ELEMENTARY MATHEMATICS

Directions: Each of the next THREE (3) items consists of two statements, one labelled as the 'Assertion two statements carefully and select the answers to these items using the codes given below:

Codes:

(a) Both A and R are individually true and R is the correct explanation A

(b) Both A and R are individaully true but R is not the correct explanation of A

(c) A is true but R is false

(d) A is false but R is true

1. *ABC* is a triangle. *AD*, *BE*, *CF* are altitudes of ΔABC.

   Assertion (A): $(AB^2 + BC^2 + CA^2) > (AD^2 + BE^2 + CF^2)$

   Reason (R): $(AE^2 - AF^2) + (BF^2 - BD^2) + (CD^2 - CE^2)$

2. *ABC* is a triangle. Let *D*, *E* denote the mid points of *BC*, *CA* respectively. Let *AD* and *BE* intersect at *G*. Let *O* be a point on *AD* such *AO* : *OD* = 2 : 7.

   Assertion (A): $AO = (2GD)/3$

   Reason (R): $OD = (2AG)/3$

3. Assertion (A):If two traingles have same primeter, than they are congruent.

   Reason (R):If under a given correspondence, the three sides of one triangle are equal to the three sides of the other triangle, then the two triangles are congruent.

4. If $x + \left(\frac{1}{x}\right) = 2\cos\alpha$, then what is the value of $x^2 + \left(\frac{1}{x^2}\right)$?

   (a) $4\cos^2\alpha$ (b) $4\cos^2\alpha - 1$

   (c) $2\cos^2\alpha - 2\sin^2\alpha$ (d) $\cos^2\alpha - \sin 2\alpha$

5. If α is the angle of first quadrant such that $\text{cosec}^4\alpha = 17 + \cot^4\alpha$, then what is the value of sin α?

   (a) 1 / 3 (b) 1 / 4

   (c) 1 / 9 (d) 1 / 16

6. Data on percentage distribution of area of land in acres owned by households in two districts of a particular state are as follows:

Land holding	District A	District B
0.01–0.99	5.62	13.53
1.0–2.49	18.35	21.84
2.5–7.49	47.12	39.32
7.5–12.49	19.34	12.15
12.5–19.99	7.21	7.43
20.0–29.99	2.36	5.73

   What is the appropriate diagram to represent the above data?

   (a) Pie diagram (b) Histogram

   (c) Bar chart (d) None of these

7. The mean of 25 observations is 36. The mean of first 13 observations is 32 and, that of last 13 observations is 39, what is the value of 13th observation?

   (a) 20 (b) 23 (c) 32 (d) 40

8. Consider the following types of data

   1. Marks of students who appeared for a test of 100 marks.
   2. Collar sizes of 200 shirts sold in a week.
   3. Monthly incomes of 250 employees of a factory.

   For which of the above data, mode is suitable measure of central tendency?

   (a) 1 and 2 only (b) 2 only

   (c) 1 and 3 only (d) 1, 2 and 3

9. The arithmetic mean of a set of 10 numbers is 20. If each number is first multiplied by 2 and then increased by 5, then what is the mean of new numbers?

   (a) 20 (b) 25 (c) 40 (d) 45

10. If $\sin x + \sin y = a$ and $\cos x + \cos y = b$, what is $\sin x \cdot \sin y + \cos x \cdot \cos y$ equal to?

    (a) $a + b - ab$ (b) $a + b + ab$

    (c) $a^2 + b^2 - 2$ (d) $\left(\frac{a^2 + b^2 - 2}{2}\right)$

11. Which one of the following represents statistical data?
    (a) The names of all owners of shops located in a shopping complex.
    (b) A list giving the names of of all states of India
    (c) A list of all European countries and their respective capital cities
    (d) The volume of a rainfall in certain geographical area, recorded every month for 24 consecutive months

12. The angle of elevation of the top of an unfinished pillar at a point 150 m from its base is 30°. If the angle of elevation at the same point to be 45°, then the pillar has to be raised to a height of how many metres?
    (a) 59.4 m (b) 61.4 m
    (c) 62.4 m (d) 63.4 m

13. A radio transmitter antenna of height 100 m stands at the top of a tall building. At a piont on the ground, the angle of elevation of bottom of the antenna is 45° and that of top of antenna is 60°. What is the height of the building?
    (a) 100 m (b) 50 m
    (c) $50(\sqrt{3}+1)$ m (d) $50(\sqrt{3}-1)$ m

14. What is the value of

$$\frac{5\sin 75^\circ \sin 77^\circ + 2\cos 13^\circ \cos 15^\circ}{\cos 15^\circ \sin 77^\circ} - \frac{7\sin 81^\circ}{\cos 9^\circ}?$$

    (a) – 1 (b) 0
    (c) 1 (d) 2

15. If $p = \tan^2 x + \cot^2 x$, the which one of the following is correct?
    (a) $p \le 2$ (b) $p \ge 2$
    (c) $p < 2$ (d) $p > 2$

16. If $0 \le x \le \pi/2$, then which one of the following is always correct?
    (a) $\sin^2 x < 1/2$ and $\cos^2 x > 1/2$
    (b) $\sin^2 x > 1/2$ and $\cos^2 x < 1/2$
    (c) $\sin^2 x < 1/2$ and $\cos^2 x < 1/2$
    (d) At least one of $\sin^2 x$, $\cos^2 x$ is less than 1

17. If $\frac{\cos x}{\cos y} = n, \frac{\sin x}{\sin y} = m,$ then what is $(m^2 - n^2)$ $\sin^2 y$ equal to?
    (a) $1 - n^2$ (b) $1 + n^2$
    (c) m^2 (d) n^2

18. If $x + y = 90°$ and $\sin x : \sin y = \sqrt{3} : 1,$ then what is $x : y$ equal to?
    (a) 1 : 1 (b) 1 : 2
    (c) 2 : 1 (d) 3 : 2

19. If $\frac{\cos x}{1+\cos ecx} + \frac{\cos x}{\cos ecx - 1} = 2,$ which one of the following is one of the values of x?
    (a) $\pi/2$ (b) $\pi/3$
    (c) $\pi/4$ (d) $\pi/6$

20. If $\tan^2 y \operatorname{cosec}^2 x - 1 - \tan^2 y$, then which one of the following is correct?
    (a) $x - y = 0$ (b) $x = 2y$
    (c) $y = 2x$ (d) $x - y = 1°$

21. If $\sin x \cos x = 1/2$, then what is the value of $\sin x - \cos x$?
    (a) 2 (b) 1
    (c) 0 (d) – 1

22. If $\cos x + \cos^2 x = 1$, then what is the value of $\sin^2 x + \sin^4 x$?
    (a) 0 (b) 1
    (c) 2 (d) 4

23. Consider the following equations
    1. $\operatorname{cosec}^2 x + \sec^2 x - \operatorname{cosec}^2 x \sec^2 x$
    2. $\sec^2 x + \tan^2 x = \sec^2 x \tan^2 x$
    3. $\operatorname{cosec}^2 x + \tan^2 x = \cot^2 x + \sec^2 x$

    Which of the above statements are correct?
    (a) 1 and 2 only (b) 2 and 3 only
    (c) 1 and 3 only (d) 1, 2 and 3

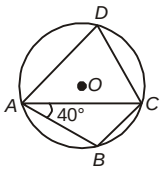
24. What is log (tan 1°) + log (tan 2°) + log (tan 3°) + ... + log (tan 89°) equal to?
    (a) 0 (b) 1
    (c) 2 (d) – 1

25. Let $\overrightarrow{AB}$ and $\overrightarrow{AC}$ be two rays intersecting at A. Let D, E be the point lying on $\overrightarrow{AB}, \overrightarrow{AC}$ respectively and P be the point such that P

divides the line DE such that $PD : PE = AD : AE$.

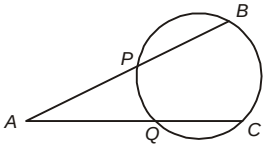
What is the locus of the point P?

(a) The angle bisector of angle A
(b) The angle trisector of angle A
(c) The perpendicular bisector of angle A
(d) None of the above

26. 

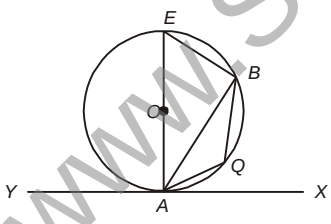
In the figure given above, O is the centre of a circle circumscribing a quadrilateral $ABCD$. If $AB = BC$ and $\angle BAC = 40°$, then what is $\angle ADC$ equal to?

(a) 50° (b) 60°
(c) 70° (d) 80°

27. 

