ELEMENTARY MATHAMATICS

1. Let $WXYZ$ be a square. Let P, Q, R be the midpoints of WX, XY and ZW respectively and K, L be the midpionts of PQ and PR respectively. What is the value of $\frac{\text{area of triangle } PKL}{\text{area of square } WXYZ}$?

   (a) $\frac{1}{32}$ (b) $\frac{1}{16}$

   (c) $\frac{1}{8}$ (d) $\frac{1}{64}$

2. Suppose the angle of elevation of the top of a tree at a point E due east of the tree is 60° and that at a point F due west of the tree is 30°. If the distance between the pionts E and F is 160 feet, then what is the height of the tree?

   (a) $40\sqrt{3}$ feet (b) 60 feet

   (c) $\frac{40}{\sqrt{3}}$ feet (d) 23 feet

3. If α is an acure angle and $\sin\alpha = \sqrt{\frac{x-1}{2x}}$, then what is $\tan\alpha$ equal to?

   (a) $\sqrt{\frac{x-1}{x+1}}$ (b) $\sqrt{\frac{x+1}{x-1}}$

   (c) $\sqrt{x^2-1}$ (d) $\sqrt{x^2+1}$

4. Consider the following data:

x	1	2	3	4 5
f	3	5	9	– 2

   The arithmetic mean of the above distribution is 2.96. What is the missing frequency?

   (a) 4 (b) 6

   (c) 7 (d) 8

5. What is the value of $\sin^2 15° + \sin^2 20° + \sin^2 25° + \ldots\ldots + \sin^2 75°$?

   (a) $\tan^2 15° + \tan^2 20° + \tan^2 25° + \ldots \tan^2 75°$

   (b) $\cos^2 15° + \cos^2 20° + \cos^2 25° + \ldots + \cos^2 75°$

   (c) $\cot^2 15° - \cot^2 20° + \cot^2 25° - \ldots + \cot^2 75°$

   (d) $\sec^2 15° + \sec^2 20° + \sec^2 25° + \ldots + \sec^2 75°$

6. For what value of k, will the roots of the euqation $kx^2 - 5x + 6 = 0$ be in the ratio of 2 : 3 ?

   (a) 0 (b) 1

   (c) – 1 (d) 2

7. ABC is a right triangle with right angle at A. If the value of $\tan B = \frac{1}{\sqrt{3}}$, then for any real k the length of the hypotenuse is of the form

   (a) $3k$ (b) $2k$

   (c) $5k$ (d) $9k$

8. Which one of the following is an infinite set?

   (a) $\{x : x$ is a whole number less than or equal to 1000$\}$

   (b) $\{x : x$ is a natural number less than 1000$\}$

   (c) $\{x : x$ is a positive integer less than or equal to 1000$\}$

   (d) $\{x : x$ is an integer and less than 1000$\}$

9. If ABC is a triangle right angled at C and having u units, v units, w units as the lengths of its sides opposite to the vertices A, B, C respectively, then what is $\tan A + \tan B$ equal to?

   (a) $\frac{u^2}{vw}$ (b) 1

   (c) $u + v$ (d) $\frac{w^2}{uv}$

10. For any integer n, what is the HCF of integers $m = 2n + 1$ and $k = 9n + 4$?

    (a) 3 (b) 1

    (c) 2 (d) 4

11. If a quantity y varies as the sum of three quantities of which the first varies as x, the second varies as $-x + x^2$, the third varies as $x^3 - x^2$, then what is y equal to?
(a) kx^3 where k is a constant
(b) $kx + lx^2 + mx^3$, where k, l, m are constants
(c) kx^2 where k is a constant
(d) kx where k is a constant

12. The wages of labourers in a factory has increased in the ratio 22 : 23 : 25 and their number decreased in the ratio 3 :2. What was the original wage bill of the factory if the present bill is 5000?
(a) Rs. 4000 (b) Rs. 6000
(c) Rs. 8000 (d) None of these

13. If 50% of $(x - y)$ = 40% of $(x + y)$ then what per cent of x is y?
(a) $10\frac{1}{9}\%$ (b) $11\frac{1}{9}\%$
(c) $13\frac{1}{9}\%$ (d) $21\frac{1}{9}\%$

14. If 6 men and 8 boys can do a piece of work in 10 days while 26 men and 48 boys can do the same in 2 days, what is the time taken by 15 men and 20 boys in doing the same type of work?
(a) 4 days (b) 5 days
(c) 6 days (d) 7 days

15. If $0 \le \theta \le \frac{\pi}{2}$ and $\cos\theta + \sqrt{3}\sin\theta = 2$, then what is the value of θ?
(a) $\frac{\pi}{3}$ (b) $\frac{\pi}{4}$ (c) $\frac{\pi}{6}$ (d) $\frac{\pi}{2}$

(For the next three questions to follow):
The arithmetic mean, geometric mean and median of 06 positive numbers a, a, b, a, b, c, c where a < b < c are 7 / 3, ,2,,2

16. What is the mode?
(a) 1 (b) 2
(c) 1, 2 and 4 (d) No mode

17. What is the value of c?
(a) 1 (b) 2 (c) 3 (d) 4

18. What is the sum of the squares of all the six numbers?
(a) 40 (b) 42
(c) 45 (d) 48

19. Let there be three simultaneous linear equations in two unknowns. What can be the number of solutions (if they do exist)?
(a) One or infinite (b) Only one
(c) Exactly two (d) Exactly three

20. The sum of roots of the equation $\frac{1}{x+a} + \frac{1}{x+b} = \frac{1}{c}$ is zero. What is the product of the roots of the equation?
(a) $-\frac{(a+b)}{2}$ (b) $\frac{(a+b)}{2}$
(c) $-\frac{(a^2+b^2)}{2}$ (d) $\frac{(a^2+b^2)}{2}$

21. What are the roots of the equation $\log_{10}(x^2 - 6x + 45) = 2$?
(a) 9, – 5 (b) – 9, 5
(c) 11, – 5 (d) – 11, 5

22. When the roots of the quadratic equation $ax^2 + bx + c = 0$ are negative of reciprocals of each other, then which one of the following is correct?
(a) $b = 0$ (b) $c = 0$
(c) $c = a$ (d) $a = -c$

23. In a triangle ABC, $\angle ABC = 90°$, $\angle ACB = 30°$, $AB = 5$ cm. What is the length of AC?
(a) 10 cm (b) 5 cm
(c) $5\sqrt{2}$ cm (d) $5\sqrt{3}$ cm

24. If $0 \le \theta < \frac{\pi}{2}$ and $p = \sec^2\theta$, then which one of the following is correct?
(a) $p < 1$ (b) $p = 1$
(c) $p > 1$ (d) $p \ge 1$

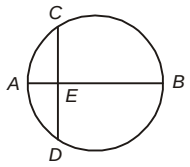
25. Consider the following statements
1. The HCF of $x + y$ and $x^{10} - y^{10}$ is $x + y$.
2. The HCF of $x + y$ and $x^{10} + y^{10}$ is $x + y$.
3. The HCF of $x - y$ and $x^{10} + y^{10}$ is $x - y$.
4. The HCF of $x - y$ and $x^{10} - y^{10}$ is $x - y$.

Which of the statements given above are correct?
(a) 1 and 2 (b) 2 and 3
(c) 1 and 4 (d) 2 and 4

26. A man bought a number of oranges at 3 for a rupee and an equal number at 2 for a rupee. At what price per dozen, should he sell them to make a profit of 20%?
(a) Rs. 4 (b) Rs. 5
(c) Rs. 6 (d) Rs. 7

27. The simple interest on a certain sum of money of 3 yr at 8% per annum is half the compound interest of Rs. 4000 for 2 yr at 10% per annum. What is the sum placed on simple interest?
(a) Rs. 1550 (b) Rs. 1650
(c) Rs. 1750 (d) Rs. 2000

28. In the figure given above, AB is a diameter of a circle and CD is perpendicular to AB, if $AB = 10$ cm and $AE = 2$ cm, then what is the length of ED?



