QR* = RS. If ∠*SRQ* = 100°, how many degrees is ∠*QPS*?

(a) 40° (b) 30°

(c) 20° (d) 15°

32. The angle of elevation and angle of depression both are measured with
(a) the vertical only
(b) the horizontal only
(c) both horizontal and vertical
(d) None of the above

33. A round ballon of unit radius subtends an angle of 90° at the eye of an observer standing at a piont, say *A*. What is the distance of the centre of the balloon from the point *A*?
(a) $1/\sqrt{2}$ (b) $\sqrt{2}$
(c) 2 (d) 1/2

34. If $\sin(x + 54°) = \cos x$, where $0 < x, x + 54° < 90°$, then what is the value of *x*?
(a) 54° (b) 36°
(c) 27° (d) 18°

35. The hypotenuse of a right triangle is $3\sqrt{10}$ unit. If the smaller side is tripled and the longer side is doubled, new hypotenuse becomes $9\sqrt{5}$ unit. What are the lengths of the smaller and longer sides of the right triangle respectively?
(a) 5 unit, 9 unit (b) 5 unit, 6 unit
(c) 3 unit, 9 unit (d) 3 unit, 6 unit

36. If $1 + \tan\theta = \sqrt{2}$, then what is the value of $\cot\theta - 1$?
(a) $1/\sqrt{2}$ (b) $\sqrt{2}$
(c) 2 (d) 1/2

37. If $\sin(x - y) = 1/2$ and $\cos(x + y) = 1/2$, then what is the value of *x*?
(a) 15° (b) 30°
(c) 45° (d) 60°

38. If α, β are the roots of $ax^2 + bx + c = 0$, then what is the value of $\left(\frac{1}{\alpha^2} - \frac{1}{\beta^2}\right)^2$?
(a) $a^2(b^2 - 4ac)/c^4$ (b) $b(b^2 - 4ac)/c^2$
(c) $(b^2 - 4ac)/c^2$ (d) $(b^2 - 4ac)/c^4$

39. The solution of the equations
$$\frac{3x - y + 1}{3} = \frac{2x + y + 2}{5} = \frac{3x + 2y + 1}{6}$$
is given by which one of the following?
(a) $x = 2, y = 1$ (b) $x = 1, y = 1$
(c) $x = -1, y = -1$ (d) $x = 1, y = 2$

40. 10 yr ago Ram was 5 times as old as Shyam, but 20 yr later from now he will be twice as old as Shyam. How many years old is Shyam?
(a) 20 yr (b) 30 yr
(c) 40 yr (d) 50 yr

41. The sum of the age of a father and the age of son is 75 yr. If the product of their ages before 5 yr was 750, then what is the present age of the father?
(a) 60 yr (b) 55 yr
(c) 52 yr (d) 50 yr

42. What is the simplified form of
$$\left(\frac{x^2 - 3x + 2}{x^3 - 8}\right) \div \left(\frac{x^2 - 9}{x^2 + 7x + 12}\right) \times \left(\frac{x^3 + 2x^2 + 4x}{x^2 + 3x - 4}\right)?$$
(a) $\frac{x}{x-3}$ (b) $\frac{x-2}{x-3}$ (c) $\frac{x}{x+3}$ (d) $\frac{x+3}{x+4}$

43. Which one of the following is a correct statement?
(a) $\phi \in \phi$ (b) $\phi \notin p(\phi)$
(c) $\phi = p(\phi)$ (d) $\phi \in P(\phi)$

44. The set {2, 4, 16, 256, ...} can be represented as which one of the following?
(a) $\{x \in N \mid x = 2^{2^n}, n \in N\}$
(b) $\{x \in N \mid x = 2^{2^n}, n = 0, 1, 2, ...\}$
(c) $\{x \in N \mid x = 2^{4n}, n = 0, 1, 2, ...\}$
(d) $\{x \in N \mid x = 2^{2n}, n = 0, 1, 2, ...\}$

45. If $y = (a^x)^{(a^x)^{\cdots\infty}}$, then which one of the following is correct?
(a) $\log y = xy \log a$ (b) $\log y = x + y \log a$
(c) $\log y = y + x \log a$
(d) $\log y = (y + x) \log a$

46. Suppose $p^* q = 2p + 2q - pq$, where p, q are natural numbers. If $8^* x = 4$, then what is the value of x?
(a) 1 (b) 2
(c) 3 (d) 4

47. The set of integers is closed with respect to which one of the following?
(a) Addition only
(b) Multiplication only
(c) Both addition and multiplication
(d) Division

48. Which one of the following is one of the two consecutive positive integers, the sum of whose is 761?
(a) 15 (b) 20
(c) 24 (d) 25

49. What is the value of
$$\frac{1}{1+\sqrt{2}}+\frac{1}{\sqrt{2}+\sqrt{3}}+...+\frac{1}{\sqrt{15}+\sqrt{16}}?$$
(a) 0 (b) 1
(c) 2 (d) 3

50. Assume that population densities of 5 major states of India are given. Which one of the following diagrams is suitable to represent the data?
(a) Single bar diagram
(b) Percentage bar diagram
(c) Pie diagram
(d) Since population density is a ratio, it cannot be represented by any diagram

51.

x	0	1	2	3	4
Frequency	4	f.	9	g	4

The table above gives the frequency distribution of a discrete variable X with two missing frequencies. If the total frequency is 25 and the arithmetic mean of X is 2, then what is the value of the missing frequency f?
(a) 4 (b) 5
(c) 6 (d) 7

52. Which one of the following statements is not correct with reference to a histogram?
(a) Frequency curve is obtained by joining the mid points of the top of the adjacent rectangles with smooth curves
(b) Historgam is drawn for continuous data
(c) The height of the bar is proportional to the frequency of that class
(d) Mode of the distribution can be obtained from the histogram

53. A sphere is cut into two equal halves and both the halves are painted from all the sides. The radius of the sphere is r unit and the rate of painting is Rs. 8 per sq unit. What is the total cost of painting the two halves of the sphere in rupees?
(a) $6\pi r^2$
(b) $32\pi r^2$
(c) $48\pi r^2$
(d) Insufficient data to answer

54. If the number of square centimetres on the surface area of a sphere is three times the number of cubic centimetres in its volume, then what is its diameter?
(a) 1 cm (b) 2 cm
(c) 3 cm (d) 6 cm

55. The ratio of the surface areas of two hemisphere is 4 : 1. What is the ratio of their volumes?
(a) 8 : 1 (b) 4 : 1
(c) 3 : 1 (d) 2 : 1

56. A container is in the form of a right circular cylinder surmounted by a hemisphere of the same radius 15 cm as the cylinder. If the volume of the container is 32400π cm^3, then the height h of the container satisfies which one of the following?
(a) 135 cm < h < 150 cm
(b) 140 cm < h < 147 cm
(c) 145 cm < h < 148 cm
(d) 139 cm < h < 145 cm

57. A hollow cylindrical iron pipe of length 1.4 m has bore radius 2.5 cm and thickness of the metal is 1 cm. What is the volume of the iron used in the pipe?
(a) 2640 cm^3 (b) 2604 cm^3
(c) 2460 cm^3 (d) None of these

58. A right circular cone is cut by a plane parallel to its base in such a way that the slant heights of the original and the smaller cone thus obtained are in the ratio 2 : 1. If V_1 and V_2 are respectively the volumes of the original cone and of the new cone, then what is $V_1 : V_2$?
(a) 2 : 1 (b) 3 :1
(c) 4 : 1 (d) 8 : 1