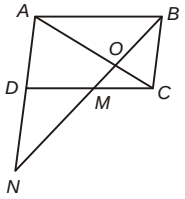
In the figure above, $AP = 3$ cm, $PB = 5$ cm, $AQ = 2$ cm and $QC = x$. What is the value of x ?

(a) 6 cm (b) 8 cm
(c) 10 cm (d) 12 cm

28. 

In the figure given above, YAX is a tangent to the circle with centre O. If $\angle BAX = 70°$ and $\angle BAQ = 40°$, then what is $\angle ABQ$ equal to?

(a) 20° (b) 30°
(c) 35° (d) 40°

29. 

In the figure given above, M is the mid point of the side CD of the parallelogram $ABCD$. What is $ON : OB$?

(a) 3 : 2 (b) 2 : 1
(c) 3 : 1 (d) 5 : 2

30. $ABCD$ is a concyclic quadrilateral. The tangents at A and C intersect each other at P. If $\angle ABC = 100°$, then what is $\angle APC$ equal to?

(a) 10° (b) 20°
(c) 30° (d) 40°

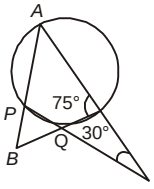
31. PQ is a common chord of two circles. APB is a secant line joining pionts A and B on the two circles. Two tangents AC and BC are drawn. If $\angle ACB = 45°$, then what is $\angle AQB$ euqal to?

(a) 75° (b) 90°
(c) 120° (d) 135°

32. A, B, C, D are four distinct points on a circle whose centre is at O.

If $\angle OBD - \angle CDB = \angle CBD - \angle ODB$, then what is $\angle A$ equal to?

(a) 45° (b) 60°
(c) 120° (d) 135°

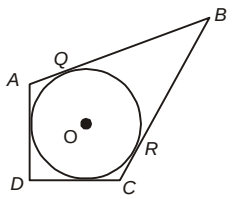
33. 

In the figure given above, what is $\angle CBA$?

(a) 30° (b) 45°
(c) 50° (d) 60°

34. In the figure given above, a circle is inscribed in a quadrilateral $ABCD$. Given that $BC = 38$ cm, $QB = 27$ cm, $DC = 25$ cm

and AD is perpendicular to DC. What is the radius of the circles?

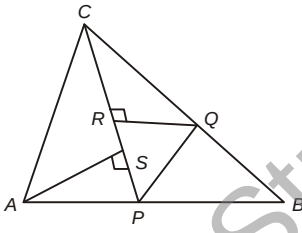


(a) 11 cm (b) 14 cm
(c) 15 cm (d) 16 cm

35. The cnetroid and the orthocenter are coincident for which one of the following triangles?
(a) Scalene triangle
(b) Isosceles triangle
(c) Equilateral triangle
(d) Right angled triangle

36. If the medians of two equilateral triangles are in the ratio 3 : 2, then what is the ratio of their sides?
(a) 1 : 1 (b) 2 : 3
(c) 3 : 2 (d) $\sqrt{3} : \sqrt{2}$

37.



In the figure given above, P is a point on AB and PQ is parallel to AC. What is the number of pairs of distinct similar triangles in the figure?
(a) 1 (b) 2
(c) 3 (d) 4

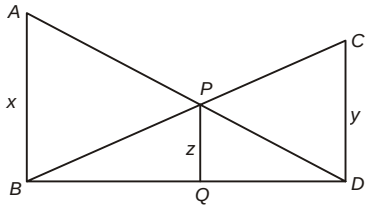
38. If C_1 and C_2 and r_1 and r_2 are respectively the centroids and radii of incircles of two congruent triangles, then which one of the following is correct?
(a) C_1 and C_2 are the same piont and $r_1 = r_2$
(b) C_1 and C_2 are not necessarily the same piont and $r_1 = r_2$
(c) C_1 and C_2 are the same point and r_1 is not necessarily equal to r_2
(d) C_1 and C_2 are not necessarily the same point and r_1 is not necessarily equal to r_2

39. A ladder 25 m long is leaing against a wall which is perpendicular to the level ground. The bottom of the ladder is 7 m from the base of the wall. If the top of the ladder slips down 4 m, how much will the bottom of the ladder slip?
(a) 7 m (b) 8 m (c) 10 m (d) 15 m

40. Which one of the following figures has only one line of symmetry?
(a) Rhombus (b) Rectangle
(c) Isosceles trapezium
(d) Parallelogram

41. ΔPQR is right angled at Q, $PR = 5$ cm and $QR = 4$ cm. If the lengths of sides of another triangle ABC are 3 cm, 4 cm and 5 cm, then which one of the following is correct?
(a) Area of ΔPQR is double that of ΔABC
(b) Area of ΔABC is double that of ΔPQR
(c) $\angle B = \dfrac{\angle Q}{2}$
(d) Both triangles are congruent

42.



In the figure given above

$\angle ABD = \angle PQD = \angle CDQ = \dfrac{\pi}{2}$. If $AB = x$, $PQ = z$ and $CD = y$, then which one of the following is correct?

(a) $\dfrac{1}{x}+\dfrac{1}{y}=\dfrac{1}{z}$ (b) $\dfrac{1}{x}+\dfrac{1}{z}=\dfrac{1}{y}$

(c) $\dfrac{1}{z}+\dfrac{1}{y}=\dfrac{1}{x}$ (d) $\dfrac{1}{x}+\dfrac{1}{y}=\dfrac{2}{z}$

43. What is the angle (in radian) included between the hands of a clock, when the time is 10 min past 5?
(a) $17\pi/36$ (b) $19\pi/36$
(c) $5\pi/9$ (d) $7\pi/12$

44. A lead pencil is in the shape of a cylinder. The pencil is 21 cm long with radius 0.4 cm and its lead is of radius 0.1 cm. What is the volume of wood in the pencil?
(a) 9 cm^3 (b) 9.4 cm^3
(c) 9.9 cm^3 (d) 10.1 cm^3

45. A cylindrical vessel of base radius 14 cm is filled with water to some height. If a rectangular solid of dimensions 22 cm × 7 cm × 5 cm is immersed in it, what is the rise in water level?
(a) 0.5 cm (b) 1.0 cm
(c) 1.25 cm (d) 1.5 cm

46. A cylinderical rod of length h is melted and cast into a cone of base radius twice that of the cylinder. What is the height of the cone?
(a) $3h/4$ (b) $4h/3$
(c) $2h$ (d) $h/2$

47. A roller of diameter 70 cm and length 2 m is rolloing on the ground. What is the area covered by the roller in 50 revolutions?
(a) 180 m^2 (b) 200 m^2
(c) 220 m^2 (d) 240 m^2

48. If A is the area of a triangle in cm^2, whose sides are 9 cm, 10 cm and 11 cm, then which one of the following is correct?
(a) $A < 40\ cm^2$
(b) $40\ cm^2 < A < 45\ cm^2$
(c) $45\ cm^2 < A < 50\ cm^2$
(d) $A > 50\ cm^2$

49. What is the area of right angled isosceles triangle whose hypotenuse is $6\sqrt{2}$ cm?
(a) 12 cm^2 (b) 18 cm^2
(c) 24 cm^2 (d) 36 cm^2

50. The diameter of the moon is approximately one-fourth of that of the earth. What is the (approximately) ratio of the volume of the moon to that of earth?
(a) 1/16 (b) 1/32
(c) 1/48 (d) 1/64

51. Three cubes each of side 5 cm are joined end to end. What is the surface area of the resulting cuboid?
(a) 300 cm^2 (b) 350 cm^2
(c) 375 cm^2 (d) 400 cm^2

52. From a rectangular metla sheet of sides 25 cm and 20 cm, a circular sheet as large as possible is cut-off. What is the area of the remaining sheet?
(a) 186 cm^2 (b) 144 cm^2
(c) 93 cm^2 (d) 72 cm^2

53. Two sides of a parallelogram are 10 cm and 15 cm. If the altitude corresponding to the side of length 15 cm is 5 cm, then what is the altitude to the side of length 10 cm?
(a) 5 cm (b) 7.5 cm
(c) 10 cm (d) 15 cm

54. A horse is tied to a pole fixed at one corner of a 50 m × 50 m square field of gass by means of a 20 m long rope. What is the area of that part of the field which the horse can graze?
(a) 1256 m^2 (b) 942 m^2
(c) 628 m^2 (d) 314 m^2

55. The difference between the area of a square and that of an equilateral triangle on the same base is $1/4\ cm^2$. What is length of side of triangle?
(a) $(4-\sqrt{3})^{1/2}$ cm (b) $(4+\sqrt{3})^{1/2}$ cm
(c) $(4-\sqrt{3})^{1/2}$ cm (d) $(4+\sqrt{3})^{1/2}$ cm

