½ **Solved Paper**

ELEMENTRY MATHEMATICS

1. What is the geometric means of the observations 125, 729, 1331?
   (a) 495 (b) 1485
   (c) 2221 (d) None of these
2. Examples of data are given below:
   I. Information on households collected by an investigator by door-to-door visits.
   II. Data on the percentage of literates, sex-wise, for the different districts of a state collected from records of the census of India.
   III. General Information about families, collected by telephonic interviews.
   Which one of the following in respect of the above is correct?
   (a) I and II are primary data
   (b) I and III are primary data
   (c) II and III are primary data
   (d) I, II and III are primary data
3. Which one of the following can be obtained from a historgam?
   (a) Mean (b) Median
   (c) Mode (d) None of these
4. The median of three positive integers, no two of which are equal, is 5. What is the least possible value of arithmetic means of these integers?
   (a) 2 (b) 3
   (c) 4
   (d) No such least possible value exists
5. Consider the following pairs of numbers:
   I. (8, 12) II. (9, 11) III. (6, 24)
   Which pairs of number have the same harmonic means?
   (a) I and II (b) II and III
   (c) I and III (d) I, II and IIII
6. If $27 \times (81)^{2n+3} - 3^m = 0$, then what is m equal to?
   (a) $2n + 5$ (b) $5n + 6$
   (c) $8n + 3$ (d) $8n + 15$
7. There are four numbers forming a GP in which the third term is greater than the first by 9 and the second term is greater than the fourth by 18. What is the first term?
   (a) 2 (b) 3
   (c) – 2 (d) – 3
8. What is the value of $[\log_{13}(10)] / [\log_{169}(10)]$?
   (a) 1 / 2 (b) 2
   (c) 1 (d) $\log_{10}13$
9. A boy travels a distance of 8 km at the rate of 4 km / h, 6 km at the rate of 3 km / h and 4 km at the rate of 2 km / h. What is the average speed for the entire journey?
   (a) 2 km / h (b) 3 km / h
   (c) 4 km / h (d) 6 km / h
10. Which one of the following is not correct in respect of the sets A and B?
    (a) If $A \subseteq B$, then $B \cup A = B$
    (b) If $A \subseteq B$, then $B \cap (A - B)$ ϕ
    (c) If $A \subseteq B$, then $B \cap A = A$
    (d) If $A \cap B = \phi$ then neither $A = \phi$ or $B = \phi$
11. If $0 < \theta < \phi < 90°$, then which one of the following is correct?
    (a) $(\sin \theta + \cos \theta)^2 > 2$
    (b) $(\sin^2 \theta + \cos^2 \phi) \leq 2$
    (c) $(\sin^2 \theta + \cos^2 \phi) < 2$
    (d) $(\sin^2 \theta + \cos^2 \phi) > 2$
12. For what value of θ is $(\sin \theta + \text{cosec } \theta) = 2.5$ where $0 < \theta \leq 90°$?
    (a) 30° (b) 45°
    (c) 60° (d) 90°
13. The angles of a triangle are in AP and the greatest angle is double the least. What is the ratio of angles in the Indian measure?
    (a) 2 : 3 : 4 (b) 1 : 2 : 3
    (c) 3 : 3 : 6 (d) 4 : 5 : 7
14. The length of the shadow of a person s cm tall when the angle of elevation of the sun is α is p cm. It is q cm when the angle of elevation of the sun is β. Which one of the following is correct when $\beta = 3\alpha$?
    (a) $p - q = s\left(\dfrac{\tan\alpha - \tan 3\alpha}{\tan 3\alpha \tan\alpha}\right)$

(b) $p - q = s\left(\frac{\tan 3\alpha - \tan\alpha}{3\tan 3\alpha \tan\alpha}\right)$

(c) $p - q = s\left(\frac{\tan 3\alpha - \tan\alpha}{\tan 3\alpha \tan\alpha}\right)$

(d) $p - q = s\left(\frac{\tan 2\alpha}{\tan 3\alpha \tan\alpha}\right)$

15. If $\cot\theta = \frac{2xy}{x^2 - y^2}$, then what is $\cos\theta$ equal to?

(a) $\frac{x^2 - y^2}{x^2 + y^2}$ (b) $\frac{x^2 + y^2}{x^2 - y^2}$

(c) $\frac{2xy}{x^2 + y^2}$ (d) $\frac{2xy}{\sqrt{x^2 + y^2}}$

16. The value of $\text{cosec}^2\theta - 2 + \sin^2\theta$ is always
(a) less than zero (b) non-negative
(c) zero (d) 1

17. If $\tan\theta = \frac{p}{q}$, then what is

$\frac{p\sec\theta - q\,\text{cosec}\,\theta}{p\sec\theta + q\,\text{cosec}\,\theta}$ equal to?