59. If C_1 is a right circular cone with base radius r_1 cm and height h_1 cm and C_2 is a right circular cylinder with base radius r_2 cm and height h_2 cm and if $r_1 : r_2 = 1 : n$ (where n is a positive integer) and their volumes are equal, then which one of the following is correct?
(a) $h_1 = 3nh_2$ (b) $h_1 = 3n^2h_2$
(c) $h_1 = 3h_2$ (d) $h_1 = n^2h_2$

Directions: Each of the following four (4) items consists of two statements, one labelled as 'Assertion (A)' and the other as 'Reason (R)'. You are to examine these two statements carefully and select the answers to these items using the code given below:

Codes:
(a) Both A and R are individually true and R is the correct explanation of A
(b) Both A and R are individually true but R is not the correct explanation of A
(c) A is true but R is false
(d) A is false but R is true

60. Assertion (A):The curved surface area of a right ciruclar cone of base radius r and height h is given by $\pi r(\sqrt{h^2 + r^2})$.
Reason (R):The right circular cone of base radius r and height h when cut opened along the slant height, froms a rectangle of length πr and breadth $\sqrt{h^2 + r^2}$.

61. Assertion (A):The volume of a cuboid is the product of the lengths of its coterminous edges.
Reason (R):The surface area of a cuboid is twice the sum of the products of lengths of its coterminous edges taken two at a time.

62. Assertion (A):Triangles on the same base and between the same parallel lines are equal in area.
Reason (R):The distance between two parallel lines is same everywhere.

63. Assertion (A):Two distinct lines cannot have more than one point in common.
Reason (R):Any number of lines can be drawn through one piont.

64. The population of a State increased from 100 million to 169 million in two decades. What is the average increase in population per decade?
(a) 20% (b) 34.5% (c) 69% (d) 30%

65. How much tea at Rs. 9 per kg must be mixed with 100 kg of superior tea at Rs. 13.50 per kg to give an average price of Rs. 11 per kg?
(a) 85 kg (b) 120 kg
(c) 125 kg (d) 130 kg

66. A truck made a trip of 200 km, covering the first 100 km at the rate of 50 km / h and the second 100 km at 40 km / h. What was the average speed of the truck in km / h?
(a) 22.2 (b) 28.2 (c) 33.3 (d) 44.4

67. Sets A, B and C contain 5 numbers each. The medians of the numbers in these sets are 3, 8 and 11 respectively. What is the median of the combined 15 numbers of the three sets?
(a) 8 (b) 7
(c) 22 / 3
(d) Median cannot be determined from the given data

68. Suppose X is some statistical variable with mean μ. Let $x_1, x_2, ..., x_n$ be its deviations from mean with the respective frequencies $f_1, f_2, ..., f_n$. What is the value of the sum $x_1f_1 + x_2f_2 + ... + x_nf_n$?
(a) 0 (b) 1 (c) μ (d) $\mu + 1$

69. If a polynomial equation has rational coefficients and has exactly three real roots, then what is the degree of the polynomial?
(a) Equal to 3
(b) Greater than or equal to 3
(c) Strictly greater than 3
(d) Less than 3

70. The value of y which will satisfy the equations
$2x^2 + 6x + 5y + 1 = 0$
and $2x + y + 3 = 0$
may be found by solving which one of the the following equations?
(a) $y^2 + 14y - 7 = 0$ (b) $y^2 + 8y + 1 = 0$
(c) $y^2 + 10y - 7 = 0$ (d) $y^2 - 8y + 7 = 0$

71. Which one of the following is the quadratic equation whose roots are reciprocal to the roots of the quadratic equation $2x^2 - 3x - 4 = 0$?
(a) $3x^2 - 2x - 4 = 0$ (b) $4x^2 + 3x - 2 = 0$
(c) $3x^2 - 4x - 2 = 0$ (d) $4x^2 - 2x - 3 = 0$

72. What is the LCM of $(x + 2)^2 (x - 2)$ and $x^2 - 4x - 12$?
(a) $(x + 2) (x - 2)$
(b) $(x + 2)^3 (x - 2) (x - 6)$
(c) $(x + 2) (x - 2)^2$
(d) $(x + 2)^2 (x - 2) (x - 6)$

73. What is the value of k for which the HCF of $2x^2 + kx - 12$ and $x^2 + x - 2k - 2$ is $(x + 4)$?
(a) 5 (b) 7
(c) 10 (d) –4

74. If $(x + k)$ is the HCF of $ax^2 + ax + b$ and $x^2 + cx + d$, then what is the value of k?
(a) $\frac{b+d}{a+c}$ (b) $\frac{a+b}{c+d}$
(c) $\frac{a-b}{c-d}$
(d) None of these

75. What should be subtracted from $27x^3 - 9x^2 - 6x - 5$ to make it exactly divisible by $(3x - 1)$?
(a) – 5 (b) – 7
(c) 5 (d) 7

76. Which one of the following is a factor of $2x^3 - 3x^2 - 11x + 6$?
(a) $x + 1$ (b) $x - 1$
(c) $x + 2$ (d) $x - 2$

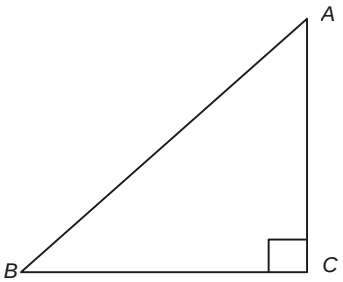
77. If $x^2 - 11x + a$ and $x^2 - 14x + 2a$ have a common factor, then what are the values of a?
(a) 0, 7 (b) 5, 20
(c) 0, 24 (d) 1, 3

78. If a is a rational number such that $(x - a)$ is a factor of the polynomial $x^3 - 3x^2 - 3x + 9$, then
(a) a can be any integer
(b) a is an integer dividing 9
(c) a cannot be an integer
(d) a can take three values

79. If $x + y + z = 6$ and $xy + yz + zx = 11$, then what is the value of $x^3 + y^3 + z^3 - 3xyz$?
(a) 18 (b) 36
(c) 54 (d) 66

80. If $x^2 - 4x + 1 = 0$, then what is the value of $x^3 + \frac{1}{x^3}$?
(a) 44 (b) 48
(c) 52 (d) 64

81. If $\sin\theta = \frac{m^2 - n^2}{m^2 + n^2}$, then what is the value of $\tan\theta$?
(a) $\frac{m^2 + n^2}{m^2 - n^2}$ (b) $\frac{2mn}{m^2 + n^2}$
(c) $\frac{m^2 - n^2}{2mn}$ (d) $\frac{m^2 + n^2}{2mn}$

82.