(a) 5 cm (b) 4 cm
(c) $\sqrt{10}$ cm (d) $\sqrt{20}$ cm

29. The speed of a boat in still water is 11 km/h. It can go 12 km upstream and return downstream to the engine point in 2h 45 min. What is the speed of stream?
(a) 5 km/h (b) 4 km/h
(c) 3 km/h (d) 2 km/h

30. The shadow of a pole 6 m high is 15 m long and at the same time the shadow of a tree is 25 m long. What is the height of the tree?
(a) 21 m (b) 10 m
(c) 35 m (d) None of the above

31. 1/48 of a work completed in half a day by 5 persons. Then, 1/40 of work can be completed by 6 persons in how may days?
(a) 1 (b) 2 (c) 3 (d) 1/2

32. The three sides of a triangle are 10, 100 and x. Which one of the following is correct?
(a) $10 < x < 100$ (b) $90 < x < 110$
(c) $90 \le x \le 110$ (d) $90 \le x < 110$

33. Three circular laminas of the same radius are cut out from a larger circular lamina. When the radius of each lamina cut out is the largest possible, then what is the ratio (approximately) of the area of the residual piece of the original lamina to its original total area?
(a) 0.30 (b) 0.35
(c) 0.40 (d) 0.45

34. What is the value of
$\sqrt{29.16} + \sqrt{0.2916} + \sqrt{0.002916} + \sqrt{0.00002916}$?
(a) 5.9949 (b) 5.9894
(c) 5.9984 (d) 5.9994

35. The remainder on dividing given integers a and b by 7 are respectively 5 and 4. What is the remainder when ab is divided by 7?
(a) 3 (b) 4
(c) 5 (d) 6

36. If median BD of the triangle ABC meets AC at D. If $BD = \frac{1}{2}AC$, then which one of the following is correct?
(a) $\angle ACB = 1$ right angle
(b) $\angle BAC = 1$ right angle
(c) $\angle ABC = 1$ right angle
(d) None of the above

37. r is a non-zero real number such that $t^{75} > t^{90}$. This is possible only when
(a) $-1 < r < 0$ (b) $0 < r < 1$
(c) $1 < r$ (d) $-1 < r < 1$

38. If r and s are any real numbers such that $0 \le s \le 1$ and $r + s = 1$, then what is the maximum value of the product rs?
(a) 1 (b) $\frac{3}{4}$
(c) $\frac{1}{2}$ (d) $\frac{1}{4}$

39. If the highest common factor of two positive integers is 24 then their least common multiple *cannot* be
(a) 72 (b) 216
(c) 372 (d) 600

40. What is the value of
$[(1 - \sin^2\theta)\sec^2\theta + \tan^2\theta](\cos^2\theta + 1)$ when $0 < \theta$ 90°?
(a) 2 (b) > 2
(c) ≥ 2 (d) < 2

41. If one of the roots of the equation $ax^2 + x - 3 = 0$ is – 1.5 then what is the value of a?
(a) 4 (b) 3
(c) 2 (d) – 2

42. What is the maximum area of rectangle, the perimeter of which is 18 cm?
(a) 20.25 cm^2 (b) 20.00 cm^2
(c) 19.75 cm^2 (d) 19.60 cm^2

43. Which one of the following numbers is *not* a square of any natural number?
(a) 5041 (b) 9852
(c) 1936 (d) 6241

44. Consider the following statements in respect of quadrilateral:
1. The line segments joining the midpoints of the two pairs of opposite sides bisect each other at the oint of intersection.
2. The area of the quadrilateral formed by joining the midpionts of the four adjacent sides is half of the total area of the quadrilateral.

Which of the statements given above is / are correct?
(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2

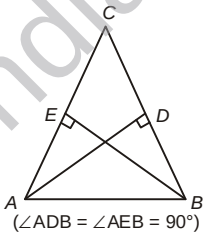
45. The inner and outer radii of a 7 m long hollow iron right circular cylindrical pipe are 2 cm and 4 cm respectively. If 1000 cm^3 of iron weights 5 kg. What is the weight of the pipe?
(a) 264 kg (b) 132 kg
(c) 396 kg (d) None of these

46. The table below gives the number of members of a club classified by sex and nativity:

Nativity / Sex	Locals	Migrants	Total
Male	85	45	120
Female	35	35	70
Total	120	80	200

The above data are represented by a pie diagram. What is the sectorial angle of the area representing male-migrant category?
(a) 45° (b) 22.5°
(c) 81° (d) 67.5°

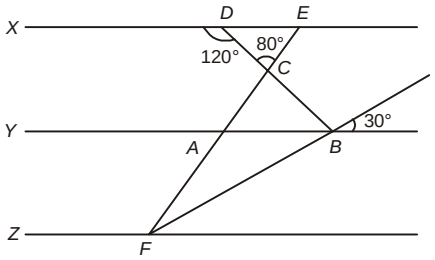
47. Consider the following in respect of the above figure:



(∠ADB = ∠AEB = 90°)

1. $\Delta DAC - \Delta EBC$ 2. $CA / CB = CD / CE$
3. $AD / BE = CD / CE$

Which of the above are correct?
(a) 1, 2, 3 (b) 1, 2
(c) 1, 3 (d) 2, 3

48. 

Three strainght lines X, Y and Z are parallel and the angles are as shown in the figure above, What is $\angle AFB$ equal to?
(a) 20° (b) 15° (c) 30° (d) 10°

49. With the help of histogram one can prepare
(a) frequency polygon
(b) frequency curve
(c) frequency distribution
(d) All of the above

50. If $7\cos^2\theta + 3\sin^2\theta = 4$ and $0 < \theta < \frac{\pi}{2}$, then what is the value of $\tan\theta$?

(a) $\sqrt{7}$ (b) $\frac{7}{3}$ (c) 3 (d) $\sqrt{3}$

51. A sphere and a cube have same surface area. What is the ratio of the square of volume of the sphere to the square of volume of the cube?

(a) $\pi : 6$ (b) $1 : 1$
(c) $6 : \pi$ (d) $3 : \pi$

52. A hemisphere is made of a sheet of a metal 1 cm thick. If the outer radius is 5 cm. What is the weight of the hemisphere (1 cm3 of the metal weights 9 g)?

(a) 54π g (b) 366π g
(c) 122π g (d) 108π g

53. A hemispherical bowl of internal radius 20 cm contains sauce. The sauce is to be filled in conical shaped bottles of radius 5 cm and height 8 cm. What is the number of bottles rquired?

(a) 100 (b) 90 (c) 80 (d) 75

54. Smaller lead shots are to be prepared by suing the material of a spherical lead shot of radius 1 cm. Some possibilities are listed in the statements given below:

1. The materal is just sufficient to prepare 8 shots each of radius 0.5 cm.
2. A shot of radius 0.75 cm and a second shot of radius 0.8 cm can be prepared from the available material.

Which of the above statements is/are correct?

(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2

55. A semi circular thin sheet of a metal of diameter 28 cm is bent and an open conical cup is made. What is the capacity of the cup?

(a) $\frac{1000}{3}\sqrt{3}\,\text{cm}^3$ (b) $300\sqrt{3}\,\text{cm}^3$
(c) $\frac{700}{3}\sqrt{3}\,\text{cm}^3$ (d) $\frac{1078}{3}\sqrt{3}\,\text{cm}^3$

56. Two similar parallelograms have corresponding sides in the ratio $1 : k$. What is the ratio of their areas?

(a) $1 : 3k^2$ (b) $1 : 4k^2$
(c) $1 : k^2$ (d) $1 : 2k^2$

57. A person travelled by train for 1 hr at a speed of 50 km/h. He, then travelled by a taxi for 30 min at a speed of 32 km/h to complete his journey. What is the average speed at which he travelled during the journey?

(a) 44 km/h (b) 42 km/h
(c) 41 km/h (d) 33 km/h

58. A parallelogram and a rectangle stand on the same base and on the same side of the base with the same height. If l_1, l_2 be the perimeters of the parallelogram and the rectangle respectively, then which one of the following is correct?

(a) $l_1 < l_2$ (b) $l_1 = l_2$
(c) $l_1 > l_2$ but $l_1 \neq 2l_2$ (d) $l_1 = 2l_2$

59. The volume of a cone is equal to that of sphere. If the diameter of base of cone is equal to the diameter of the sphere, what is the ratio of height of cone to the diameter of the sphere?

(a) 2 : 1 (b) 1 : 2
(c) 3 : 1 (d) 4 : 1

60. Consider the following statements in respect of any triangle:

1. The three medians of a triangle divide it into six traingles of equal area.
2. The perimeter of a triangle is greater than the sum of the lengths of its three medians.

Which of the statements given above is/are correct?