56. A man walking at the rate 3 km/h croses a square field diagonally in 1 min. What is the area of the field?
(a) 1000 m^2 (b) 1250 m^2
(c) 2500 m^2 (d) 5000 m^2

57. What is the greatest number which divides 392, 486 and 627 so as to leave the same remainder in each case?
(a) 47 (b) 43
(c) 37 (d) 34

58. What is the sum of positive integers less than 100 which leave a remainder 1 when divided by 3 and leave a remainder 2 when dividend by 4?
(a) 416 (b) 620
(c) 1250 (d) 1314

59. What is the total number of three digit numbers with unit digit 7 and divisible by 11?
(a) 6 (b) 7
(c) 8 (d) 9

60. There are some coins and rings of either gold or silver in a box, 60% of the objects are coins, 40% of the rings are of gold and 30% of the coins are of silver. What is the percentage of gold articles?
(a) 16 (b) 27
(c) 58 (d) 70

61. A train of length 150 m takes 10 s to cross anotgher train 100 m long coming from the opposite direction. If the speed of first train is 30 km / h. What is the speed of second train?
(a) 72 km / h (b) 60 km / h
(c) 54 km / h (d) 48 km / h

62. Suppose y is equal to the sum of two quantities of which one varies directly as x and the other inversely as x. If $y = 6$ when $x = 4$, and $y = 10/3$ when $x = 3$, then what is the relation between x and y?
(a) $y = x + (4/x)$ (b) $y = -2x + (4/x)$
(c) $y = 2x + (8/x)$ (d) $y = 2x - (8/x)$

63. If $\frac{1}{x+1}+\frac{2}{y+2}+\frac{1009}{z+1009}=1,$ then what is the value of $\frac{x}{x+1}+\frac{y}{y+2}+\frac{z}{z+1009}$?
(a) 0 (b) 2
(c) 3 (d) 4

64. If $(x/y) = (z/w)$, then what is $(xy + zw)^2$ equal to ?
(a) $(x^2 + z^2)(y^2 + w^2)$ (b) $x^2y^2 + z^2w^2$
(c) $x^2w^2 + y^2z^2$ (d) $(x^2 + w^2)(y^2 + z^2)$

65. Two taps can fill a tub in 5 min and 7 min respectively. A pipe can empty it in 3 min. If all the three are kept open simultaneously, when will the tube be full?
(a) 60 min (b) 85 min
(c) 90 min (d) 105 min

66. A bag contains Rs. 114 in the form of 1 rupees, 50 paise and 10 paise coins in the ratio 3 : 4 : 10. What is the number of 50 paisa coins?
(a) 76 (b) 72
(c) 56 (d) 48

67. If $a : b = 1\frac{1}{2} : 2\frac{1}{4}$ and $b : c = 2 : 3\frac{1}{2}$, then what is a: b: c equal to?
(a) 12 : 8 : 21 (b) 8 : 21 : 12
(c) 8 : 12 : 21 (d) 21 : 8 : 12

68. A person A sells a table costing Rs. 2000 to a person B and earns a profit of 6%. The person B sells it to another the person C at a loss of 5%. At what price did B sell the table?
(a) Rs. 2054 (b) Rs. 2050
(c) Rs. 2024 (d) Rs. 2014

69. What the seller marked the printed price of a watch purchased at Rs. 380, so that after giving 5% discount, there is 25% profit?
(a) Rs. 400 (b) Rs. 450
(c) Rs. 500 (d) Rs. 600

70. A person invested part of Rs. 45000 at 4% and the rest at 6%. If his annual income from both are equal, then what is the average rate of interest?
(a) 4.6% (b) 4.8%
(c) 5.0% (d) 5.2%

71. The compound interest on a sum for 2 yr is Rs. 832 and the simple interest on the same sum at the same rate for the same period is Rs. 800. What is the rate of interest?
(a) 6% (b) 8%
(c) 10% (d) 12%

72. A and B can do a piece of work in 8 days, B and C can do the same work in 12 days. If A, B and C can complete the same work in 6

days, in how many days can A and C complete the same work?
(a) 8 days (b) 10 days
(c) 12 days (d) 16 days

73. A boy walks from his house to school at 2.5 km / h and arrives 12 min late. The next day he walks at 4 km / h and reaches the school 15 min earlier. What is the distance from his house to school?
(a) 2 km (b) 2.5 km
(c) 3 km (d) 3.5 km

74. Two person P and Q start at the same time form city A for city B, 60 km away. P travels 4 km / h slower than Q. Q reaches city B and at once turns back meeting P , 12 km from city B. What is the speed of P?
(a) 8 km / h (b) 12 km / h
(c) 16 km / h (d) 20 km / h

75. What is the value of

$$\left(\frac{1}{\sqrt{9}-\sqrt{8}}-\frac{1}{\sqrt{8}-\sqrt{7}}+\frac{1}{\sqrt{7}-\sqrt{6}}-\frac{1}{\sqrt{6}-\sqrt{5}}+\frac{1}{\sqrt{5}-\sqrt{4}}\right)?$$

(a) 0 (b) 1 / 3
(c) 1 (d) 5

76. Nine numbers are written in ascending order. The middle number is the average of the nine numbers. The average of the first five larger numbers is 68 and that of five smaller numbers is 44. What is the sum of all numbers?
(a) 450 (b) 501
(c) 504 (d) 540

77. If we divivde a positive integer by another positive interger, what is the resulting number?
(a) It is always a natural number
(b) It is always an integer
(c) It is a rational number
(d) It is an irrational number

78. A dishonest dealer professes to sell his goods at cost price, but use a false weight and thus gains 20%. For a kilogram he uses a weight of
(a) 700 g (b) 750 g
(c) 800 g (d) 850 g

79. If the unit of weight is 15 / 4 kg. What number will 3 / 2 quintal represent?
(a) 25 (b) 6
(c) 1 / 9 (d) None of these

80. If $x \cos 60° + \cos 0° = 3$ and $4x \sin 30° - y \cot 45° = 2$, then what is the value of x?
(a) – 1 (b) 0
(c) 1 (d) 2

81. What is the value of 2 log (5 / 8) + log (128 / 125) + log (5 / 2)?
(a) 0 (b) 1
(c) 2 (d) 5

82. A ball is dropped from a height 64 m above the ground and every time if hits the ground it rises to a height equal to half of the previous. What is the height attained after it hits the ground for the 16th time?
(a) 2^{-12} m (b) 2^{-11} m
(c) 2^{-10} m (d) 2^{-9} m

83. If $a^x = c^q = b$ and $c^y = a^z = d$, then which one of the following is correct?
(a) $x/y = q/z$ (b) $x + y = q + z$
(c) $xy = qz$ (d) $x^y = q^z$

84. The product of two alternate odd integers exceeds three times the smaller by 12. What is the larger number?
(a) 3 (b) 5
(c) 7 (d) 9

85. A person bought 5 tickets from a station P to a station Q and 10 tickets from the station P to a station R. He paid Rs. 350. If the sum of a ticket from P to Q and a ticket from P to R is Rs. 42, then what is the fare from P to Q?
(a) Rs. 12 (b) Rs. 14
(c) Rs. 16 (d) Rs. 18

86. Pooja started her job with certain monthly salary and gets a fixed increment every year. If her salary was Rs. 4200 after 3 yr and Rs.

6800 aftter 8 yr of service, then what are her initial salary and the annual increment respectively?
(a) Rs. 2640, Rs. 320 (b) Rs. 2460, Rs. 320
(c) Rs. 2460, Rs. 520 (d) Rs. 2640, Rs. 520

87. What is the solution of the equations $x - y = 0.9$ and $11(x + y)^{-1} = 2$?
(a) $x = 32$ and $y = 2.3$ (b) $x = 1$ and $y = 0.1$
(c) $x = 2$ and $y = 1.1$ (d) $x = 1.2$ and $y = 0.3$

88. What is the magnitude of difference of the roots of $x^2 - ax + b = 0$?
(a) $\sqrt{a^2 - 4b}$ (b) $\sqrt{b^2 - 4a}$
(c) $2\sqrt{a^2 - 4b}$ (d) $\sqrt{b^2 - 4ab}$

89. If $x = 1 + \sqrt{2}$, then what is the value of $x^3 - 4x^3 + 4x^2$?
(a) – 1 (b) 0
(c) 1 (d) 2

90. If $3^x + 27(3^{-x}) = 12$, then what is the value of x?
(a) 1 only (b) 2 only
(c) 1 or 2 (d) 0 or 1

91. What is the LCM of $(x^2 - y^2 - z^2 - 2yz)$, $(x^2 - y^2 - x^2 + xz)$ and $(x^2 - y^2 - z^2 - 2xy)$?
(a) $(x + y + z)(x + y - z)(x - y + z)$
(b) $(x + y + z)(x - y - z)(x - y + z)$
(c) $(x + y + z)(x + y - z)(x - y - z)$
(d) $(x + y - z)(x - y - z)(x - y + z)$