In the given figure, $BC = 15$ cm and $\sin B = 4/5$. What is the value of AB?
(a) 25 cm (b) 20 cm
(c) 5 cm (d) 4 cm

83. If $\sin x = \cos y$ and angle x and angle y are acute, then what is the relation between x and y?
(a) $x - y = \pi/2$ (b) $x + y = 3\pi/2$
(c) $x + y = \pi/2$ (d) $x + y = \pi/4$

84. If $\cos A = \frac{5}{13}$, then what is the value of $\frac{\sin A - \cot A}{2\tan A}$?
(a) 395 / 3644 (b) 395 / 3844
(c) 395 / 3744 (d) 385 / 3744

85. If $\sin^2 60° + \cos^2 (3x - 9°) = 1$, then what is the value of x?
(a) 24° (b) 23°
(c) 22° (d) 21°

86. What is the value of $\cot^2 \theta - (1 / \sin^2\theta)$?
(a) 1 / 2 (b) – 1
(c) – 1 / 2 (d) 3 / 2

87. What is the value of $\log_{100} 0.1$?
(a) 1 / 2 (b) – 1 / 2
(c) – 2 (d) 2

88. 21 mango trees, 42 apple trees and 56 orange trees have to be planted in rows such that each row contains the same number of trees of one variety only. What is the minimum number of rows in which the above trees may be planted?
(a) 3 (b) 15
(c) 17 (d) 20

89. What least value must be given to * so that the number 8798546*5 is divisible by 11?
(a) 0 (b) 1
(c) 2 (d) 3

90. What is the sum of all prime numbers between 100 and 120?
(a) 652 (b) 650
(c) 644 (d) 533

91. What is the value of the polynomial $r(x)$ so that $f(x) = g(x)\, q(x) + r(x)$ and deg $r(x)$ < deg $g(x)$, where $f(x) = x^2 + 1$ and $g(x) = x + 1$?
(a) 1 (b) – 1
(c) 2 (d) – 2

92. The radius and height of a right circular cone are in the ratio 3 : 4 and its volume is $96\pi cm^3$. What is its lateral surface area?
(a) $24\ \pi cm^2$ (b) $36\ \pi cm^2$
(c) $48\ \pi cm^2$ (d) $60\ \pi cm^2$

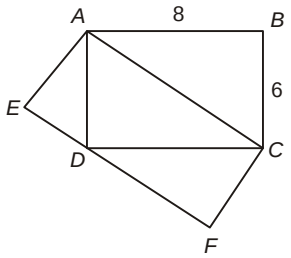
93. From a solid cube of edge 3 m, a solid of largest sphere is carved out. What is the volume of solid left?
(a) $(27 - 2.25\pi)\ m^3$ (b) $(27 - 4.5\pi)\ m^3$
(c) $2.25\ \pi m^3$ (d) $4.5\pi\ m^3$

94. The dimensions of a rectangular box are in the ratio 2 : 3 : 4 and the difference between the cost of covering it with a sheet of some metal at Rs. 8 and Rs. 10 per square metre is Rs. 234. What are the length, breadth and height of the box respectively.
(a) 2 m, 3 m, 4 m (b) 3 m, 4.5 m, 6 m
(c) 4 m, 6 m, 8 m (d) 5 m, 7.5 m, 10 m

95. A solid metallic cube of edge 4 cm is melted and recast into solid cubes of edge 1 cm. If x is the surface area of the melted cube and y is the total surface area of all the cubes recast, then what is $x : y$?
(a) 2 : 1 (b) 1 : 2
(c) 1 : 4 (d) 4 : 1

96. What is the total surface area of a one-side open cubical box of outer side of length 5 cm and thickness 0.5 cm?
(a) $125\ cm^2$ (b) $222\ cm^2$
(c) $180\ cm^2$ (d) $205\ cm^2$

97. A field is 125 m log and 15 m wide. A tank 10 m × 7.5 m × 6 m was dug in it and the earth thus dug out was spread equally on the remaining field. The level of the field thus raised is equal to which one of the following?
(a) 15 cm (b) 20 cm
(c) 25 cm (d) 30 cm

98. 

ABCD is a rectangular of dimensions 8 unit and 6 unit. *AEFC* is a rectangle drawn in such a way that diagonal *AC* of the first rectangle

is one side and side opposite to it is touching the first rectangle at *D* as shown in the figure given above. What is the ratio of the area of rectangle *ABCD* to that of *AEFC*?

(a) 2 (b) 3/2

(c) 1 (d) 8/9

99. If the area of a circle, inscribed in an equilateral traingle is 4π cm^2, then what is the area of the triangle?

(a) $12\sqrt{3}$ cm^2 (b) $9\sqrt{3}$ cm^2

(c) $8\sqrt{3}$ cm^2 (d) 18 cm^2

100. If *x* and *y* are respectively the areas of a square and a rhombus of sides of same length, then what is $x : y$?

(a) 1 : 1 (b) $2 : \sqrt{3}$

(c) $4 : \sqrt{3}$ (d) 3 : 2

ANSWERS

1. (c)	2. (d)	3. (d)	4. (a)	5. (b)	6. (b)	7. (a)	8. (b)	9. (b)	10. (c)
11. (b)	12. (c)	13. (d)	14. (d)	15. (a)	16. (d)	17. (b)	18. (a)	19. (c)	20. (b)
21. (d)	22. (c)	23. (b)	24. (b)	25. (c)	26. (c)	27. (a)	28. (b)	29. (a)	30. (c)
31. (c)	32. (b)	33. (b)	34. (d)	35. (c)	36. (b)	37. (c)	38. (a)	39. (b)	40. (a)
41. (b)	42. (a)	43. (d)	44. (b)	45. (a)	46. (b)	47. (c)	48. (b)	49. (d)	50. (c)
51. (a)	52. (d)	53. (c)	54. (b)	55. (a)	56. (a)	57. (a)	58. (d)	59. (b)	60. (c)
61. (b)	62. (a)	63. (b)	64. (b)	65. (c)	66. (d)	67. (d)	68. (a)	69. (a)	70. (c)
71. (b)	72. (d)	73. (a)	74. (d)	75. (b)	76. (c)	77. (c)	78. (d)	79. (a)	80. (c)
81. (c)	82. (a)	83. (c)	84. (c)	85. (b)	86. (b)	87. (b)	88. (c)	89. (d)	90. (d)
91. (c)	92. (d)	93. (b)	94. (b)	95. (c)	96. (b)	97. (c)	98. (c)	99. (a)	100. (a)

EXPLANATIONS

1. Let *x* number be added to 49: 68, then it becomes 3 : 4.

$$\therefore \quad \frac{49+x}{68+x} = \frac{3}{4}$$

$$\Rightarrow \quad 196 + 4x = 204 + 3x$$

$$\Rightarrow \quad 4x - 3x = 204 - 196$$

$$\Rightarrow \quad x = 8$$

2. Suppose *x* number added to each term, they become same proportional.

i.e., $(4 + x)$ $(10 + x)$, $(12 + x)$, $(24 + x)$ are in proportional.

$\therefore (4 + x) : (10 + x) :: (12 + x) : (24 + x)$

$$\Rightarrow \quad \frac{4+x}{10+x} = \frac{12+x}{24+x}$$

$$\Rightarrow \quad (4 + x)(24 + x) = (10 + x)(12 + x)$$

$$\Rightarrow \quad 96 + 28x + x^2 = 120 + 22x + x^2$$

$$\Rightarrow \quad 6x = 24$$

$$\Rightarrow \quad x = 4$$

3. Given marked price of machine = Rs. 18000

$$\therefore \quad \text{Discount} = \frac{20}{100} \times 18000$$

$$= \text{Rs. } 3600$$

$$\therefore \quad SP = 18000 - 3600$$

$$= \text{Rs. } 14400$$

If Loss is 4%, then

$$\text{CP} = \frac{100 \times SP}{100 - r}$$

$$= \frac{100 \times 14400}{100 - 4}$$

$$= \frac{100 \times 14400}{96}$$

$$= \text{Rs. } 15000$$

4. Let principal amount = p
As amount = $3p$, $T = 25$ yr
$\therefore$ SI = $3p - p = 2p$

Q Rate = $\frac{100 \times \text{SI}}{\text{Pricipal} \times T}$

$= \frac{100 \times 2p}{p \times 25} = 8\%$

5.