(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2

61. The middle points of the parallel sides *AB* and *CD* of a parallelogram *ABCD* are *P* and *Q* respectively. If *AQ* and *CP* divide the diagonal *BD* into three parts *BX*, *XY* and *YD*, then which one of the following is correct.

(a) $BX \neq XY \neq YD$ (b) $BX = YD \neq XY$
(c) $BX = XY = YD$ (d) $XY = 2BX$

62. Consider the following data:

Year	1911-21	1921-31	1931-41	1941-51	1951-61	1961-71	1971-81
Birth rate	48.1	46.4	45.2	39.9	41.7	41.1	37.1
Death rate	38.5	36.3	31.2	27.4	22.8	35.9	14.8

For which period is the natural growth rate minimum?

(a) 1911-21 (b) 1921-31
(c) 1951-61 (d) 1961-71

63. The following sets of conditions relate to two triangles *ABC* and *DEF*. Which set of conditions does not guarantee the congruence of *ABC* and *DEF*?

(a) $a = d, b = e, c = f$
(b) $\angle B = \angle E, \angle C = \angle F, a = d$
(c) $c = f, b = e, \angle A = \angle D$
(d) $c = f, b = e, \angle B = \angle E$

64. ABC is a triangle. The internal bisector of ∠ABC and the external bisector of ∠ACB meet at D. Which one of the following is correct?

(a) $\angle BDC = \angle BAC$ (b) $\angle BDC = \frac{1}{2}\angle BAC$

(c) ∠BDC = ∠DBC (d) $\angle BDC = \frac{1}{2}\angle ABC$

65. A ladder of 17 ft length reaches a window which is 15 ft above the ground on one side of the street. Keeping its foot at the same point the ladder is turned to the other side of the street and how it reaches a window 8ft hight. What is the width of the street?

(a) 23 ft (b) 15 ft
(c) 25 ft (d) 30 ft

66. A new frequency distribution is constructed by doubling each frequency of the original distribution keeping the other entires intact. The following are computed for both the tables:

1. Arithmetic mean
2. Medina
3. Harmonic mean

Which of the following statements with reference to above is correct?

(a) Corresponding values of 1 and 2 only are in both the distributions
(b) Corresponding values of 1 and 3 only are equal in both the distributions
(c) Corresponding values of 2 and 3 only are equal to both the distributions
(d) Corresponding values of 1, 2 and 3 are equal in both the distributions

67. *ABCD* is a square, *P, Q, R*. S are points on the sides *AB, BC, CD, DA* respectively such that *AP = BQ = CR = DS*. What is ∠*SPQ* equal to?

(a) 30° (b) 45°
(c) 60° (d) 90°

68. Three lines intersect each other in pairs. What is the number of angles so formed?

(a) 3 (b) 6
(c) 9 (d) 12

69. What is the value of $1.\overline{34} + 4.1\overline{2}$?

(a) $\frac{133}{90}$ (b) $\frac{371}{90}$

(c) $5\frac{219}{990}$ (d) $5\frac{461}{990}$

70. One saree was purchased for Rs. 564 after getting a discount of 6% and another saree was purchased for Rs. 396 after getting a discount of 1%. Taking both the items as a single transaction, what is the percentage of discount?

(a) 3.5 (b) 4
(c) 7 (d) 7.5

71. $x^4 + 4y^4$ is divisible by which one of the following?

(a) $(x^2 + 2xy + 2y^2)$ (b) $(x^2 + 2y^2)$
(c) $(x^2 - 2y^2)$ (d) None of these

72. What is the value of

$$\left(\frac{1}{3}\log_{10} 125 - 2\log_{10} 4 + \log_{10} 32 + \log_{10} 1\right)?$$

(a) 0 (b) $\frac{1}{5}$

(c) 1 (d) $\frac{2}{5}$

73. What is the least integral value of k he for which the equation $x^2 - 2(k-1)x + (2k+1) = 0$ has both the roots positive?

(a) 1 (b) $-\frac{1}{2}$

(c) 4 (d) 0

74. Which one of the following is a correct statement?

(a) $\{a\} \in \{\{a\}, \{b\}, c\}$ (b) $\{a\} \subseteq \{\{a\}, b, c\}$

(c) $\{a, b\} \subseteq \{\{a\}, b, c\}$ (d) $a \subseteq \{\{a\}, b, c\}$

75. Which one of the following is correct?

(a) There is only one θ with $0° < \theta < 90°$ such that $\sin\theta = a$, where a is a real number

(b) There is more than one θ with $0° < \theta < 90°$ such that $\sin\theta = a$, where a is a real number

(c) There is no θ with $0° < \theta < 90°$ such that $\sin\theta = a$, where a is a real number

(d) There are exactly two θs with $0° < \theta < 90°$ such that $\sin\theta = a$, where a is real number

76. If u, v, w, are real numbers such that $u^3 - 8v^3 - 27w^3 = 18uvw$, which one of the following is correct?

(a) $u - v + w = 0$ (b) $u = -v = -w$

(c) $u - 2v = 3w$ (d) $u + 2v = -3w$

77. A three digit number is divisble by 11 and has its digit in the unit's place equal to 1. The number is 297 more than the number obtained by reversing the digits. What is the number?

(a) 121 (b) 231

(c) 561 (d) 451

78. What is the median of the ages of minor children?

(a) 3 yr (b) 5 yr

(c) 7 yr

(d) Cannot be determined due to insufficient data

79. What is the mean of ages of minor children?

(a) 3 yr (b) 4 yr

(c) 5 yr (d) 6 yr

80. Statement I:Let LMN be a triangle. Let P, Q be the mid points of the sides LM, LN respectively. If $PQ^2 = MP^2 + NQ^2$, then LMN is a right angled triangle right angled at L.

Statemment II:If in a triangle ABC, $AB^2 > BC^2 + CA^2$, then $\angle ACB$ is obtuse.

Which of the following is correct in the light of the above statements?

(a) Both statement-I and statement-II are correct and statement-II is the reason for statement-I

(b) Both statement-I and statement-II are correct and statement-II is not the reason for statement-I

(c) Statement-I is true, but statement-II is false

(d) Statement-I is false, but statement-II is ture

(For the next two questions to follows):

The average age of 6 persons living in a house is 23.5 yr.

Three of them are majors and their average age is 42 yr.

The difference in ages of the three minor children is same.

81. Consider the following assumption and two statements:

Assumption: A number 'ABCDE' is divisible by 11.

Statement-I: E – D + C – B + A is divisible by 11.

Statement-II: E – D + C – B + A = 0

Which one of the following is correct?

(a) Only statement-I can be drawn from the assumption

(b) Only statement-II can be drawn from the assumption

(c) Both the statement can be drawn from the assumption

(d) Neither of the statements can be drawn from the assumption

82. Consider those numbers between 300 and 400 such that when each number is divided by 6,9 and 12, it leaves 4 as remainder in each case. What is the sum of the numbers?

(a) 692 (b) 764

(c) 1080 (d) 1092

83. If $\cos 1° = p$ and $\cos 89° = q$, then which one of the following is correct?
(a) p is close to zero and q is close to 1
(b) $p < q$ (c) $p = q$
(d) p is close to 1 and q is close to zero

84. What is the locus of centres of circles which touch a given line at a given point?
(a) A line perpendicular to the given line, passing through the given point
(b) A line parallel to the given line
(c) A circle tangent to the given line at the given point
(d) A close curve other than a circle

85. What can be said about the expansion of $2^{12n} - 6^{4n}$, where n is a positive integer?
(a) Last digit is 4
(b) Last digit is 8
(c) Last digit is 2
(d) Last two digits are zero

86. A cylinder having base of circumference 60 cm is rolling without sliding at a rate of 5 rounds per second. How much distance will the cylinder roll in 5 *s* ?
(a) 15 m (b) 1.5 m
(c) 30 m (d) 3 m

87. What is volume of the frustum of a cone with height h and radii r_1, r_2?
(a) $\frac{1}{3}\pi h(r_1^2 - r_2^2)$
(b) $\frac{1}{3}\pi h(r_1^2 - r_2^2 + r_1 r_2)$
(c) $\frac{1}{3}\pi h(r_1^2 - r_2^2 - r_1 r_2)$
(d) $\frac{1}{3}\pi h(r_1^2 - r_2^2)$

88. What is the weighted mean of first 10 natural numbers whose weights are equal to the corresponding number?
(a) 7 (b) 5.5
(c) 5 (d) 4.5