92. If $(x + 2)$ is the HCF of $x^2 + ax + b$ and $x^2 + cx + d$ $(a \neq c$ and $b \neq d)$, then which one of the following is correct?
(a) $a + c = b + d$ (b) $2a + b = 2c + d$
(c) $b + 2c = 2a + d$ (d) $b - 2c = 2a - d$

93. What are the values of c when the HCF of $x^3 + cx^2 - x + 2c$ and $x^2 + cx - 2$ over the rationals is a linear polynomial?
(a) ± 1 (b) ± 2
(c) ± 3 (d) ± 4

94. When $(x^3 - 2x^2 + px - q)$ is divided by $(x^2 - 2x - 3)$, the remainder is $(x - 6)$. What are the values of p, q respectively?
(a) – 2, – 6 (b) 2, – 6
(c) – 2, 6 (d) 2, 6

95. If the remainder of the polynomial $a_0 + a_1x + a_2x^2 + \ldots\ldots\ldots, + a_nx^n$ when divivded by $(x - 1)$ is 1, then which one of the following is correct?
(a) $a_0 + a_2 + \ldots = a_1 + a_3 + \ldots$
(b) $a_0 + a_2 + \ldots = 1 + a_1 + a_3 + \ldots$
(c) $1 + a_0 + a_2 + \ldots - (a_1 + a_3 + \ldots)$
(d) $1 - a_0 - a_2 - \ldots = a_1 + a_3 + \ldots$

96. What is $x(y - z)(y + z) + y(z - x)(z + x) + z(x - y)(x + y)$ equal to?
(a) $(x + y)(y + z)(z + x)$
(b) $(x - y)(y + z)(z + x)$
(c) $(x + y)(z - y)(x - z)$
(d) $(y - x)(x - y)(x - z)$

97. Consider the following statements:
A number $a_1a_2a_3a_4a_5$ is divisible by 9 if
1. $a_1 + a_2 + a_3 + a_4 + a_5$ is divisible by 9.
2. $a_1 - a_2 + a_3 - a_4 + a_5$ is divisible by 9.
Which of the above statements is/are correct?
(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2

98. If $\log_r 6 = m$ and $\log_r 3 = n$, then what is $\log_r (r/2)$ equal to?
(a) $m - n + 1$ (b) $m + n - 1$
(c) $1 - m - n$ (d) $1 - m + n$

99. What is the last digit in the expansion of $(2457)^{754}$?
(a) 3 (b) 7
(c) 8 (d) 9

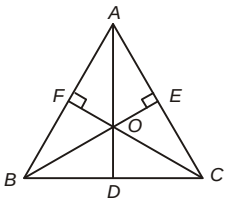
100. When a natural number n is divided by 4, the remainder is 3. What is the remainder when $2n$ is divided by 4?
(a) 1 (b) 2
(c) 3 (d) 6

ANSWERS

1. (b)	2. (c)	3. (d)	4. (c)	5. (a)	6. (c)	7. (b)	8. (b)	9. (d)	10. (d)
11. (d)	12. (d)	13. (c)	14. (b)	15. (b)	16. (d)	17. (a)	18. (c)	19. (c)	20. (a)
21. (c)	22. (b)	23. (c)	24. (a)	25. (a)	26. (d)	27. (c)	28. (b)	29. (b)	30. (b)
31. (d)	32. (b)	33. (d)	34. (b)	35. (c)	36. (c)	37. (c)	38. (a)	39. (b)	40. (c)
41. (d)	42. (a)	43. (b)	44. (c)	45. (c)	46. (a)	47. (c)	48. (b)	49. (b)	50. (d)
51. (b)	52. (a)	53. (b)	54. (d)	55. (c)	56. (b)	57. (a)	58. (a)	59. (c)	60. (d)
61. (b)	62. (d)	63. (b)	64. (a)	65. (d)	66. (a)	67. (c)	68. (d)	69. (a)	70. (c)
71. (b)	72. (a)	73. (c)	74. (a)	75. (d)	76. (c)	77. (c)	78. (c)	79. (d)	80. (d)
81. (a)	82. (c)	83. (c)	84. (c)	85. (b)	86. (d)	87. (a)	88. (a)	89. (c)	90. (c)
91. (b)	92. (c)	93. (a)	94. (c)	95. (d)	96. (b)	97. (a)	98. (d)	99. (d)	100. (b)

EXPLANATIONS

1. (A) we know that in a right angled triangle, ihypotenuse is a longest side.



In ΔABD,

$AB^2 > AD^2$ ...(i)

In $\Delta BEC, BC^2 > BE^2$ ...(ii)

In $\Delta ACF, AC^2 > CF^2$ ...(iii)

On adding Eqs. (i), (ii) and (iii), we get

$(AB^2 + BC^2 + AC^2) > (AD^2 + BE^2 + CF^2)$

(R) Now,

$(AE^2 - AF^2) + (BF^2 - BD^2) + (CD^2 - CE^2)$

$= (OA^2 - OE^2) - (OA^2 - OF^2)$

$+ (OB^2 - OF^2) - (OB^2 - OD^2)$

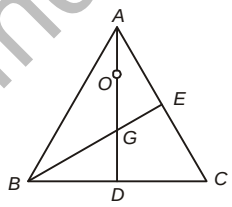
$+ (OC^2 - OD^2) - (OC^2 - OE^2)$

$= 0$

Hence, both (A) and (R) are individually true and (R) is not the correct explanation of (A).

2. Given, $AO : OD = 2 : 7$

We know that controid makes a ratio 2 : 1 on the media.



So, $AG = \frac{2}{9}AD, GD = \frac{1}{3}AD$ ... (i)

(A) $OA = \frac{2}{9}AD$

$OA = \left(2 \cdot \frac{1}{3}AD\right)\frac{1}{3}$

$= (2GD)\frac{1}{3}$ [from Eq. (i)]

$= \frac{2GD}{3}$

(R) $OD = \frac{7}{8}AD$

$= \left(7 \cdot \frac{2}{3}AD\right) \cdot \frac{1}{3 \times 2}$

$= \frac{7AG}{6}$

So, (A) is true but (R) is false.

3. (A) If two triangles have same perimeter, then it is not necessary that they have same area, so, they need not be congruent.
(R) This condition is true, becuase two triangles are congruent by (SSS) property.
Hence. option (d) is correct.

4. Given, $x+\frac{1}{x} = 2\cos\alpha$

$$\Rightarrow \quad x^2+\frac{1}{x^2}+2 = 4\cos^2\alpha$$

$$\Rightarrow \quad x^2+\frac{1}{x^2} = 2(2\cos^2\alpha-1)$$

$$= 2\cos^2\alpha - 2\sin^2\alpha$$

5. Given, $\text{cosec}^4\alpha - \cot^4\alpha = 17$
$\Rightarrow (\text{cosec}^2\alpha - \cot^2\alpha)(\text{cosec}^2\alpha \cot^2\alpha) = 17$

$$\Rightarrow \quad 1\cdot\left(\frac{1+\cos^2\alpha}{\sin^2\alpha}\right) = 17$$

$$\Rightarrow \quad 2-\sin^2\alpha = 17\sin^2\alpha$$

$$\Rightarrow \quad \sin^2\alpha = \frac{1}{9}$$

$$\Rightarrow \quad \sin^2\alpha = \frac{1}{3}$$

6. Because there is a gap between two adjacent bars so both the district can be represented by bar chart.

7. Sum of 25 observations = 25 × 36 = 900
Sum of first 13 observations = 13 × 32 = 416
Sum of last 13 observation = 13 × 39 = 507
Q 13th observation = 416 + 507 – 900
= 923 – 900
= 23

8. Collar sizes of 200 shirts sold in a week, is a suitable measure of central tendency.

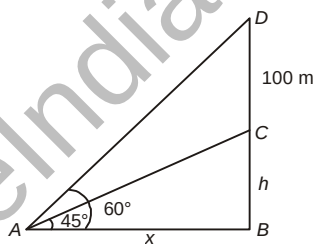
9. We know that arithmetic mean is independent of change of origin and scale.
∴ Required mean = 20 × 2 + 5
= 45