Men	Days
m	p
$(m+r)$	x

$\therefore \quad \frac{x}{p} = \frac{m}{m+r}$

$\Rightarrow \quad x = \frac{mp}{m+r}$ days

6. 1 man = 2 women = 3 boys

$\therefore$ 1 ma + 1 women + 1 boy = 3 boys + $\frac{3}{2}$ boys

+ 1 boy = $\frac{11}{2}$ boys

$\therefore$

Boys	Days
3	55
11 / 2	x

$\Rightarrow \quad \frac{x}{55} = \frac{3}{11/2}$

$\Rightarrow \quad x = \frac{3 \times 2}{11} \times 55$

$= 30$ days

7. Let the speed of a train be x m / s and length be y metre.

$\therefore \quad \text{Time} = \frac{\text{Distance}}{\text{Speed}}$

$\Rightarrow \quad 8 = \frac{y}{x}$

$\Rightarrow \quad y = 8x \quad \ldots$ (i)

IInd condition, when $t = 24$ s, speed = x, relative distance = $y + 200$

$\therefore \quad \text{Time} = \frac{\text{Relative Distance}}{\text{Speed}}$

$\Rightarrow \quad 24 = \frac{y + 200}{x}$

$\Rightarrow \quad 24x = y + 200$

$\Rightarrow \quad 24x = 8x + 200$ [from equ. (i)]

$\Rightarrow \quad 16x = 200$

$\Rightarrow \quad x = \frac{25}{2}$

$\therefore$ From Eq. (i),

$y = 8x = 8 \times \frac{25}{2}$

$= 100$ m

8. Since, train and man running opposite to each other.
$\therefore$ Relative speed = 60 + 6 = 66 km / h

$= \frac{66 \times 5}{18}$ m / s

Given, distance = 110 m

$\therefore \quad \text{Time} = \frac{\text{Distance}}{\text{Speed}}$

$= \frac{100}{\frac{66 \times 5}{18}}$

$= \frac{110 \times 18}{66 \times 5} = 6s$

10. Given, $\sqrt{1 + \frac{93}{196}} = 1 + \frac{x}{14}$

$\Rightarrow \quad \sqrt{\frac{289}{196}} = 1 + \frac{x}{14}$

$\Rightarrow \quad \frac{17}{14} = 1 + \frac{x}{14}$

$\Rightarrow \quad \frac{x}{14} = \frac{17}{14} - 1$

$\Rightarrow \quad x = 3$

11. $9\sqrt{2}-\sqrt{8}-4\sqrt{2}$

$= 9\sqrt{2}-2\sqrt{2}-4\sqrt{2}$

$= 3\sqrt{2}$

12. Let the remainder be x, then according to the Equation

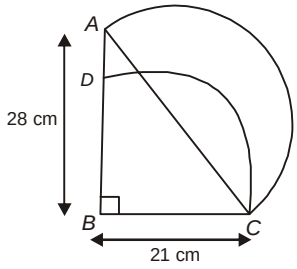
$987 + 59 = 21x$

$\Rightarrow \quad x = \frac{1046}{21}$

$\Rightarrow \quad x = 17$

13. It is always correct that the product of two square numbers is a square number.

14. In ΔABC,



$AC^2 = \sqrt{28^2+21^2}$

$= \sqrt{784+441}$

$= \sqrt{1225}$

$\Rightarrow \quad AC = 35$

Area of shaded portion = Area of semicircle ACE + Area of ΔABC – Area of quadrant circle BCD

$= \frac{\pi r^2}{2}+\frac{1}{2}\times BC\times BA-\frac{\pi}{4}\times r_1^2$

$= \frac{22}{7}\times\frac{1}{2}\times\frac{35}{2}\times\frac{35}{2}+\frac{1}{2}\times 21\times 28-\frac{22}{7\times 4}\times 21\times 21$

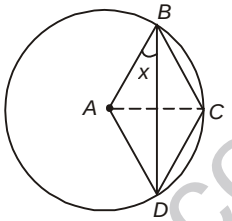
$= \frac{5\times 11\times 35}{4}+\frac{1}{2}[21\times 28-33\times 21]$

$= \frac{1925}{4}+\frac{1}{2}[-105]$

$= 481.25 - 52.50$

$= 428.75 \text{ cm}^2$

16. In a given figure we do not determined the value of x.

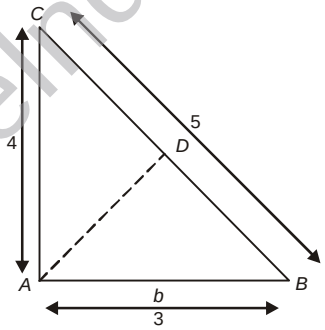


17. Using Pythagorus theorem,

$CB^2 = AC^2 + AB^2$

$= 16 + 9 = 25$

$\Rightarrow CB = 25$



Area of $\Delta ABC = \frac{1}{2}\times 3\times 4 = 6$

Again, in ΔABC,

$\text{Area} = \frac{1}{2}\times BC\times AD$

$6 = \frac{5}{2}AD$

$\Rightarrow \quad AD = \frac{12}{5}$

In right angled ΔADB,

$AB^2 = AD^2 + BD^2$

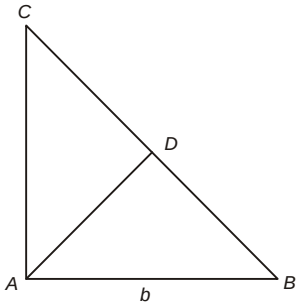
$\Rightarrow \quad (3)^2 = \left(\frac{12}{5}\right)^2 + BD^2$

$\Rightarrow \quad BD = \sqrt{9-\frac{144}{25}} = \frac{9}{5}$

$\therefore$ Area of $\Delta ABD = \frac{1}{2} \times BD \times AD$

$= \frac{1}{2} \times \frac{9}{5} \times \frac{12}{5} = \frac{54}{25}$

18. In ΔABC,



$A = \frac{1}{2} \times$ base $\times$ altitude

$A = \frac{1}{2} b \times AC$

$AC = \frac{2A}{b}$

Using Pythagorus theorem

$AC^2 + AB^2 = BC^2$

$BC = \sqrt{\frac{4A^2}{b^2} + b^2}$

Again, in ΔABC,

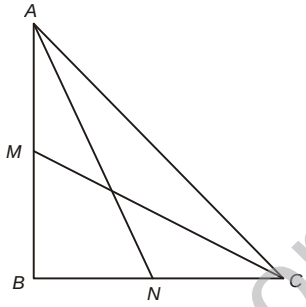
$A = \frac{1}{2} \times BC \times AD$

$\Rightarrow \quad AD = \frac{2A}{\sqrt{\frac{4A^2 + b^4}{b^2}}}$

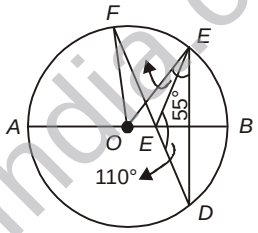
$= \frac{2Ab}{\sqrt{4A^2 + b^4}}$

19. In right angled triangle ABC,

$\Rightarrow \quad 5AC^2 = 4(AN^2 + CM^2)$



20. Given, $EC = ED$



$\Rightarrow \quad \angle EDC = \angle ECD$

$\Rightarrow \quad 35° = 35°$

Since, $\angle OCD = 55°$

Then, $\angle OCE = 20°$

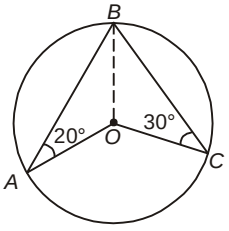
By using the theorem that triangle on the same segment of a circle makes an equal angles.