89. A rectangular tank whose length and breadth are 2.5 m and 1.5 m respectively is half full of water. If 750 L more water is poured into the tank, what is the height through which water level further goes up?
(a) 20 cm (b) 18 cm
(c) 15 cm (d) 200 cm

90. The length, breadth and height of a rectangular parallelopiped are in ratio 6 : 3 : 1. If the surface area of a cube is equal to surface area of this parallelopiped, then what is the ratio of the volume of the cube to the volume of the parallelopiped?
(a) 1 : 1 (b) 5 : 4
(c) 7 : 5 (d) 3 : 3

91. The marks of the students of a class who appread for a test in English are represented in the following frequency table:

Class Interval	1-10	11-20	21-30	31-40	41-50	51-60	
Frequency	9	22	–	20	12	8	100 (toal (frequency

What is / are the modal class(es)?
(a) 10.5 – 20.5 only
(b) 20.5 – 30 only
(c) 10.5 – 20.5 and 20.5 – 30.5
(d) There is no modal class

92. In the given figure, square *ABCD* is 7 cm. What is the area of the shaded portion, formed by the arcs $\overset{\frown}{BD}$ of the circles with centre at *C* and *A*?
(a) 7 cm^2 (b) 28 cm^2
(c) 14 cm^2 (d) 21 cm^2

93. A man is watching from the top of a tower a boat speeding away from the tower. The boat makes an angle of depression of 45° with the man's eye when at distance of 60 m from the bottom of tower. After 5 *s*, the angle of depression becomes 30°. What is the approximate speed of the boat assuming that it it running in still water?
(a) 31.5 km / h (b) 36.5 km / h
(c) 38.5 km / h (d) 40.5 km / h

94. A person inversted some amount at the rate of 12% simple interest and the remaining at 10%. He received yearly an interest and the

remaining at 10%. He received yearly an interest of Rs. 130. Had he interchanged the amounts invested, he would have received an interest of Rs. 134. How much money did he invest at different rates?

(a) Rs. 500 @ 10%, Rs. 800 @ 12%
(b) Rs. 700 @ 10%, Rs. 600 @ 12%
(c) Rs. 800 @ 10%, Rs. 400 @ 12%
(d) Rs. 700 @ 10%, Rs. 500 @ 12%

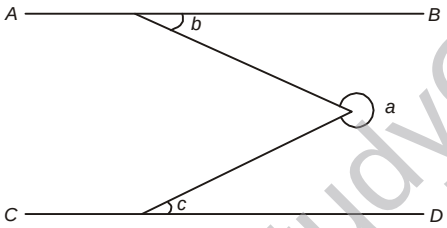
95. If $x\,(x + y + z) = 9$, $y\,(x + y + z) = 16$ and $z\,(x + y + z) = 144$, then what is x equal to

(a) $\frac{9}{5}$ (b) $\frac{9}{7}$

(c) $\frac{9}{13}$ (d) $\frac{16}{13}$

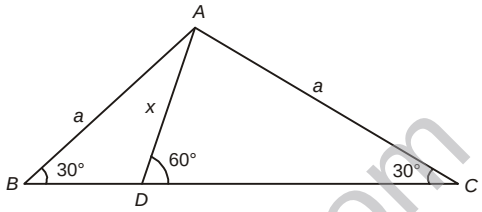
96. If the expressions $px^3 + 3x^2 - 3$ and $2x^3 - 5x + p$ when divided by $x - 4$ leave the same remainder, then what is the value of p?

(a) – 1 (b) 1
(c) – 2 (d) 2

97. 

In the figure given above, AB is parallel to LM. What is angle a equal to?

(a) $\pi + b + c$ (b) $2\pi - b + c$
(c) $2\pi - b - c$ (d) $2\pi + b - c$

98. 

In the above figure, what is x equal to

(a) $\frac{a}{3}$ (b) $\frac{a}{2}$

(c) $\frac{a}{\sqrt{3}}$ (d) $\frac{a}{\sqrt{2}}$

99. Two circles touch ech other externally at P. Two secants APB and CPD are drawn through P to meet the circle at A, C and B, D respectively. Then, which one of the following is correct?

(a) AC is perpendicular to BD
(b) AC intersects BD
(c) AC is parallel to BD
(d) None of the above

100. What is the number of circles passing through a given pair of pionts?

(a) One
(b) Only two
(c) More than two, but finite
(d) Infinitely many

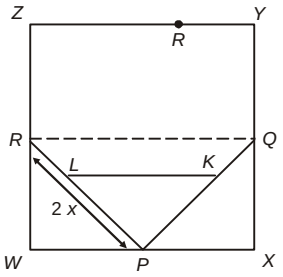
ANSWERS

1. (b)	2. (a)	3. (a)	4. (b)	5. (b)	6. (b)	7. (b)	8. (d)	9. (d)	10. (b)
11. (b)	12. (a)	13. (b)	14. (a)	15. (a)	16. (d)	17. (d)	18. (b)	19. (d)	20. (c)
21. (c)	22. (c)	23. (a)	24. (d)	25. (c)	26. (c)	27. (c)	28. (b)	29. (a)	30. (a)
31. (d)	32. (b)	33. (b)	34. (d)	35. (d)	36. (c)	37. (b)	38. (d)	39. (c)	40. (b)
41. (c)	42. (b)	43. (b)	44. (c)	45. (b)	46. (c)	47. (a)	48. (b)	49. (d)	50. (d)
51. (c)	52. (b)	53. (c)	54. (a)	55. (d)	56. (d)	57. (a)	58. (c)	59. (a)	60. (c)
61. (c)	62. (d)	63. (d)	64. (b)	65. (a)	66. (d)	67. (d)	68. (d)	69. (d)	70. (b)
71. (a)	72. (c)	73. (c)	74. (a)	75. (a)	76. (c)	77. (d)	78. (b)	79. (c)	80. (b)
81. (c)	82. (a)	83. (d)	84. (a)	85. (d)	86. (a)	87. (b)	88. (a)	89. (a)	90. (d)
91. (b)	92. (b)	93. (a)	94. (d)	95. (c)	96. (b)	97. (c)	98. (c)	99. (d)	100. (d)

EXPLANATIONS

1. Area $(PRQ) = \frac{1}{2}$ are $(WXOR)$

$$= \frac{1}{2}\left[\frac{1}{2}\text{area}(WXYZ)\right] = \frac{1}{4}\text{ area }(WXYZ) \quad \ldots(i)$$



$$\frac{\text{Area}(PRQ)}{\text{Area}(PLK)} = \frac{RP^2}{LP^2}$$

(by properties of similar triangle)

$$\Rightarrow \quad \frac{\text{Area }(PRQ)}{\text{Area }(PLK)} = \frac{(2x)^2}{x^2}$$

$$\Rightarrow \quad \text{Area }(PRQ) = 4 \text{ area }(PLK)$$

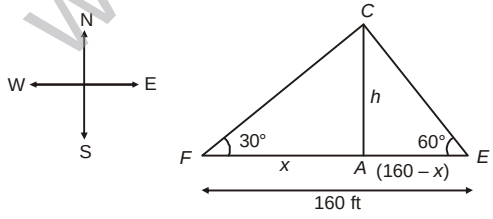
$$\Rightarrow \quad \frac{1}{4}(WXYZ) = 4(PLK)$$

[from Eq. (i)]

$$\Rightarrow \quad \frac{1}{16}\text{ area }(WXYZ) = \text{Area }(PLK)$$

$$\Rightarrow \quad \frac{1}{16} = \frac{\text{Area}(PLK)}{\text{Area}(WXYZ)}$$

2. In ΔAFC,



$$\tan 30° = \frac{h}{x}$$

$$\Rightarrow \quad x = \sqrt{3}h \quad \ldots(i)$$

and in ΔAEC,

$$\tan 60° = \frac{h}{160 - x}$$

$$\Rightarrow \quad \sqrt{3}(160 - x) = h$$

$$\Rightarrow \quad \sqrt{3}(160 - \sqrt{3}) = h \quad \text{[from Eq. (i)]}$$

$$\Rightarrow \quad 160\sqrt{3} - 3h = h$$

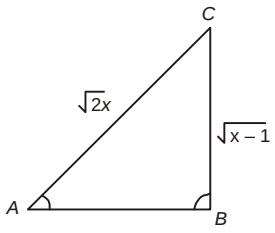
$$\Rightarrow \quad 4h = 160\sqrt{3}$$

$$\Rightarrow \quad h = 40\sqrt{3} \text{ ft}$$

3. Given,

$$\sin \alpha = \sqrt{\frac{x-1}{2x}}$$

In ΔABC, using Pythagoras theorem



$$AC^2 = AB^2 + BC^2$$

$$\Rightarrow \quad 2x = AB^2 + (x + 1)$$

$$\Rightarrow \quad AB^2 = x + 1$$

$$\Rightarrow \quad AB = \sqrt{x+1}$$

$$\therefore \quad \tan \alpha = \frac{BC}{AB} = \frac{\sqrt{x-1}}{\sqrt{x+1}}$$

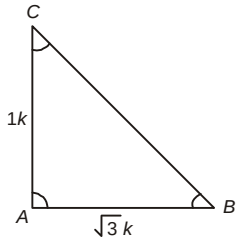
4.