10. Given, six $x + \sin y = a$ and $\cos x + \cos y = b$
$\Rightarrow \sin^2 x + \sin^2 y + 2\sin x \sin y = a^2$
and $\cos^2 x + \cos^2 y + 2\cos x \cos y = b^2$
On adding Eqs. (i) and (ii), we get
$(\sin^2 x + \cos^2 x) + (\sin^2 y + \cos^2 y)$
$+ 2(\sin x \sin y + \cos x \cos y) = a^2 + b^2$

$$\Rightarrow (\sin x \sin y + \cos x \cos y) = \frac{a^2+b^2-2}{2}$$

11. Option (d) is correct.

13. Let BC be a building of height h metre and CD = 100 m be a height of antennna.



In ΔABC,

$$\tan 45° = \frac{h}{x} \Rightarrow x = h$$

And in ΔABD,

$$\tan 60° = \frac{100+h}{x}$$

$$\Rightarrow \quad \sqrt{3} = \frac{100+h}{h}$$

$$\Rightarrow \quad h = \frac{100}{(\sqrt{3}-1)} \times \frac{(\sqrt{3}+1)}{(\sqrt{3}+1)}$$

$$= 50(\sqrt{3}+1)\,\text{m}$$

14. $$\frac{5\sin 75°\sin 77° + 2\cos 13°\cos 15°}{\cos 15°\sin 77°} - \frac{7\sin 81°}{\cos 9°}$$

$$= \frac{5\cos 15°\sin 77° + 2\sin °\cos 15°}{\cos 15°\sin 77°} - \frac{7\cos 9°}{\cos 9°}$$

= 7 – 7
= 0

15. Given, $p = \tan^2 x + \cot^2 x$

At $x = \frac{\pi}{4}$

$p = 1 + 1 = 2$

Again at $x = \frac{\pi}{6}$

$p = \frac{1}{3} + 3 = \frac{10}{3} = 3.3$

$\Rightarrow \quad p > 2$

Hence, $p \geq 2$

16. For $\leq x + \frac{\pi}{2}$

$\cos^2 x$ and $\sin^2 x$ lies between 0 to 1.

Hence, option (d) is correct.

17. Given, $\frac{\cos x}{\cos y} = n, \frac{\sin x}{\sin y} = m$

Now, $(m^2 - n^2)\sin^2 y$

$= \left(\frac{\sin^2 x}{\sin^2 y} - \frac{\cos^2 x}{\cos^2 y}\right)\sin^2 y$

$= \frac{(1-\cos^2 x)\cos^2 y - \cos^2 x(1-\cos^2 y)}{\cos^2 y}$

$= \frac{\cos^2 y - \cos^2 x}{\cos^2 y} = 1 - n^2$

18. $\sin x : \sin y = \sqrt{3} : 1$

$= \frac{\sqrt{3}}{2} : \frac{1}{2}$

$= \sin 60° : \sin 30°$

$\therefore \quad x : y = 60 : 30$

$\Rightarrow \quad x : y = 2 : 1$

19. Give,

$\frac{\cos x}{1+\cos ecx} + \frac{\cos x}{\cos ec - 1} = 2$

$\Rightarrow \quad \frac{2\cos x \cos ecx}{\cos ec^2 x - 1} = 2$

$\Rightarrow \quad \frac{\cos x \cos ecx}{\cot^2 x} = 1$

$\Rightarrow \quad \tan x = 1$

$\Rightarrow \quad x = \frac{\pi}{4}$

20. Given, $\tan^2 y \operatorname{cosec}^2 x - 1 = \tan^2 y$

$\Rightarrow \quad \tan^2 y (\operatorname{cosec}^2 x - 1) = 1$

$\Rightarrow \quad \tan^2 y \cdot \cot^2 x = 1$

$\Rightarrow \quad x = y$

21. Now,

$(\sin x - \cos x)^2 = (\sin^2 x + \cos^2 x) - 2\sin x \cos x$

$= 1 - 2\left(\frac{1}{2}\right) \left(\text{Q } \sin x \cos x = \frac{1}{2}\right)$

$= 0$

$\Rightarrow \sin x - \cos x = 0$

22. Given, $\cos x + \cos^2 x = 1$

$\Rightarrow \quad \cos x = \sin^2 x$

On squaring both sides, we get

$\cos^2 x = \sin^4 x$

$\Rightarrow \quad 1 = \sin^2 x + \sin^4 x$

23. 1. LHS = $\operatorname{cosec}^2 x + \sec^2 x$

$= \frac{\cos^2 x + \sin^2 x}{\sin^2 x \cos^2 x}$

$= \operatorname{cosec}^2 x \sec^2 x = \text{RHS}$

2. LHS = $\sec^2 x + \tan^2 x$

$= \frac{1 + \sin^2 x}{\cos^2 x} \neq \text{RHS}$

3. LHS = $\operatorname{cosec}^2 x + \tan^2 x$

$= \cot^2 x + 1 + \tan^2 x$

$= \cot^2 x + \sec^2 x = \text{RHS}$

Hence, option (c) is correct.

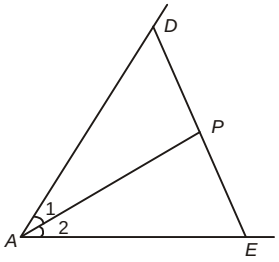
24. $\log(\tan 1°) + \log(\tan 2°) + \ldots + \log(\tan 89°)$

$= \log(\tan 1° \tan 2° \ldots \tan 45° \ldots \tan 88° \tan 89°)$

$= \log [(\tan 1° \cot 1°)(\tan 2° \cot 2°) \ldots \tan 45°(]$

$= \log (1.1 \ldots 1) = 0$

25. ΔDAP and ΔAPE are similar

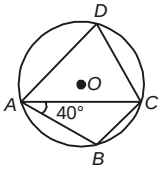


Since, $\frac{PD}{PE} = \frac{AD}{AE} = \frac{AP}{AP}$

AP is bisector of $\angle A$.

Hence, the locus of p is the triangle bisector of angle A.

26. Since, $AB = CB$



$\therefore \quad \angle CAB = \angle ACB = 40°$

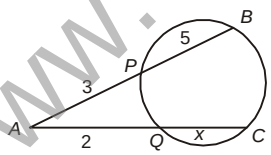
$\Rightarrow \quad \angle ABC = 180° - 2(40°) = 100$

We know that in cyclic quadrilateral, the sum of opposite angles are equal.

$\therefore \quad \angle B + \angle D = 180°$

$\angle D = 180° - 100° = 80°$

27. By theorem, we have



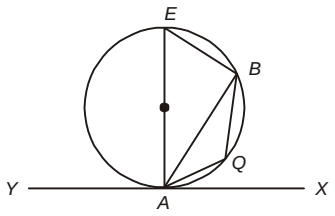
$AB \times AP = AC \times AQ$

$\Rightarrow \quad 8 \times 3 = (2 + x) \times 2$

$\Rightarrow \quad \frac{8 \times 3}{2} = 2 + x$

$\Rightarrow \quad x = 10 \text{ cm}$

28. $\angle QAX = 70° - 40° = 30°$



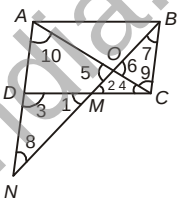
$\angle QAX = \angle AQB = 30°$ (by theorem)

29. In ΔDMN and ΔBMC,

$DM = MC$ (given)

$\angle 1 = \angle 2z$ (vertically opposite)

$\angle 3 = \angle 4$ (alternate interio angle)



$\Delta DMN \cong \Delta BMC (AS A)$

$DN = BC = AD$

So, $AN = 2BC \Rightarrow \frac{AN}{BC} = \frac{2}{1}$ ...(i)

In ΔOAN and ΔOBC,

$\angle 5 = \angle 6$ (vertically opposite)

$\angle 7 = \angle 8$ (alternate interior angle)

$\angle 9 = \angle 10$ (rest angle)

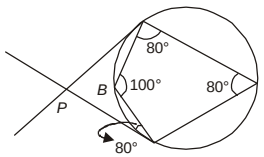
$\therefore \quad \Delta OAN \sim \Delta OBC$

So, the sides will be in same ratio

$$\frac{AN}{BC} = \frac{ON}{OB}$$

$\Rightarrow \quad \frac{2}{1} = \frac{ON}{OB}$ [from Eq. (i)]

30. We know that the sum of opposite angles of a concyclic quadrilateral is always 180°.



$\angle B + \angle D = 180°$

$\Rightarrow \quad 100 + \angle D = 180°$

$\Rightarrow \quad \angle D = 80°$

$\therefore \quad \angle ACP = \angle PAC = 80°$

(by theorem of alternate interior segment)

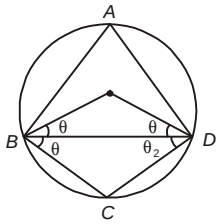
In ΔPAC,

$\angle P + \angle PAC + \angle PCA = 180°$

$\Rightarrow \quad \angle P + 80° + 80° = 180°$

$\Rightarrow \quad \angle P = 180° - 160° = 20°$

32. Given, $\angle OBD + \angle ODB = \angle CBD + \angle CDB$



Let $\angle OBD = \angle ODB$

and $\angle DBC = \theta_1, \angle BDC = \theta_2$

$\therefore \quad \theta + \theta_1 + \theta_2$ ...(i)