Here, OE is a segment, which makes a triangle OFE and OCE.

Therefore, $\angle OCE = \angle OFE$

$= 20°$

21. In ΔOAB,



$OA = OB$ (radius of circle)

$\therefore \quad \angle OAB = \angle OBA = 20°$

(Since, two sides of a triangle are equal, then angles are also equal)

Again, in ΔOCB,

$OC = OB$

$\therefore \quad \angle OCB = \angle OBC = 30°$

Now, $\angle ABC = \angle ABO + \angle CBO$

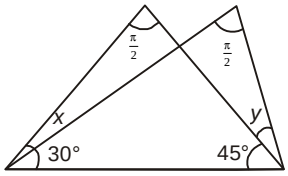
$= 20° + 30° = 50°$

$\therefore \quad \angle AOC = 2\angle ABC$

(by properties of circle)

$= 2 \times 50° = 100°$

23. In ΔADB,



$\angle DAB + \angle ABD + \angle BDA = 180°$

$\Rightarrow \quad 30° + x + 45° + 90° = 180°$

$\Rightarrow \quad x = 15°$

Again, in ΔACB,

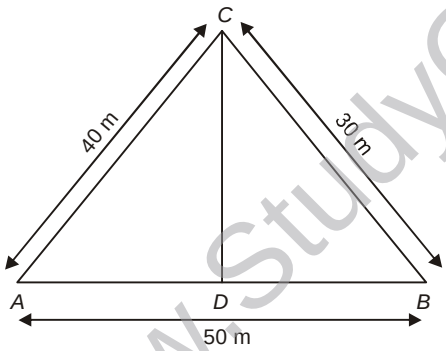
$\angle CAB + \angle ABC + \angle BCA = 180°$

$\Rightarrow \quad 30° + 45° + y + 90° = 180°$

$\Rightarrow \quad y = 15°$

$\therefore \quad 2x - y = 2(15°) - 15° = 15°$

24. Let $a = 30$ m, $b = 40$ m, $c = 50$ m



Now $2s = 30 + 40 + 50$

$\Rightarrow \quad s = 60$ m

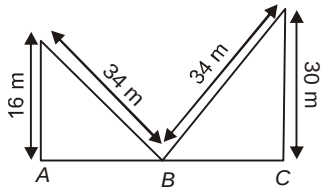
$\therefore \Delta = \sqrt{60(60-30)(60-40)(60-50)}$

$= \sqrt{60 \times 30 \times 20 \times 10} = 600$

$\Rightarrow \quad \frac{1}{2} \times AB \times CD = 600$

$\Rightarrow \quad CD = \frac{600 \times 2}{50} = 24$ m

25. In ΔBCD,



$BC = \sqrt{34^2 - 30^2}$

$= \sqrt{1156 - 900} = 16$ m

And in ΔBAE,

$AB = \sqrt{(34)^2 - (16)^2}$

$= \sqrt{1156 - 256} = 30$ m

$\therefore$ Breadth of line $= AB + BC$

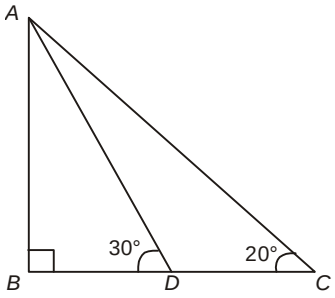
$= 30 + 16 = 46$ m

26. Since, $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$

$\therefore \sin 90° : \sin 60° : \sin 30° = 1 : \frac{\sqrt{3}}{2} : \frac{1}{2}$

$= 2 : \sqrt{3} : 1$

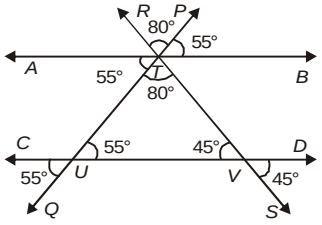
27. In ΔADC,



$\angle D = 180° - 30° = 150°$

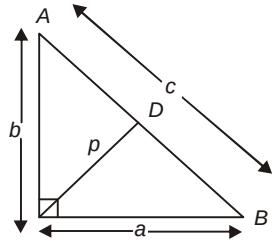
$\therefore \quad \angle DAC = 180° - (150° + 20°) = 10°$

28. Since, $\angle PTB = 55°$



Then, $\angle TUV = 55°$
(by corresponding angle)
Also, $\angle PTB = \angle UTA = 55°$
(vertically opposite angle)
Also give, $\angle DVS = 45°$
(vertically opposite angle)
In ΔUTV
$\angle T = 180° - (55° + 45°) = 80°$
$\Rightarrow$ $\angle T = \angle PTR = 80°$
(by vertically opposite angle)
$\therefore$ $\angle CUQ + \angle RTP = 55° + 80° = 135°$

29. In right triangle ABC,



$$\text{Area} = \frac{1}{2} \times a \times b \quad ...(i)$$

Again, in right triangle ABC,

$$\text{Area} = \frac{1}{2} \times AB \times DC$$

$$\Rightarrow \quad \frac{1}{2}ab = \frac{1}{2} \times c \times p$$

$$\Rightarrow \quad ab = p(\sqrt{a^2 + b^2}) \quad (\text{Q } c^2 = a^2 + b^2)$$

$$\Rightarrow \quad a^2b^2 = p^2(a^2 + b^2)$$

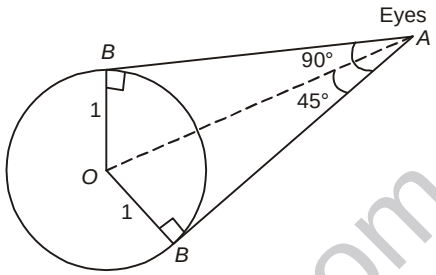
30. The clock revolve in
360 min = 180°

$$1 \text{ min} = \frac{180°}{360}$$

$$225 \text{ min} = \frac{180°}{360} \times 225 = 112.5°$$

32. The angle of elevation and angle of depression both are measured with the horizontal line only.

33. In ΔOAB,



$$\sin 45° = \frac{OB}{OA}$$

$$\Rightarrow \quad \frac{1}{\sqrt{2}} = \frac{1}{OA}$$

$$\Rightarrow \quad OA = \sqrt{2}$$

34. Given $\sin(x + 54°) = \cos x$
$\Rightarrow$ $\sin(x + 54°) = \sin(90° - x)$
$\Rightarrow$ $x + 54° = 90° - x$
$\Rightarrow$ $2x = 36°$
$\Rightarrow$ $x = 18°$

35. Let the smaller and larger sides of a right triangle be x and y respectively.
According to the equation,

$$x^2 + y^2 = (3\sqrt{10})^2$$

$\Rightarrow$ $x^2 + y^2 = 90$ ...(i)
and $9x^2 + 4y^2 = 405$ ...(ii)
On solving Eqs. (i) and (ii), we get
$x = 3$ unit, $y = 9$ unit