x	f	xf
1	3	3
2	5	10
3	9	27
4	f_1	$4f_1$
5	2	10
Total	$19 + f_1$	$50 + 4f_1$

5. $\sin^2 15° + \sin^2 20° + \sin^2 25° + ... + \sin^2 75°$
$= \sin^2 (90° - 75°) + \sin^2 (90° - 70°) + ... + \sin^2 (90° - 15°)$
$= \cos^2 75° + \cos^2 70° + ... + \cos^2 15°$

7. Given,

$$\tan B = \frac{k}{\sqrt{3}k}$$



In ΔABC,
By Pythagoras theorem,

$$AB^2 + AC^2 = BC^2$$

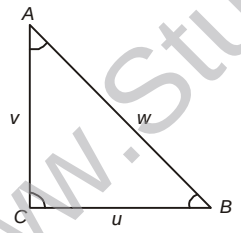
$$\Rightarrow (\sqrt{3}k)^2 + (1k)^2 = BC^2$$

$$\Rightarrow BC^2 = 4k$$

$$\Rightarrow BC = 2k$$

8. In a given option only
(x : x is an integer and less than 1000)
i.e., $x \in (-\infty, 1000)$ is an infinite set.

9. In ΔABC,



$$\tan A = \frac{BC}{AC} = \frac{u}{v}$$

and $$\tan B = \frac{v}{u}$$

Also, $u^2 + y^2 = w^2$ ... (i)
(by Pythagoras theorem)

$$\therefore \quad \tan A + \tan B = \frac{u}{v} + \frac{v}{u} = \frac{u^2 + y^2}{uv}$$

$$= \frac{w^2}{uv}$$

10. Since, $m = 2n + 1$ is an odd integer so it may be factors 1 or 3. And $k = 9n + 4$ its factor may be 1, 2 and 4. Hence, HCF of (m, l) is 1.

11. Since, first term = x $\Rightarrow$ first term = $c_1 x$
Second term $\propto (-x + x^2) \Rightarrow$ second term $= c_2 (-x + x_2)$
and third term $\propto (x^2 - x^2) \Rightarrow$ third term $= c_3 (x^3 - c^2)$
Also, $y \propto [c_1 x + c^2 (-x + x^2) + c^3 (x^3 - x^2)]$

$$\Rightarrow \quad y = c_4[(c_1 - c_2)\, x\, (c^2 - c^3)\, x^2 + c^3 x^3]$$
$$= c_4 (c_1 - c_2)\, x + (c_2 - c_3)\, c_4 x^2 + c_3 c_4 x^3$$
$$= kx + lx^2 + mx^3$$

where
$k = c_4 (c_1 - c_2)$,
$l = (c_2 - c_3)\, c_4$ and $m = c_3 c_4$
Hence, option (b) is correct.

12. Let initial salary = Rs. $22x$
Final salary = Rs. $25x$
Let initial number o employees = $3y$
Final number of employees = $2y$
Present bill = Final salary × Final number of employees

$$\Rightarrow \quad 5000 = 25\, x \times 2y$$

$$\Rightarrow \quad \frac{5000}{50} = xy$$

$$\Rightarrow \quad xy = 100$$

Original bill = Initial salary × Initial number of employees
$= 22x \times 3y$
$= 66\, xy = 66 \times 100$
$=$ Rs. 6600

13. Given,

$$50\% \text{ of } (x - y) = 40\% \text{ of } (x + y)$$

$$\Rightarrow \quad \frac{50}{100} \times (x - y) = \frac{40}{100} \times (x + y)$$

$$\Rightarrow \quad 5x - 5y = 4x + 4y$$

$\Rightarrow \quad x = 9y$

Let $r\%$ of $x = y$

$\Rightarrow \quad \frac{r}{100} \times x = y$

$\Rightarrow \quad \frac{r}{100} \times 9y = y$

$\Rightarrow \quad r = \frac{100}{9} = 11\frac{1}{9}\%$

14. Since,

$6 \text{ men} + 8 \text{ boys} = 10 \text{ days}$

$\Rightarrow \quad 3 \text{ men} + 4 \text{ boys} = 20 \text{ days}$

$\therefore \quad 15 \text{ men} + 20 \text{ boys} = 4 \text{ days}$

15. Given,

$\cos\theta + \sqrt{3}\sin\theta = 2$

$\Rightarrow \quad \frac{1}{2}\cos\theta + \frac{\sqrt{3}}{2}\sin\theta = 1$

$\Rightarrow \quad \sin 30° \cos\theta + \cos 30° \sin\theta = 1$

$\Rightarrow \quad \sin(30° + 0) = \sin 90°$

$\Rightarrow \quad 30° + 0 = 90°$

$\Rightarrow \quad \theta = 60°$

16. Mode = 3 (median) – 2 (mean)

$= 3(2) - 2\left(\frac{7}{3}\right)$

$= \frac{18-14}{3} = \frac{4}{3}$

Hence, option (d) is correct.

17. The value of c is 4.

18. Required sum $= 2(a)^2 + 2(b)^2 + 2(c)^2$

$= 2(1)^2 + 2(2)^2 + 2(4)^2$

$= 2 + 8 + 32 = 42$

19. The number of solution for three simultaneously linear equation in two unknown variable is exactly three solutions.

Solution 83 to 85

Since, $a < b < c$

Therefore, series in increasing order

a, a, b, b, c, c

$\therefore \text{ Median} = \frac{\left(\frac{6}{2}\right)\text{th term} + \left(\frac{6}{2}+1\right)\text{th term}}{2}$

$= \frac{3\text{rd term} + 4\text{th term}}{2}$

$\Rightarrow \quad 2 = \frac{b+b}{2} = 2$

Also, arithmetic mean $= \frac{a+a+b+b+c+c}{6}$

$\Rightarrow \quad \frac{7}{3} = \frac{a+b+c}{3}$

$\Rightarrow a + c = 7 - 2 = 5 \ldots(i)$

and geometric mean $= (a^2 \times b^2 \times c^2)^{1/6}$

$\Rightarrow \quad 2 = (abc)^{1/3}$

$\Rightarrow \quad abc = 8$

$\Rightarrow \quad ac = \frac{8}{2} = 4 \qquad \ldots(ii)$

$\therefore$ From Eqs. (i) and (ii), we get

$a = 1, c = 4$ and $b = 2$

20. Given, $\frac{1}{x+a} + \frac{1}{x+b} = \frac{1}{c}$

$\Rightarrow \frac{(x+b)+(x+a)}{(x+a)(x+b)} = \frac{1}{c}$

$\Rightarrow \quad 2cx + (a + b)c = x^2 + (a + b)x + ab$

$\Rightarrow x^2 + (a + b - 2c) + ab - ac - bc = 0$

Let, the roots of equation above be α and β.