$20 = \theta_1 + \theta_2$

$\therefore \quad \angle BOD = 180° - 20$

$$\angle BCD = \frac{360° - (180 - 20)}{2}$$

$\Rightarrow \quad 180° - (\theta_1 + \theta_2) = 90° + \theta$

$\Rightarrow \quad 180° - 2\theta = 90° + \theta$

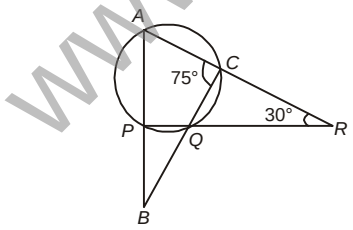
$\Rightarrow \quad 90° = 30$

$\Rightarrow \quad \theta = 30°$

$\therefore \quad \angle BOD = 120°$

$\angle BAD = 60°$

33. We know that the sum of opposite angles in a cyclic quadrilateral is always 180°.



$\angle ACQ + \angle APQ = 180°$

$\Rightarrow \quad 75° + \angle APQ = 180°$

$\Rightarrow \quad \angle APQ = 180° - 75°$

$\Rightarrow \quad \angle APQ = 105°$

Since, $\angle APQ + \angle BPQ = 180°$ (straight line)

$\therefore \quad 105 + \angle BPQ = 180°$

$\Rightarrow \quad \angle BPQ = 180° - 105°$

$= 75°$

Since, $\angle ACQ$ is outer angle of ΔRCQ,

$\therefore \quad \angle ACQ = \angle CRQ + \angle CQR$

$\Rightarrow \quad 75° = 30° + \angle CQR$

$\Rightarrow \quad \angle CQR = 45°$

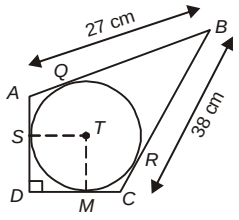
In ΔBPQ,

$\angle B + \angle P + \angle Q = 180°$

$\Rightarrow \angle B + 75° + 45° = 180°$

$\Rightarrow \quad \angle B = 60°$

34. We know that the tangents drawn to the circle from a point outside the circle are always equal.



$\therefore \quad BQ = BR = 27$

$\Rightarrow \quad RC = 38 - 27 = 11$ cm

$\therefore \quad RC = CM = 11$ cm

Now, $DM = 25 - 11$

$= 14$ cm

35. In an equilateral triangle, the centroid and the orthocentre are coincident.

36. Median of an equilateral triangle $= \frac{\sqrt{3}}{2}a$

According to the equation,

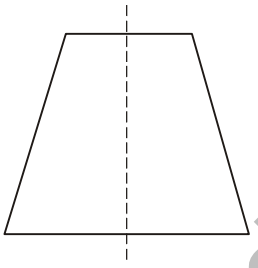
$$\frac{\frac{\sqrt{3}}{2}a_1}{\frac{\sqrt{3}}{2}a_2} = \frac{3}{2}$$

$\therefore \quad \frac{a_1}{a_2} = \frac{3}{2}$

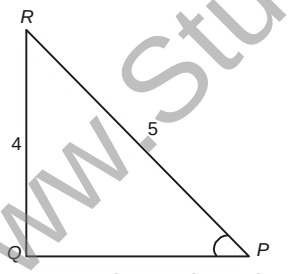
37. In a given figure, there are three distinct similar right angled triangles.
i.e., ΔPSA, ΔPRQ and ΔCRQ.

38. Since, in congruent triangles corresponding sides and angles of one triangle are equal to that of other triangle. So, there medians also will be equal and interest at the same point so $C_1 = C_2$.
Since, both the tringles have equal angles so there angle bisectors, will be equal and intersect at the same point. Hence, the radii of incircles will be equal
So $r_1 = r_2$

40. Isosceles trapezium has only one line os symmetry.



41. In right angle ΔPQR,



$QP^2 = (5)^2 - (4)^2$
$= 9$
$\Rightarrow \quad QP = 3$
In second triangle ABC whose sides are 3 cm, 4 cm and 5 cm. So, the sides of both triangle are same, hence they are congrunent.

43. In 60 min hour hand gains = 5 min
In 1 min hour hand gains $= \frac{5}{60}$ min
In 10 min hour hand gains $= \frac{5}{60} \times 10$
$= \frac{5}{6}$ min
There is 15 min gap between hours and minutes hands but in 10 min hour and gains $\frac{5}{6}$ min more. So, the actual gap
$= 15 + \frac{5}{6} = \frac{95}{6}$ min
In 1 min there are 6°
In $\frac{95}{6}$ min there are $\frac{95}{6} \times 6°$
$= \frac{95}{6} \times 6 \times \frac{\pi}{180} = \frac{19\pi}{36}$ radian

44. Volume of wood = Volume of lead pencil – Volume of lead
$= \pi (0.4)^2\, 21 - \pi (0.1)^2 \times 21$
$= 21 \times \frac{22}{7}(0.16 - 0.01)$
$= 66\ (0.15)$
$= 9.9\ \text{cm}^3$

45. Volume of solid $= l \times b \times h$
$= 22 \times 7 \times 5 = 770\ \text{cm}^3$
Let the water rise in height $= h$
Q Volume of water rise in vessel
= Volume of solid $\pi r^2 h = 770$
$\Rightarrow \frac{22}{7} \times 14 \times 14 \times h = 770$
$\Rightarrow \quad h = \frac{770 \times 7}{22 \times 14 \times 14}$
$= \frac{5}{4} = 1.25$ cm

46. Let the radius of cylinder be r and radius of cone be R.

$\therefore R = 2r$ (given)

Also, volume of cylinder = Volume of cone

$$\therefore \quad \pi r^2 h = \frac{1}{3}\pi R^2 H$$

$$\Rightarrow \quad r^2 h = \frac{1}{3}\pi (2r)^2 H$$

$$\Rightarrow \quad H = \frac{3h}{4}$$

47. The radius of roller is 0.35 m.

The area covered in one revolution = Curved surface area of roller

$$= 2 \times \frac{22}{7} \times 0.35 \times 2$$

$$= 4.4 \text{ m}^2$$

$\therefore$ Total area covered in 50 revolutions

$$= 4.4 \times 50$$

$$= 220 \text{ m}^2$$

48. Let $a = 9$ cm, $b = 10$ cm and $c = 11$ cm

$$\therefore \quad s = \frac{9+10+11}{2} = 15 \text{ xm}$$

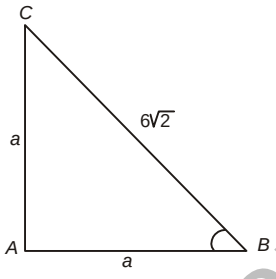
$$\therefore \quad A = \sqrt{s(s-a)(s-b)s-c)}$$

$$= \sqrt{15(15-9)(15-10)(15-11)} = 42.3 \text{ cm}^2$$

Hence, option (b) is correct.

49. Let the other sides of a right isosceles triangle be a cm.

In ΔABC



$$\Rightarrow \quad \sqrt{a^2 + a^2} = 6\sqrt{2}$$

$$a\sqrt{2} = 6\sqrt{2}$$

$$\Rightarrow \quad a = 6$$

$$\therefore \quad \text{Area of } \Delta ABC = \frac{1}{2} \times a^2$$

$$= \frac{1}{2} \times 6 \times 6 = 18 \text{ cm}^2$$

50. $$\frac{\text{Volume of moon}}{\text{Volume of earth}} = \frac{\frac{4}{3}\pi\left(\frac{x}{8}\right)^3}{\frac{4}{3}\pi\left(\frac{x}{2}\right)^3}$$

$$= \frac{1}{64}$$

51. When we join three cubes the length of the cuboid becomes 15 cm and height, width remains the 5 cm each.

$\therefore$ Surface area of cuboid = 2 ($lb + bh + hl$)