36. Give, $1 + \tan\theta = \sqrt{2}$

$$\therefore \quad \cot\theta - 1 = \frac{1}{\sqrt{2} - 1} - 1$$

$$= \frac{\sqrt{2} + 1}{1} - 1 = \sqrt{2}$$

37. Given, $\sin(x - y) = \frac{1}{2}$ and $\cos(x + y) = \frac{1}{2}$
$\Rightarrow x - y = 30°$ and $x + y = 60°$
$\Rightarrow x = 45°$ and $y = 15°$

38. Since, α and β are the roots of the equation $ax^2 + bx + c = 0$

$$\therefore \alpha + \beta = -\frac{b}{a} \text{ and } \alpha\beta = \frac{c}{a}$$

$$\therefore \quad \left(\frac{1}{\alpha^2}-\frac{1}{\beta^2}\right)^2 = \left(\frac{\beta^2-\alpha^2}{\alpha^2\beta^2}\right)^2$$

$$= \frac{(\alpha+\beta)^2\{(\alpha+\beta)^2-4\alpha\beta\}}{(\alpha^2\beta^2)^2}$$

$$= \frac{\frac{b^2}{a^2}\left(\frac{b^2}{a^2}-\frac{4c}{a}\right)}{\left(\frac{c^2}{a^2}\right)^2}$$

$$= \frac{b^2}{c^4}(a^2-4ac)$$

39. Given equations are

$$\frac{3x-y+1}{3} = \frac{2x+y+2}{5} = \frac{3x+2y+1}{6}$$

Taking Ist and IInd terms

$5(3x-y+1) = 3(2x+y+2)$

$\Rightarrow \quad 9x-8y = 1 \quad ...(i)$

Taking IInd and IIIrd terms

$6(2x+y+2) = 5(3x+2y+1)$

$\Rightarrow \quad 3x+4y = 7 \quad ...(ii)$

On solving Eqs. (i) and (ii), we get

$y = 1, x = 1$

40. Let the present age of Ram and Shyam be x and y respectively.

According to the question,

$(x-10) = 5(y-10)$

$\Rightarrow \quad x-5y = -40 \quad ...(i)$

and $(x+20) = 2(y+20)$

$\Rightarrow \quad x-2y = 20 \quad ...(ii)$

On solving Eqs. (i) and (ii), we get

$y = 20$ yr

41. Let the present age of son and father be x and y years respectively.

According to the question,

$x+y = 75 \quad ...(i)$

and $(x-5)(y-5) = 750$

$\Rightarrow \quad xy-5(x+y)+25 = 750$

$\Rightarrow \quad xy = 750-25+5(75)$

$= 1100$

Now, $y-x = \sqrt{(x+y)^2-4xy}$

$= \sqrt{(75)^2-4\times1100}$

$= 35$

$\Rightarrow \quad y-x = 35 \quad ...(ii)$

On solving Eqs. (i) and (ii), we get

$x = 20$ and $y = 55$

42. $\therefore \left(\frac{x^2-3x+2}{x^3-8}\right)\div\left(\frac{x^2-9}{x^2+7x+12}\right)\times\left(\frac{x^3+2x^2+4x}{x^2+3x-4}\right)$

$$= \left(\frac{x^2-3x+2}{x^3-8}\times\frac{x^2+7x+12}{x^2-9}\right)\times\frac{x^3+2x^2+4x}{x^2+3x-4}$$

$$= \frac{(x-1)(x-2)}{(x-2)(x^2+4+2x)}\times\frac{(x+4)(x+3)}{(x-3)(x+3)} \times\frac{x(x^2+2x+4)}{(x-1)(x+4)}$$

$$= \frac{x}{x-3}$$

43. In a given options, the correct statement is $\phi \in P(\phi)$.

44. Let $A = \{2, 4, 16, 256, ...\}$

This set can be rewritten as

$\{x \in N \mid x = 2^{2^n}, n = 0, 1, 2, ...\}$

45. Given, $y = (a^x)^{(a^x)^{...\infty}}$

$\therefore \quad y = (a^x)^y$

$\Rightarrow \quad \log y = xy \log a$

46. Given $p^* q = 2p + 2q - pq$

Q $8^* x = 4$

$\Rightarrow 2(8) + 2(x) - 8x = 4$

$\Rightarrow \quad -6x = -12$

$\Rightarrow \quad x = 2$

47. The set of integers is closed with respect to addition and mulitiplication.

48. Let two positive integers be x and $x+1$.

According to the question,

$x^2 + (x+1)^2 = 761$

$\Rightarrow 2x^2 + 2x - 760 = 0$

$\Rightarrow x^2 + x - 380 = 0$

$\Rightarrow (x+20)(x-19) = 0$

$\Rightarrow x = 19 \quad (\text{Q } x \neq -20)$

49. $\frac{1}{1+\sqrt{2}} + \frac{1}{\sqrt{2}+\sqrt{3}} + ... + \frac{1}{\sqrt{15}+\sqrt{16}}$

$= \frac{1-\sqrt{2}}{1-2} + \frac{\sqrt{2}-\sqrt{3}}{2-3} + ... + \frac{\sqrt{15}-\sqrt{16}}{15-16}$

$= -1[1-\sqrt{2}+\sqrt{2}-\sqrt{3}+...+\sqrt{15}-\sqrt{16}]$

$= 1[1-4] = 3$

50. To determine the population of 5 major states of India, the best suitable data is pie diagram.

51. Arithmetic mean

$\bar{X} = \frac{4\times0+1\times f+2\times9+3\times\times g+4\times4}{25}$

$\Rightarrow 50 = 34 + f + 3g$

$\Rightarrow f + 3g - 16 = 0 \quad ...(i)$

Also given,

$4 + f + 9 + g + 4 = 25$

$\Rightarrow f + g - 8 = 0 \quad ...(ii)$

On solving Eqs. (i) and (ii), we get

$f = g = 4$

52. Mode of the distribution cannot be determined from the histogram.

53. Total surface area $= 3\pi r^2 + 3\pi r^2$

$= 6\pi r^2$

$\therefore$ The total cost of painting the two havles

$= 6\pi r^2 \times 8$

$= 48\pi r^2$

54. According to the question,

Surface area of cube = 3 (Volume of cube)

$\Rightarrow 4\pi r^2 = 3 \times \frac{4}{3}\pi r^3$

$\Rightarrow r = 1$

$\therefore$ Diameter $= 2r$

$= 2$ cm

55. According to the question,

$\frac{4\pi r_1^2}{4\pi r_2^2} = \frac{4}{1}$

$\Rightarrow \frac{r_1}{r_2} = \frac{2}{1}$

$\therefore$ Required ratio $= \frac{\frac{4}{3}\pi r_1^3}{\frac{4}{3}\pi r_2^3} = \frac{8}{1}$

57. $\therefore$ Volume, $V = \pi(r_1^2 - r_2^2)\times h$

$= \frac{22}{7}[(3.5)^2 - (2.5)^2] \times 140$

$= \frac{22}{7}(12.25 - 6.25) \times 140$

$= 22 \times 6 \times 20 = 2640 \text{ cm}^3$

59. Let $r_1 = 1k$ and $r_2 = nk$

Since, $V_1 = V_2$

$\therefore \frac{1}{3}\pi r_1^2 h_1 = \pi r_2^2 h_2$

$\Rightarrow \frac{1}{3}\pi k^2 \times h_1 = \pi n^2 k^2 h_2$

$\Rightarrow h_1 = 3n^2 h_2$

60. (A) It is a true statement.
(R) It is a false statement.

61. Both statements are true but (R) is not a correct explanation of (A).

62. (A) By the properties of triangle, it is true.
(B) It is also true, that the distance between two parallel lines is same everywhere.