Given,

$\alpha + \beta = 0$

$\Rightarrow \quad -(a + b - 2c) = 0$

$\Rightarrow \quad a + b = 2c \qquad \ldots(i)$

Now, $\alpha\beta = ab - ac - bc$

$= ab - (a + b)\frac{(a+b)}{2}$

(from Eq. (i))

$$= \frac{2ab-(a^2+b^2+2ab)}{2}$$

$$= -\frac{(a^2+b^2)}{2}$$

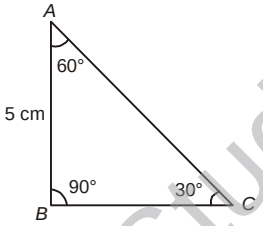
21. Given,

$$\log 10\,(x^2-6x+45) = 2$$
$$\Rightarrow (x^2-6x+45) = 10^2$$
$$\Rightarrow x^2-6x-55 = 0$$
$$\Rightarrow x^2-11x+5x-55 = 0$$
$$\Rightarrow x(x-11)+5(x-11) = 0$$
$$\Rightarrow (x+5)(x-11) = 0$$
$$\Rightarrow x = 11, -5$$

22. Let the roots of equation $ax^2 + bx + c = 0$ arc $-\alpha$ and $-\frac{1}{\alpha}$

$$\therefore \quad (-\alpha)\left(-\frac{1}{\alpha}\right) = \frac{c}{a}$$
$$\Rightarrow \quad c = a$$

23. In ΔBAC,



$$\cos 60° = \frac{AB}{AC}$$
$$\Rightarrow \frac{1}{2} = \frac{5}{AC}$$
$$\Rightarrow AC = 10 \text{ cm}$$

24. We know in the interval $\theta \in \left[0, \frac{\pi}{2}\right]$ $\sec^2\theta$ is increasing from 1 to ∞.
$\therefore p \geq 1$

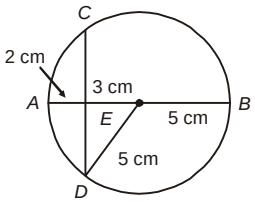
25. We know $(x-y)$ is a factor of $x^{10}-y^{10}$ and $(x+y)$ is a factor of $x^{10}-y^{10}$.

27. Let the principal amount be Rs. P.
According to the given condition,

$$\text{SI} = \frac{1}{2}\text{ CI}$$
$$\Rightarrow \frac{p\times 8\times 3}{100} = \frac{1}{2}\left[4000\left(1+\frac{10}{100}\right)^2 - 4000\right]$$
$$\Rightarrow \frac{24P}{100} = \frac{1}{2}\left[4000\times\frac{121}{100} - 4000\right]$$
$$\Rightarrow \frac{24P}{100} = 420$$
$$\Rightarrow P = \frac{420\times 100}{24}$$
$$\Rightarrow P = \text{Rs. } 1750$$

28. In ΔOED,

$$(OD)^2 = (DE)^2 + (EO)^2$$
$$\Rightarrow (5)^2 = (DE)^2 + (3)^2$$



$$\Rightarrow (DE)^2 = 25-9 = 16$$
$$\Rightarrow DE = 4 \text{ cm}$$

29. Let speed of boar, $S_1 = 11$ km/h and speed of stream be S_2.

In upstream; $11 - S_2 = \frac{12}{t_1}$

and in downstream, $11 + S_2 = \frac{12}{t_2}$

$$\Rightarrow t_2 = \frac{12}{11+S_2}$$

Also, $t_1 + t_2 = 2 + \frac{45}{60}$

$$\Rightarrow \frac{122}{11-S_2} + \frac{12}{11+S_2} = 2.75$$

$\Rightarrow \quad 12\left(\frac{22}{121-S_2^2}\right) = 2.75$

$\Rightarrow \quad 121 - S_2^2 = \frac{22\times 12}{2.75}$

$\Rightarrow \quad 121 - S_2^2 = 96$

$\Rightarrow \quad S_2^2 = 25$

$\Rightarrow \quad S_2 = 5\text{km/h}$

30. For 15 m legnth of shadow, height of pole = 6 m

For 1 m length of shadow = $\frac{6}{15}$ m

For 25 m length of shadow = $\frac{6}{15}\times 25$

= 10 m

Hence, the length of tree is 10 m.

31. $\therefore \quad \frac{M_1D_1}{W_1} = \frac{M_2D_2}{W_2}$

$\Rightarrow \quad \frac{5\times\frac{1}{2}}{\frac{1}{48}} = \frac{6\times D_2}{\frac{1}{40}}$

$\Rightarrow \quad D_2 = \frac{\frac{1}{2}\times 5\times 48}{40\times 6} = \frac{1}{2}$

32. We know, the sum of two sides is always greater than this sides.

$\therefore$ 10 + 100 > x, 10 + x > 100 x > 10

$\Rightarrow$ 110 > x.

and x > – 110, but x can not be negative

$\therefore$ 90 < x, x < 110

33. In ΔADC,

$(2r)^2 = r^2 + DC^2$

$\Rightarrow \quad 4r^2 - r^2 = DC^2$

$\Rightarrow \quad DC = \sqrt{3}r$

Q $\quad OC = \frac{2}{3}DC$

$= \frac{2}{3}\times\sqrt{3}r = \frac{2r}{\sqrt{3}}$

Radius of larger circular lamina = OE

$= OC + CE = \frac{2r}{\sqrt{3}} + r$

$= \frac{(2+\sqrt{3})r}{\sqrt{3}}$

Area of 3 laminas = $3\pi r^2$

Area of larger lamina $= \pi\left[\frac{(2+\sqrt{3})}{\sqrt{3}}r\right]^2$

$= \pi\frac{(4+3+4\sqrt{3})}{3}r^2$

$= \frac{(7+4\sqrt{3})}{3}\pi r^2$

Residual area $= \left[\frac{7+4\sqrt{3}}{3} - 3\right]\pi r^2$

$= \frac{(4\sqrt{3}-2)}{3}\pi r^2$

Ratio $= \left(\frac{4\sqrt{3}-2}{3}\right)\pi r^2$

Ratio $= \frac{3}{\frac{7+\sqrt{3}}{3}\pi r^2}$

$= \frac{4\sqrt{3}-2}{7+4\sqrt{3}}\times\frac{7-4\sqrt{3}}{7-4\sqrt{3}}$

$= \frac{28\sqrt{3}-48-14+8\sqrt{3}}{49-48}$

$= 36\sqrt{3} - 62 \quad = 36 \times 1.732 - 62$

$= 62.352 - 62 = 0.35$

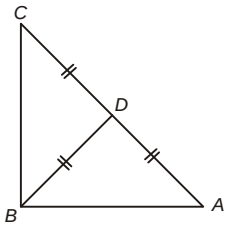
34. $\sqrt{29.16} + \sqrt{0.2916} + \sqrt{0.002916} + \sqrt{0.00002916}$

$= 5.4 + 0.54 + 0.054 + 0.0054$

$= 5.9994$

35. Let $a = 7p + 5$ and $b = 7q + 4$
where p and q are natural numbers.
$ab = (7p + 5)(7q + 4)$
$ab = 49pq + (4p + 5q)\,7 + 20$
when ab is divided by 7, we get the remainder 6.

36. Here we see that,



This is possible only when ABC is right angle triangle.

37. $r^{75} > r^{90}$ is possible only when $0 < r < 1$

38. Given $r + s = 1$

For maximum product, $r = s = \frac{1}{2}$

$\therefore\ rs = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$

39. Q In the given options, 372 is not divisible by 24.
Therefore, LCM of numbers cannot be 372.
Hence, option (c) is correct.

40. $[(1 - \sin^2\theta)\sec^2\theta + \tan^2\theta](\cos^2\theta + 1)$
$= (\sec^2\theta - \tan^2\theta + \tan^2\theta)(\cos^2\theta + 1)$
$= 1 + \sec^2\theta > 1 + 1 > 2$
[Since, $\sec^2\theta > 1$ for $0° < \theta < 90°$]

41. Since, -1.5 is a root of $ax^2 + x - 3 = 0$
$\therefore a(-1.5)^2 + (-1.5) - 3 = 0$
$\Rightarrow \quad 2.25a - 4.5 = 0$
$\Rightarrow \quad a = \frac{4.5}{2.25}$
$\Rightarrow \quad a = 2$

42. Let sides of a rectangle be l and b.
$\therefore \quad 2(l + b) = 18$
$\Rightarrow \quad l + b = 9$
Area of rectangle $= l \times b$
For maximum area of rectangle $l = 5$ and $b = 4$
$\therefore$ Area of rectangle $= 5 \times 4 = 20\ \text{cm}^2$

43. Any number is not a square, if the unit's place digit of number may be 2, 3, 7, 8.

45. Volume of hollow cylindirical pipe

$$= \pi(r_2^2 - r_1^2) \times h$$

$$= \frac{22}{7}\{(4)^2 - 2(2)^2\} \times 700$$

$$= \frac{22}{7} \times 12 \times 700$$

$$= 26400\ \text{cm}^3$$

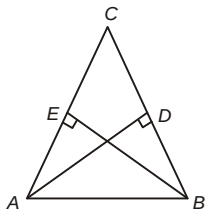
$$= 26400 \times \frac{5}{1000}\ \text{kg}$$

(Q $1000\ \text{cm}^3 = 5$ kg; given)

$$= 132\ \text{kg}$$

46. The sectorial angle of male-migrant category in pie diagram $= \frac{45}{200} \times 360°$
$= 81°$

47. In ΔCAD and ΔCEB,
$\angle C = \angle C$ (common)
$\angle CEB = \angle ADC$ (each 90°)
$\angle CAD = \angle CBE$ (rest angle)
$\therefore \quad \Delta CAD - \Delta CEB$



Sides will be in same proportion,

$$\frac{CA}{CB} = \frac{CD}{CE}$$

and $$\frac{AD}{BE} = \frac{CD}{CE}$$

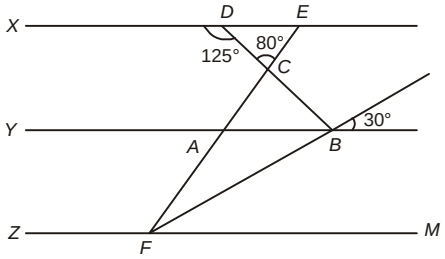
Hence, option (a) is correct.