$= 2(15 \times 5 + 5 \times 5 + 15 \times 5)$

$= 2\ (75 + 25 + 75)$

$= 350 \text{ cm}^2$

52. Here width of sheet is 20 cm, which is the maximum diameter of the circular sheet.

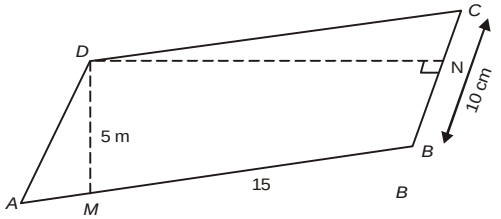
$\therefore$ Remaining area of sheet = Area of rectangle sheet – Area of circular sheet

$$= 25 \times 20 - \pi\ (10)^2$$

$$= 500 - 314$$

$$= 186 \text{ cm}^2$$

53. Area of parallelogram = base × height



Area of parallelogram = base × height

$$= 10 \times DN$$

$$\therefore \quad 10 \times DN = 75$$

$$\Rightarrow \quad DN = \frac{75}{10}$$
$$= 7.5 \text{ cm}$$

55. Let the side of an square be a cm.
According to the question,
Area of square – Area of an equilateral triangle
$$= \frac{1}{4}$$
$$\Rightarrow \quad a^2 - \frac{\sqrt{3}}{4}a^2 = \frac{1}{4}$$
$$\Rightarrow \quad a^2(4-\sqrt{3}) = 1$$
$$\Rightarrow \quad a = (4-\sqrt{3})^{-1/2} \text{ cm}$$

56. Since, the distance covered by a man diagonally is
$$d = \frac{3\times1000}{60}\times1 = 50 \text{ m}$$
$$\therefore \text{ Area of field} = \frac{1}{2}d^2 = \frac{1}{2}\times(50)^2 = 1250 \text{ m}^2$$

57. When we divide the given numbers by 47 it will give the remainder 16 in each case.

58. Required numbers are
10, 22, 34, 46, 58, 70, 82, 94
$\therefore$ Total sum is 416.

59. The total number of three digit numbers with unit digit7 and divisible by 11 are
187, 297, 407, 517, 627, 737, 847, 957

60. Coin in object = 60
Gold ring = 40
Coins of sivler = 30
$\therefore$ Gold coins = 60 – 30 = 30
Total gold articles percentage of
= 30 + 40
= 70

61. Time taken to cross the trains $= \frac{150+100}{\frac{25}{3}+x}$
where x is the speed of second train.
$$\Rightarrow \quad 10 = \frac{250\times3}{25+3x}$$
$$\Rightarrow \quad 250 + 50x = 750$$
$$\Rightarrow \quad 30x = 500$$
$$\Rightarrow \quad x = \frac{50}{3} \text{ m/s}$$
$$\therefore \quad x = \frac{50}{3}\times\frac{18}{5}$$
$$= 60 \text{ km/h}$$

62. According to the question,
$$y = lx + \frac{m}{x} \qquad \text{...(i)}$$
where l and m are proportionally constant.
When $y = 6$, $x = 4$, then
$$6 = 4l + \frac{m}{4}$$
$$\Rightarrow \quad 16l + m = 24 \qquad \text{...(ii)}$$
When $y = \frac{10}{3}$, $x = 3$, then
$$\frac{10}{3} = 3l + \frac{m}{3}$$
$$\Rightarrow \quad 9l + m = 10 \qquad \text{... (iii)}$$
On solving Eqs. (ii) and (iii), we get
$l = 2$, $m = 8$
From Eq. (i),
$$y = 2x - \frac{8}{x}$$

63. Given, $\frac{1}{x+1} + \frac{2}{y+2} \mid \frac{1009}{z+1009} = 1$
$$\Rightarrow \frac{1}{x+1} - 1 + \frac{2}{y+2} - 1 + \frac{1009}{z+1009} - 1 = 1 - 3$$
$$\Rightarrow -\frac{x}{x+1} - \frac{y}{y+2} - \frac{z}{z+1009} = -2$$

$$\Rightarrow \frac{x}{x+1}+\frac{y}{y+2}+\frac{z}{z+1009} = 2$$

64. Given, $\frac{x}{y} = \frac{z}{w} \Rightarrow xw = yz$

Now, $(xy + zw)^2 = x^2y^2 + z^2w^2 + 2\,(xy\ zw)$
$= x^2y^2 + z^2w^2 + 2\,(yz\ .\ yz)$
$= x^2y^2 + y^2z^2 + z^2w^2 + y^2z^2$
$= y^2\,(x^2 + z^2) + z^2w^2 + x^2w^2$
$= y^2\,(x^2 + z^2) + w^2\,(x^2 + z^2)$
$= (x^2 + z^2)\,(y^2 + w^2)$

65. Net filling in 1 min

$$= \frac{1}{5}+\frac{1}{7}-\frac{1}{3}$$

$$= \frac{21+15-35}{105}$$

$$= \frac{1}{105}$$

Hence, the tub will be full in 105 min.

66. Ratio of values of the coins $= \frac{3}{1}:\frac{4}{2}:\frac{10}{10}$

$= 30 : 20 : 10$

Values of 50 paise coin $= 114\times\frac{20}{60}$ = Rs. 38

Number of 50 paise coins = 38 × 2 = 76

67. $a : b = \frac{3}{2}:\frac{9}{4}$ and $b : c = 2 : \frac{7}{2}$

$\Rightarrow a : b = 6 : 9$ and $b : c = 4 : 7$
$\Rightarrow a : b = 2 : 3$ and $c = 12 : 21$
$\Rightarrow a : b : c = 8 : 12 : 21$

68. The cost price of table for person

$$B = 2000 + 6 \times \frac{2000}{100}$$

$= 2000 + 120 =$ Rs. 2120

Its selling price $= 2120-\frac{2120\times 5}{100}$

$= 2120 - 106$
$=$ Rs. 2014

69. Let marked price be Rs. x.

$\therefore$ Discount = 5% of x

$$= \frac{5}{100}\times x = \frac{x}{20}$$

Since, SP = MP – Discount

$$\Rightarrow \quad 380 = x-\frac{x}{20}$$

$$\Rightarrow \quad 380 = \frac{19x}{20}$$

$\Rightarrow \quad x =$ Rs. 400

70. The average rate of interest $= \left(\frac{4+6}{2}\right)\%$

$= 5\%$

71. Since, $CI = P\left\{\left(1+\frac{R}{100}\right)^n - 1\right\}$

$$\Rightarrow \quad 832 = P\left\{\left(1+\frac{R}{100}\right)^2 - 1\right\} \ldots \text{(i)}$$

Also, $SI = \frac{P\times R\times T}{100}$

$$\Rightarrow \quad 800 = \frac{P\times R\times 2}{100}$$

$$\Rightarrow \quad P = \frac{4000}{R}$$

$\therefore$ From Eq. (i),

$$832 = \frac{40000}{R}\left(\frac{R^2}{10000}+\frac{2R}{100}\right)$$

$$832 = 4R + 800$$

$$\Rightarrow \quad R = \frac{32}{4} = 8\%$$

72. Number of days taken by A and B to do the work = 8

Number of days takenby B and C to the work = 12

A's and B's one day work = $\frac{1}{8}$

B's and C's one day work = $\frac{1}{12}$

Number of days taken by A, B and C to do work = 6

A's, B's and C's one day work = $\frac{1}{6}$

B's one day work = $\frac{1}{8}+\frac{1}{12}-\frac{1}{6}$

A's and C's one day work = $\frac{1}{6}-\frac{1}{24}$

$$= \frac{3}{24} = \frac{1}{8}$$

Hence, A and C can do the work in 8 days.

73. Let the usual speed of boy was x km / h and his usual time was t hour.

According to the question,

$$d = \left(t+\frac{12}{60}\right)\times 2.5 \quad ...(i)$$

and $$d = \left(t-\frac{15}{60}\right)\times 4 \quad ... (ii)$$

On dividing Eq. (i) by Eq. (ii) we get

$$\frac{1}{1} = \frac{\left(t+\frac{1}{5}\right)\times 2.5}{\left(t-\frac{1}{4}\right)\times 4}$$

$$\Rightarrow \quad 8\left(t-\frac{1}{4}\right) = 5\left(t\frac{1}{5}\right)$$

$$\Rightarrow \quad 8t-2 = 5t+l$$

$$\Rightarrow \quad 3t = 3 \Rightarrow t = 1$$

On putting the value of t in Eq. (ii), we get

$$d = \left(1-\frac{15}{60}\right)\times 4$$

$$= \frac{3}{4}\times 4 = 3 \text{ km}$$

74. P ⟷ Q

City A 60 km City B

Distance between P and Q = 60 km

Let the speed of $Q = x$ km / h

Distance travelled by Q till they meet = 60 + 12 = 72 km

Time taken by Q till they meet = $\frac{72}{x}$

Distance travelled by P till they meet = 60 – 12 = 48 km

Time taken by P till they meet = $\frac{48}{x-4}$

Since, time taken by both will be equal.