63. (A) It is true that two distiner lines interest only one point.
(R) It is true that, from a one point we can draw any number of lines.

64. The differece of population in two decades = 169 – 100 = 69 million

$\therefore$ Population in first decade $= \frac{69}{2} = 34.5\%$

65. Let x kg of tea of Rs 9 per kg.
According to the question,

$\frac{9\times x + 13.5\times100}{x+100} = 11$

$\Rightarrow \quad 9x + 1350 = 11x + 1100$

$\Rightarrow \quad 2x = 250$

$\Rightarrow \quad x = 125$ kg

66. Average speed $= \dfrac{2uv}{u+v}$

$= \dfrac{2\times 50\times 40}{50+40}$

$= \dfrac{400}{9} = 44.4$ km / h

67. Median cannot be determined from the given data.

68. As we know, the sum of deviation from their mean is zero.

$\therefore x_1f_1 + x_2f_2 + ... + x_nf_n = 0$

69. If a polynomial equation has rational coefficient and has exactly three real roots, then degree of the polynomial must be 3.

70. Given that,

$2x^2 + 6x + 5y + 1 = 0$

and $2x + y + 3 = 0$

$\Rightarrow \quad x = \dfrac{-y-3}{2}$

On putting the value of x in Eq. (i), we get

$$2\left(\frac{-3-y}{2}\right)^2 + 6\left(\frac{-3-y}{2}\right) + 5y + 1 = 0$$

$\Rightarrow \quad \dfrac{9+y^2+6y}{2} - \dfrac{(18+6y)}{2} + 5y + 1 = 0$

$\Rightarrow \quad y^2 + 10y - 7 = 0$

71. Given equation is

$2x^2 - 3x - 4 = 0.$

For getting a reciprocal roots, we replace x by $\dfrac{1}{x}$, we get

$$2\left(\frac{1}{x}\right)^2 - 3\left(\frac{1}{x}\right) - 4 = 0$$

$\Rightarrow \quad -4x^2 - 3x + 2 = 0$

$\Rightarrow \quad 4x^2 + 3x - 2 = 0$

72. Let $f(x) = (x + 2)^2 (x - 2)$

and $g(x) = x^2 - 4x - 12$

$= (x - 6) (x + 2)$

$\therefore$ LCM of $\{f(x), g(x)\} = (x + 2)^2 (x - 2) (x - 6)$

73. Since, $(x + 4)$ is HCF, so it will divide both the expression, *i.e.*, $x = -4$ will make each one zero.

$\therefore 2(-4)^2 + k(-4) - 12 = 0$ and $(-4)^2 + (-4) - 2k - 2 = 0$

$\Rightarrow 32 - 12 = 4k$ and $16 - 6 = 2k$

$\Rightarrow k = 5$ and $k = 5$

74. Since, $x + k$ is hte HCF of given expression, then $x = -k$ will make each one zero.

$\therefore a(-k)^2 - ak + b = 0 = k^2 - ck + d$

$\Rightarrow k^2(a - 1) - (a - c)k + b - d = 0$

$$\Rightarrow k = \frac{(a-c) + \sqrt{(a-c)^2 - 4(a-1)(b-d)}}{1(a-1)}$$

Hence, option (d) is correct.

75.
$$3x-1\overline{)\,27x^3 - 9x^2 - 6x - 5\,(\,9x^2 - 2}$$
$$27x^3 - 9x^2$$
$$- \quad +$$
$$-6x - 5$$
$$-6x + 2$$
$$+ \quad -$$
$$7$$

Hence, option (b) is correct.

76. Let $f(x) = 2x^3 - 3x^2 - 11x + 6$

Put $x = -2$, we get

$f(-2) = 2(-2)^3 - 3(-2)^2 - 11(-2) + 6$

$= -16 - 12 + 22 + 6 = 0$

Hence, $(x + 2)$ is a factor of $f(x)$.

77. Let α be the common factors of the given equations.

$\therefore \alpha^2 - 11\alpha + \alpha = 0$ and $\alpha^2 - 14\alpha + 2\alpha = 0$

Thus, it represents the same equation.

$\therefore \quad \dfrac{\alpha^2}{-22a+14a} = \dfrac{\alpha}{a-2a} = \dfrac{1}{-14+11}$

$\Rightarrow \quad \dfrac{a^2}{-8a} = \dfrac{a}{-a} = \dfrac{1}{-3}$

$\Rightarrow \quad \frac{\alpha}{a} = -\frac{1}{3} \Rightarrow \alpha = \frac{a}{3}$

and $\frac{\alpha^2}{-8a} = -\frac{1}{3} \Rightarrow \frac{a^2/9}{-8a} = -\frac{1}{3}$

$\Rightarrow \quad a = 0, a = 24$

78. $x^3 - 3x^2 - 3x + 9 = (x^2 - 3)(x - 3)$

Here, factors are $\sqrt{3}, -\sqrt{3}, 3$.

Hence, option (d) is correct.

79. Given, $x + y + z = 6$ and $xy + yz + zx = 11$.

$\therefore x^3 + y^3 + z^3 - 3xyz$

$= (x + y + z)[(x + y + z)^2 - 3(xy + yz + zx)]$

$= 6[6^2 - 3(11)] = 6 \times 3 = 18$

80. Given equation is

$x^4 - 4x + 1 = 0$

$\therefore \quad x = \frac{4 \pm \sqrt{16 - 4 \times 1 \times 1}}{2 \times 1}$

$= \frac{4 \pm 2\sqrt{3}}{2} = 2 + \sqrt{3}$

When $x = 2 + \sqrt{3}$, them

$x^3 + \frac{1}{x^3} = (2 + \sqrt{3}) + \left(\frac{1}{2 + \sqrt{3}}\right)^3$

$= (2 + \sqrt{3})^3 + (2 - \sqrt{3})^3$

$= 2^3 + (\sqrt{3})^3 + 3 \times 2 \times \sqrt{3}(2 + \sqrt{3})$

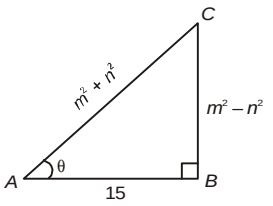
$+ (2)^3 - (\sqrt{3})^3 - 3 \times 2 \times \sqrt{3}(2 - \sqrt{3})$

$= 8 + 18 + 8 + 18 = 52$

Similary, when $x = 2 - \sqrt{3}$

then $x^3 + \frac{1}{x^3} = 52$

81. In ΔABC,



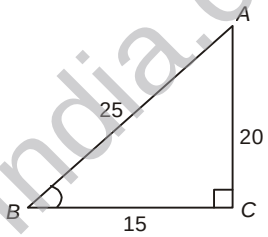
$AB = \sqrt{(AC)^2 - (BC)^2}$

$= \sqrt{m^4 + n^4 + 2m^2n^2 - (m^4 + n^4 - 2m^2n^2)}$

$= \sqrt{4m^2n^2} = 2mn$

$\therefore \tan\theta = \frac{m^2 - n^2}{2mn}$

82. Given, $BC = 15$ cm and $\sin B = \frac{4}{5} = \frac{20}{25}$



Hence, value of AB is 25 cm.