48. $\angle CDE = 180° - 125° = 55°$
In ΔDCE,
$\angle CED = 180° - 55° - 80°$
and $\angle B = \angle ABF = 30°$

(by vertically opposite)

Also $\angle ABF = \angle BFM = 30°$

(by alternate angle)



From figure,

$\angle DEF = \angle EFM$

(by alternate angle)

$\angle EFM = 45°$

$\Rightarrow \angle EFB + \angle BFM = 45°$

$\Rightarrow \angle EFB + \angle BFM = 45°$

$\Rightarrow \angle AFB = 15°$

49. With the help of histogram we can prepare frequency polygon, frequency curve and frequency distribution. Hence option (d) is correct

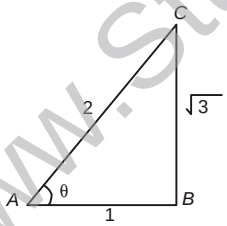
50. Given, $7\cos^2\theta + 3\sin^2\theta = 4$

$\Rightarrow 7(1 - \sin^2\theta) + 3(\sin^2\theta) = 4$

$\Rightarrow 7 - 4\sin^2\theta = 4$

$\Rightarrow 4\sin^2\theta = 3$

$\Rightarrow \sin\theta = \pm\frac{\sqrt{3}}{2}$



For $0 < \theta < \frac{\pi}{2}$,

We take $\sin\theta = \frac{\sqrt{3}}{2}$

$\therefore \tan\theta = \frac{\sqrt{3}}{1}$

51. Since,

Surface area of sphere = Surface area of cube

$\Rightarrow 4\pi r^2 = 6a^2$

$\Rightarrow \left(\frac{r}{a}\right)^2 = \frac{3}{2\pi}$

Now, $\frac{(\text{Volume of sphere})^2}{(\text{Volume of cube})^2} = \frac{\left(\frac{4}{3}\pi r^3\right)^2}{(a^3)^2}$

$= \frac{16}{9}\pi^2\left[\left(\frac{r}{a}\right)^2\right]^3$

$= \frac{16}{9}\pi^2\left(\frac{3}{2\pi}\right)^3$

$= \frac{16}{9}\pi^2 \times \frac{27}{8\pi^3}$

$= \frac{6}{\pi}$

53. Volume of hemispherical bowl

$= \frac{2}{3}\pi(20)^3$

$= \frac{16000}{3}\pi\,\text{cm}^3$

Volume of conical shape bottle

$= \frac{1}{3}\pi(5)^2 8$

$= \frac{200\pi}{3}$

$\therefore$ Required number of bottles $= \frac{16000\pi/3}{200\pi/3}$

$= 80$

54. Volume os spherical lead $= \frac{4}{3}\pi(1)^3$

$= \frac{4}{3}\pi\,\text{cm}^3$

1. Volume of 8 shots $= \frac{4}{3}\pi(0.5)^2 \times 8$

$= \frac{4}{3}\pi \text{cm}^3$

2. Volume of both shots

$= \frac{4}{3}\pi(0.75)^3 + \frac{4}{3}\pi(0.8)^3$

$= \frac{4}{3}\pi\left[\left(\frac{3}{4}\right)^3 + \left(\frac{4}{5}\right)^3\right]$

$= \frac{4}{3}\pi\left[\frac{27}{64} + \frac{64}{125}\right]$

$= \frac{4}{3}\pi\left[\frac{3375 + 4096}{8000}\right]$

$= \frac{4}{3}\pi\left(\frac{7471}{8000}\right)$

$= \frac{4}{3}\pi\ (0.93)\ \text{cm}^3$

Hence, only statement 1 is true.

55. Let $r = 14$ cm,
For Conical Cup, $l = 14$ cm,
According to questions circumference of base cone = Circumference of semi cricle

$\Rightarrow \quad 2\pi R = \pi r$

$\Rightarrow \quad 2R = r$

$\Rightarrow \quad 2R = 14$

$\Rightarrow \quad R = 7$ cm

$\therefore \quad l^2 = R^2 + h^2$

$\Rightarrow \quad (14)^2 = (7)^2 + h^2$

$\Rightarrow \quad h^2 = 196 - 49 = 147$

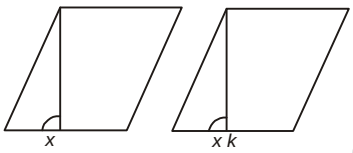
$\Rightarrow \quad h = 7\sqrt{3}$

$\therefore$ Capacity of cup $= \frac{1}{3}\pi R^2 h$

$= \frac{1}{3} \times \frac{22}{7} \times 7 \times 7 \times 7\sqrt{3}$

$= \frac{1078}{3}\sqrt{3}\ \text{cm}^3$

56. Their height will be in same ratio
$1 : k$ or $y : yk$



$\therefore$ Ratio of areas of two similar parallelograms

$= \frac{x \times y}{xk \times yk} = \frac{1}{k^2}$

57. Average speed $= \frac{\text{Total distance}}{\text{Total time}}$

$= \frac{50 \times 1 + 32 \times \frac{1}{2}}{1 + \frac{1}{2}}$

$= \frac{50 + 16}{3/2}$

$= \frac{66 \times 2}{2}$

$= 44$ km / h

58. If a parallelogram and a rectangle stand on the same base and on the same side of the base with the height, then perimeter of parallelogram is greater than perimeter of rectangle.

$\therefore \quad l_1 > l_2$

59. Let the radius of cone and sphere be r.
Q Volume of cone = Volume of sphere

$\Rightarrow \quad \frac{1}{3}\pi r^2 h_1 = \frac{4}{3}\pi(r)^3$

$\Rightarrow \quad \frac{h_1}{2r} = \frac{2}{1}$

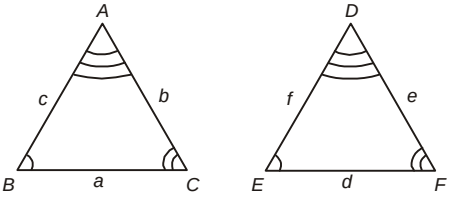
60. 1. It is true that the three medians of a triangle divide it into six triangles of equal area.
2. It is also true that, the perimeter of a triangle is greater than the sum of its three medians.

62.

Year	Growth rate = Birth rate – Death rate
1911–1921	9.6
1921–1931	10.1
1931–1941	14
1941–1951	12.5
1951–1961	18.9
1961–1971	5.2
1971–1981	22.3

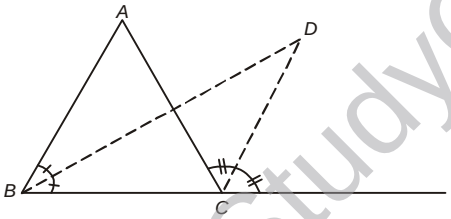
It is clear from the above table that minimum growth rate is 5.2 in the year 1961–1971.

63. Hence the set $c = f$, $b = e$, $\angle B = \angle E$



does not guarantee the congruence of ΔABC and ΔDEF.