$$\therefore \quad \frac{72}{x} = \frac{48}{x-4}$$

$$\Rightarrow \quad 72x-288 = 48x$$

$$\Rightarrow \quad 24x = 288$$

$$\Rightarrow \quad x = 12$$

Sped of $P = x-4$

$= 12-4 = 8$ km / h

75. $\left(\frac{1}{\sqrt{9}-\sqrt{8}}-\frac{1}{\sqrt{8}-\sqrt{7}}+\frac{1}{\sqrt{7}-\sqrt{6}}\right.$

$$\left.-\frac{1}{\sqrt{6}-\sqrt{5}}+\frac{1}{\sqrt{5}-\sqrt{4}}\right)$$

$$= \left(\frac{\sqrt{9}+\sqrt{8}}{9-8}-\frac{\sqrt{8}-\sqrt{7}}{8-7}+\frac{\sqrt{7}+\sqrt{6}}{7-6}\right.$$

$$\left.\frac{\sqrt{6}+\sqrt{5}}{6-5}+\frac{\sqrt{5}+\sqrt{4}}{5-4}\right)$$

$= \sqrt{9}+\sqrt{4} = 3+2 = 5$

76. Since, 5th temr = Average of 9 number = x

Sum of first five larger numbers = 68×5 = 340

Sum of first five smaller numbers = 44×5 = 220

Sum of first five smaller numbers = 44×5 = 220

Average of 5 numbers = $\frac{340 + 220 - \bar{x}}{9}$

($\therefore \bar{x}$ is subtracted because 5th term repeated twice)

$$\bar{x} = \frac{560 - \bar{x}}{9}$$

$$\Rightarrow \quad 9\bar{x} + \bar{x} = 560$$

$$\Rightarrow \quad \bar{x} = 56$$

$\therefore$ Sum of 9 numbers = $56 \times 9 = 504$

77. When we divide a positive integer by another positive integear, the resultant will be a rational number.

78. Let SP = CP = Rs. x

$$\text{Gain percentage} = x \times \frac{20}{100} = \frac{x}{5}$$

$$\text{Gain weight} = \frac{x}{5} \times 1000 = 200 \text{ g}$$

Hence, the uses the weight 800 g.

79. 1 quintal = 100 kg

$$\frac{3}{2} \text{ quintal} = 100 \times \frac{3}{2} = 150 \text{ kg}$$

Now, given $\frac{15}{4}$ kg = 1 unit

$$1 \text{ kg} = \frac{4}{15} \text{ unit}$$

$$150 \text{ kg} = \frac{4}{15} \times 150 = 40 \text{ unit}$$

80. Given, $x \cos 60° + y \cos 0° = 3$

$$\Rightarrow \quad \frac{x}{2} + y = 3$$

$$\Rightarrow \quad x + 2y = 6 \qquad \text{...(i)}$$

and $4x \sin 30° - y \cot 45° = 2$

$$\Rightarrow \quad 4x \times \frac{1}{2} - y \cdot 1 = 2$$

$$\Rightarrow \quad 2x - y = 2 \qquad \text{... (ii)}$$

On solving Eqs. (i) and (ii), we get

$$x = y = 2$$

81. $2 \log\left(\frac{5}{8}\right) + \log\left(\frac{128}{125}\right) + \log\left(\frac{5}{2}\right)$

$= (2 \log 5 - 6 \log 2)(7 \log 2 - 3 \log 5) + \log 5 - \log 2$

$= 3 \log 5 - 3 \log 5 - 7 \log 2 + 7 \log 2$

$= 0$

83. Given $a^x = c^q = b$ and $c^y = a^z = d$

$\Rightarrow a = b^{1/x}$ and $c = b^{1/q}$

$\therefore b^{y/q} = b^{z/x} = d$

$$\Rightarrow \quad \frac{y}{q} = \frac{z}{x}$$

$$\Rightarrow \quad xy = zq$$

84. Let the first odd number be x and the alternate odd number is $x + 4$.

Since, $x(x + 4) = 3x + 12$

$$\Rightarrow \quad x^2 + 4x = 3x + 12$$

$$\Rightarrow \quad x^2 + x - 12 = 0$$

$$\Rightarrow \quad (x + 4)(x - 3) = 0$$

$$\Rightarrow \quad x = 3, \qquad (\text{Q } x \neq -4)$$

Hence, larger number is $x + 4 = 7$

85. Let the fare from station P to station Q is Rs x and the fare from station P to station R is Rs. y.

According to the question,

$$x + y = 42 \qquad \text{... (i)}$$

and $5x + 10y = 350$ ... (ii)

on solving equ. (i) & (ii), we get

$x = 14$, $y = 28$

Hence, fare from station P to station Q is Rs. 14.

86. Let Pooja initial salary is Rs x and fixed increment every year is Rs. y.

According to the question,

$x + 3y = 4200$ ... (i)

and $x + 8y = 6800$ ... (ii)

on solving equ. (i) & (ii), we get

x = Rs. 2640 and y = Rs. 520

87. Given, $x - y = 0.9$

and $11(x + y)^{-1} = 2$ ... (i)

$\Rightarrow 2x + 2y = 11$

On multiplying Eq. (i) by 2 and adding Eqs. (i) and (ii), we get

$4x = 12.8$

$\Rightarrow x = 3.2$

From Eq. (i), $y = 3.2 - 0.9$

$= 2.3$

88. Let the roots of the given equation $x^2 - ax + b = 0$ be α and β.

$\therefore |a + b| = a$ and $\alpha\beta = b$

Now, $\alpha - \beta = \sqrt{(\alpha-\beta)^2 - 4\alpha\beta}$

$= \sqrt{a^2 - 4b}$

89. Given, $x = 1+\sqrt{2}$

$\therefore x^4 - 4x^3 + 4x^2 = x^2(x^2 - 4x + 4)$

$= x^2(x-2)^2$

$= (1+\sqrt{2})^2(1+\sqrt{2}-2)^2$

$= (1+\sqrt{2})^2(\sqrt{2}-1)^2$

$= (2-1)^2 = 1$

91. LCM of

$[\{x^2 - (y+z)^2\}, \{(x+z)^2 - y^2\}, \{(x-y)^2 - z^2\}]$

= LCM of $[(x + y + z)(x - y - z), (x + z + y)(x + z - y), (x - y + z)(x - y - z)]$

$= (x + y + z)(x - y - z)(x - y + z)$

92. Let $f(x) = x^2 + ax + b$ and $g(x) = x^2 + cx + d$

$\therefore f(-2) = 4 - 2a + b = 0 \Rightarrow b - 2a = -4$

and $g(-2) = 4 - 2c + d = 0 \Rightarrow d - 2c = -4$

$\therefore b - 2a = d - 2c \Rightarrow b + 2c = d + 2a$

93. Let $f(x) = x^3 + cx^2 - x + 2c$ and $g(x) = x^2 + cx - 2$

$\therefore f(x) = x^3 + x^2 - x + 2$ $g(x) = x^2 + x - 2$

$= (x + 2)(x^2 - x + 1) = (x + 2)(x - 1)$

Here $(x + 2)$ is HCF of $f(x)$ and $g(x)$ which is linear.

95. Let $f(x) = a_0 + a_1 + a_2x^2 + ... + a_nx^n$

$\therefore f(1) = a_0 + a_1 + a_2 + ... + a_n$

$\Rightarrow 1 = a_0 + a_1 + a_2 + ... + a_n$

$\Rightarrow 1 - a_0 - a_2 - ... = a_1 + a_3 + ...$

96. $x(y - z)(y + z) + y(z - x)(z + x) + z(x - y)(x + y)$

$= x(y^2 - z^2) + y(z^2 - x^2) + z(x^2 - y^2)$

$= x(y^2 - z^2) + yz^2 - yx^2 + zx^2 - zy^2$

$= x(y - z)(y + z) + x^2(z - y) + yz(z - y)$

$= (y - z)(xy + xz - x^2 - yz)$

$= (y - z)[y(x - z) + x(z - x)]$

$= (y - z)(z - x)(x - y)$

$= (x - y)(x - z)(z - y)$

97. As we know that a number $a_1\,a_2\,a_3\,a_4\,a_5$ is divisible by 9 if sum of the digit *i.e.*, $a_1 + a_2 + a_3 + a_4 + a_5$ is divisible by 9.

Hence, only statement (1) is true.

98. Given, $\log_r 6 = m \log_r 3 = n$

$\therefore \log_r 3 + \log_r 2 = m$

$\Rightarrow n + \log_r 2 = m \Rightarrow \log_r 2 = m - n$

$\therefore \log_r\left(\frac{r}{2}\right) = \log_r = r - \log_r 2$

$= 1 - m + n$

99. The last digit in the expansion of $(2457)^{754}$ is depend on $(7)^{754}$

$= (7)^2 = 49$

Hence, last digit is 9.

100. Suppose $n = 15$, when we divide by 4 it will give a remainder 3.

Now, $2n = 30$, when we divide by 4 it will give a remainder 2.