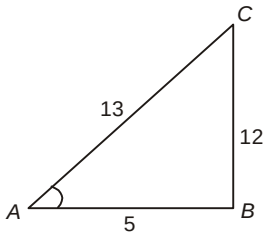
83. Since, $\sin x = \cos y$

As x and y are acute angles, then

$x = y = \frac{\pi}{4}$

$\therefore \quad x + y = \frac{\pi}{2}$

84. Given, $\cos A = \frac{5}{13}$



$\therefore \quad \frac{\sin A - \cot A}{2\tan A} = \frac{\frac{12}{13} - \frac{5}{12}}{2 \times \frac{12}{5}}$

$= \frac{144 - 65}{13 \times 12 \times 2 \times \frac{12}{5}} = \frac{395}{3744}$

85. Given equation can be rewritten as

$$\cos^2(3x - 9°) = 1 - \sin^2 60°$$
$$\Rightarrow \cos^2(3x - 9°) = \cos^2 60°$$
$$\Rightarrow 3x - 9° = 60°$$
$$\Rightarrow 3x = 69°$$
$$\Rightarrow x = 23°$$

86. $\cot^2\theta - \dfrac{1}{\sin^2\theta}$

$$= \frac{\cos^2\theta}{\sin^2\theta} - \frac{1}{\sin^2\theta} = \frac{\sin^2\theta}{\sin^2\theta} = -1$$

87. $\log_{100} 0.1 = \log_{10^2}\left(\dfrac{1}{10}\right)$

$$= \frac{1}{2}\log_{10}\left(\frac{1}{10}\right)$$

$$= -\frac{1}{2}$$

88. The HCF of (21, 42, 56) is 7.
∴ The minimum number of rows
= 3 + 6 + 8 = 17

89. Any number is divisible by 11, if difference between the sum of odd digits and sum of even digits is zero or multiple of 11.
Let N = 8 7 9 8 5 4 6 * 5
Sum of odd digit places
= 8 + 9 + 5 + 6 + 5 = 33
Sum of even digit places
= 7 + 8 + 4 + * = 19 + *
Now, 33 – (19 + *)
= 14 –*, it is divisible by 11, if value of * is 3.

90. The prime numbers between 100 and 120 are 101, 103, 107, 109, 113 = 533

91. Now,

$$x+1 \overline{)\, x^2 + 1}$$
$$x^2 + x$$
$$- \quad -$$
$$-x + 1$$
$$-x - 1$$
$$+ \quad +$$
$$2$$

Hence, $r(x) = 2$

92. Let radius and height of a cone be r and h.

Q $\dfrac{r}{h} = \dfrac{3}{4}$

Volume of cone = $\dfrac{1}{3}\pi r^2 h$

$$\therefore 96\pi = \frac{1}{3}\pi \times r^2 \times \frac{4r}{3}$$

$$\Rightarrow r^3 = \frac{96 \times 3 \times 3}{4} = 216$$

$\Rightarrow r = 6$ and $h = 8$

∴ Lateral surface area = $\pi r\sqrt{r^2 + h^2}$

$= \pi \times 6\sqrt{36 + 64} = 60\pi$ cm^2

93. The maximum diameter of a sphere in a cube is of 3 m.

∴ Volume of sphere, $V_1 = \dfrac{4}{3}\pi(1.5)^3$
$= 4.5\ \pi$ m^3
∴ Volume of cone, $V_2 = (3)^3$
$= 27$ m^2
∴ Volume of solid left = $V_2 - V_1$
$= (27 - 4.5\ \pi)$ m^3

94. Let the dimensions be $2k$, $3k$ and $4k$.
According to the question,
$(10 - 8)\ 2\ [2k \times 3k + 3k \times 4k + 4k \times 2k] = 234$

$$\Rightarrow 4[26k^2] = 234$$

$$\Rightarrow k^2 = \frac{234}{4 \times 26} = 2.25$$

$$\Rightarrow k = 1.5$$

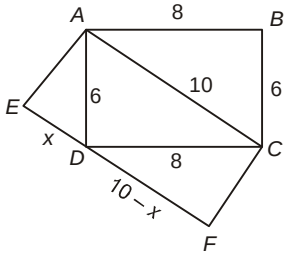
95. Volume of solid cube = $(4)^3 = 64$ cm^3
Volume of recast cube = $(1)^3 = 1$ cm^3
∴ Total surface area of cube: total surface area of recast cube = $x : y$

$$\Rightarrow x : y = 6(4)^2 : 6(1)^2 \times 64$$
$$= 1 : 4$$

96. Since, the outer edges of a cubical box is 5 cm.
∴ Surface area of the outer cubical box
= 5 (edge)^2
= 5(5)^2 = 125 cm^2

Surface area of the inner cubical box
$= 4\,(4.5 \times 4) + 4 \times 4 = 72 + 16 = 88\ \text{cm}^2$
$\therefore$ Total surface area $= 125 + 88 + (18 \times 0.5)$
$= 222\ \text{cm}^2$

98. Let $ED = x$



Now, $AC = \sqrt{8^2 + 6^2}$

In ΔAED,

$$AE^2 = AD^2 - x^2$$
$$= 36 - x^2 \ldots \text{(i)}$$

And in ΔCFD,

$$CF^2 = (8)^2 - (10 - x)^2 \quad \ldots\text{(ii)}$$

From Eqs. (i) and (ii), we get

$$36 - x^2 = 64 - (10 - x)^2$$
[Q $AE = FC$]

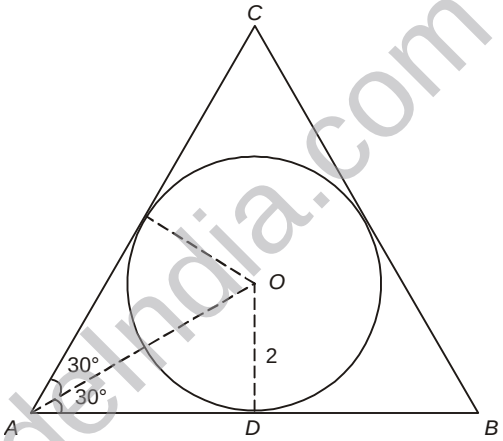
$$\Rightarrow \quad 36 - x^2 = 64 - (100 + x^2 - 20x)$$
$$\Rightarrow \quad 20x = 72$$
$$\Rightarrow \quad x = \frac{18}{5}$$

$\therefore$ From Eq. (i), $AE^2 = 36 - \left(\frac{18}{5}\right)^2$

$$\Rightarrow \quad AE^2 = 36 - \frac{324}{25} = \frac{900 - 324}{25}$$
$$\Rightarrow \quad AE^2 = \frac{576}{25}$$
$$\Rightarrow \quad AE = \frac{24}{5}$$

$$\therefore \frac{\text{Area of rectangle } ABCD}{\text{Area of rectangle } AEFC} = \frac{8 \times 6}{10 \times \frac{24}{5}} = 1.$$

99. Since, area of circle $= 4\pi$
$\Rightarrow \pi r^2 = 4\pi \Rightarrow r = 2$ cm



In ΔOAD,

$$\tan 30° = \frac{OD}{AD}$$
$$\Rightarrow \quad AD = 2\sqrt{3}\ \text{cm}$$

Now, $AB = 2\,AD$
$= 4\sqrt{3}$ cm

$\therefore$ Area of equilateral ΔABC

$$= \frac{\sqrt{3}}{4}(AB)^2$$
$$= \frac{\sqrt{3}}{4}(4\sqrt{3})^2$$
$$= 12\sqrt{3}\ \text{cm}^2$$

100. As. we know that, if the length of square and rhombus are same, then the area should be same.

$\therefore \quad x : y = 1 : 1$