64. By using the properties of triangle.



$$\angle BDC = \frac{1}{2}\angle BAC$$

66. If we double each value of original frequency distribution. then mean, median and harmonic mean remain same.

68. We know, when two lines intersect each other it makes 4 angles.
The total number of pairs = 3
$\therefore$ Total number of angles = 3×4
$= 12$

69. Q $1.\overline{34} = \frac{134-1}{90} = \frac{133}{99}$

and $4.1\overline{2} = \frac{412-41}{90}$

$= \frac{371}{90}$

$\therefore \quad 1.3\overline{4} = \frac{133}{99} + \frac{371}{90}$

$= \frac{1330+4081}{990}$

$= \frac{5411}{990} = 5\frac{461}{990}$

70. Let marked price of two sarees are x and y respectively.

$\therefore \quad x - \frac{6x}{100} = 564$

$\Rightarrow \quad \frac{94x}{100} = 564$

$\Rightarrow \quad x = \text{Rs. } 600$

and $\quad y - \frac{y}{100} = 396$

$\Rightarrow \quad \frac{99y}{100} = 396$

$\Rightarrow \quad y = \text{Rs. } 400$

$\therefore$ Total MP amount = 600 + 400
= Rs. 1000
Total amount after discount = 564 + 396
= Rs. 960

$\therefore$ Discount% = $\frac{1000-960}{1000} \times 100$

$= \frac{40}{10}\% = 4\%$

71. $x^4 + 4y^4$
$= x^3 + 4y^4 + 4x^2y^2 - 4x^2y^2$
$= (x^2 + 2y^2)^2 - (2xy)^2$
$= (x^2 + 2y^2 - 2xy)(x^2 + 2y^2 + 2xy)$
From above it is clear that $x^4 + 4y^4$ is divisible by $x^2 + 2y^2 + 2xy$.

72. $\frac{1}{3}\log_{10} 125 - 2\log_{10} 4 + \log_{10} 32 + \log_{10} 1$

$= \frac{1}{3}\log_{10} (5)^3 - 2\log_{10} (2)^2 + \log_{10}(2)^5 + 0$

$= \log_{10} 5 - 4 \log_{10} 2 + 5\log_{10} 2$

$= \log_{10} 5 + \log_{10} 2 = \log_{10} 10$

$= 1$

73. Q The condition for both the roots of the equation $ax^2 + bx + c = 0$ are positive, is

$-\frac{b}{a} > 0$ and $\frac{c}{a} > 0$

Given equation is $x^2 - 2(k - 1)\, x + (2k + 1) = 0$ whose roots are positive

$$-\frac{b}{a} = +\frac{2(k-1)}{1} > 0$$

$\Rightarrow \quad k > 1$

and $\quad \frac{c}{a} = \frac{2k+1}{1} > 0$

$\Rightarrow \quad k > -\frac{1}{2}$

$\therefore \quad k > 1$

Hence, option (c) is correct.

74. It is true that.

$\{a\} \in \{\{a\}, \{b\}, c\}$

75. It is true that, for $0° < \theta < 90°$ there exist only one θ such that $\sin \theta = a$.

76. Given,

$(u)^3 + (-2y)^3 + (-3w)^3 = 3 \times (-2)(-3)\, uvw$

$\therefore \; u + (-2v) + (-3w) = 0$

$\Rightarrow \quad u - 2y - 3w = 0$

$\Rightarrow \quad u - 2y = 3w$

77. Taking option (d).

The reverse digit of 451 is 154.

Now, 154 + 297 = 451 is equal to the original number.

Hence, option (d) is correct.

78. Median age of minor children is 5 yr.

79. Mean age of minor children $= \frac{15}{3} = 5$ yr

80. Given $\quad PQ^2 = MP^2 + NQ^2$

$= LP^2 + LQ^2$

$\Rightarrow \quad \angle NLP = 90°$

It means,

$\Delta\, NLM$ be a right angled.

Statement-II is also true.

Solution for Question 22 to 23

Total age of six persons = 23.5 × 6

= 141 yr

Total age of three major persons = 42 × 3

= 126 yr

$\therefore$ Remaining age of three minor children

= 141 – 126

= 15 yr

Since the difference in ages of the three minor children is same.

Therefore, we take ages may be,

5, 5, 5; 3, 5, 7; 2, 5, 8 and 1, 5, 9.

So, in all the case median will be 5.

81. We know that, if the difference of the sum of odd digits and sum of even digits is either 0 or multiple of 11, then the number is divisible by 11.

Here $A + C + E - (B + D) = 0$ or divisible by 11

Hence, both statements are true.

82. LCM of (6, 9, 13) = 36

$\therefore$ Number is the form of 36p + 4.

Since the required number between 300 and 400.

$\therefore\, p = 9$ and 10

$\therefore$ Required sum = 328 + 364

= 692

83. We know that the value of cos θ is decreasing from 0 to 90°

$\therefore \quad \cos 1° > \cos 89°$

$\Rightarrow \quad p > q$

Also cos 1° is close to 1 and cos 89° is close to 0.

Hence, option (d) is correct.

84. A line perpendicular to the given line, passing through the given point is the required focus.

85. $2^{12n} - 6^{4n} = (2^{12})^n - (6^4)^n$

$= (4096)^n - (1296)^n$

$= (4096 - 1296)\,[(4096)^{n-1} + (4096)^{n-2}(1296) \ldots - (1296)^n]$
$= 2800\,(k)$

88. Q Weighted mean $= \dfrac{1.1 + 2.2 + \ldots + 10.10}{1 + 2 + 3 + \ldots + 10}$

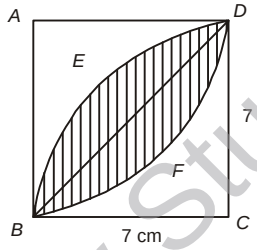
$= \dfrac{1^2 + 2^2 + \ldots + 10^2}{1 + 2 + \ldots + 10}$

$= \dfrac{\dfrac{10(10+1)(20+1)}{6}}{10\left(\dfrac{10+1}{2}\right)}$

$= \dfrac{10 \times 11 \times 21}{6 \times 55} = 7$

91. Total frequency $= 9 + 22 + f_1 + 20 + 12 + 8$
$\Rightarrow 100 = 71 + f_1$
$\Rightarrow f_1 = 29$
$\therefore$ Highest frequency is 29 which lies in the interval 20.5 – 30.5 only

92. Area of curve $BCDE = \dfrac{1}{4}\pi(7)^2$



$= \dfrac{22}{7 \times 4} \times 7 \times 7 = \dfrac{77}{2}\text{cm}^2$

Area of $\Delta BCD = \dfrac{1}{2} \times 7 \times 7 = \dfrac{49}{2}\text{cm}^2$

$\therefore$ Required area of shaded region = 2 area of curve $BEDF$

$= 2\left[\dfrac{77}{2} - \dfrac{49}{2}\right]$

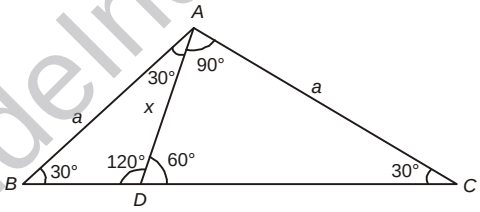
$= 2\left[\dfrac{28}{2}\right] = 28\text{ cm}^2$

94. Let the person invest amount x and y into two different rates of interest.

$\therefore \dfrac{x \times 12 \times 1}{100} + \dfrac{y \times 10 \times 1}{100} = 130 \left(\text{Q SI} = \dfrac{PRT}{100}\right)$
$\Rightarrow 12x + 10y = 13000 \quad \ldots$ (i)

and $\dfrac{y \times 12 \times 1}{100} + \dfrac{x \times 10 \times 1}{100} = 134$
$\Rightarrow 12y + 10x = 13400 \quad \ldots$(ii)
On solving Eqs. (i) and (ii), we get
x = Rs. 500 and y = Rs. 700
Hence, option (d) is correct.

98.

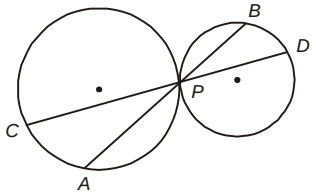


In ΔADC,
$\angle DAC = 180° - 60° - 30° = 90°$
Again in right ΔDAC,

$\tan 60° = \dfrac{AC}{AD}$

$\Rightarrow \sqrt{3} = \dfrac{a}{x} \qquad \Rightarrow x = \dfrac{a}{\sqrt{3}}$

99. It is clearly from the figure that none of the options are correct.



100. An infinite number of circles can be drawn to pass through two given points.