(a) $\frac{p-q}{p+q}$ (b) $\frac{q^2 - p^2}{q^2 + p^2}$

(c) $\frac{p^2 - q^2}{q^2 + p^2}$ (d) 1

18. What is the value of
$\sin^3 60° \cot 30° - 2\sec^2 45° + 3\cos 60° \tan 45° - \tan 2\,60°$?
(a) 35/8 (b) – 35/8
(c) –11/8 (d) 11/8

19. If $0 < x < 45°$ and $45° < y < 90°$, then which one of the following is correct?
(a) $\sin x = \sin y$ (b) $\sin x < \sin y$
(c) $\sin x > x\sin y$ (d) $\sin x \le \sin y$

20. The difference of the two angles in degree measure is 1 and their sum in circular measure is also I. What are the angles in circular measure?

(a) $\left(\frac{1}{2} - \frac{\pi}{360}\right), \left(\frac{1}{2} + \frac{\pi}{360}\right)$

(b) $\left(\frac{1}{2} - \frac{90}{\pi}\right), \left(\frac{1}{2} + \frac{90}{\pi}\right)$

(c) $\left(\frac{1}{2} - \frac{\pi}{180}\right), \left(\frac{1}{2} + \frac{\pi}{180}\right)$

(d) None of the above

21. $\frac{\cos\theta}{1 - \sin\theta} - \frac{\cos\theta}{1 + \sin\theta} = 2$ is satisfied by which one of the following values of θ?
(a) $\pi/2$ (b) $\pi/3$ (c) $\pi/4$ (d) $\pi/6$

22. Consider the following

I. $\frac{\cos^2\theta - \sin^2\theta}{\cos^2\theta + \sin^2\theta} = \cos^2\theta\,(1 + \tan\theta)(1 - \tan\theta)$

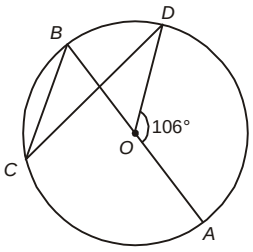
II. $\frac{1 + \sin\theta}{1 - \sin\theta} = (\tan\theta + \sec\theta)^2$

Which of the statements given above is / are correct?
(a) I only (b) II only
(c) Both I and II (d) Neither I nor II

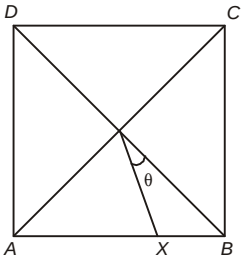
23. What is the angle subtended at the centre of a circle of radius 8 m after traversing 4π m along its circumference?
(a) $\pi/3$ (b) $\pi/2$
(c) $2\pi/3$ (d) $3\pi/4$

24. If $\cot\theta = 8/15$, then what is the value of $\sqrt{\frac{1 - \cos\theta}{1 + \cos\theta}}$ where θ is a positive acute angle?
(a) 1/5 (b) 2/5
(c) 3/5 (d) 4/5

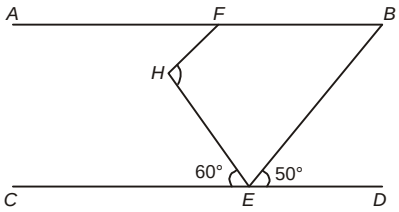
25. In the figure given above, O is the centre of the circle $\angle AOD = 106°$. What is $\angle BCD$ equal to?
(a) 53° (b) 43° (c) 40° (d) 37°



26. The quadrilateral formed by joining the mid-points of the sides *AB, BC, CD, DA* of a quadrilateral *ABCD* is
    (a) a trapezium but not a parallelogram
    (b) a quadrilateral but not a trapezium
    (c) a parallelogram only
    (d) a rhombus

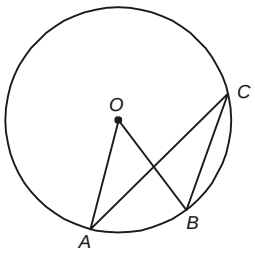
27. 

    In the figure given above, *ABCD* is a square in which *AO* = *AX*. What is ∠*XOB*?
    (a) 22.5° (b) 25°
    (c) 30° (d) 45°

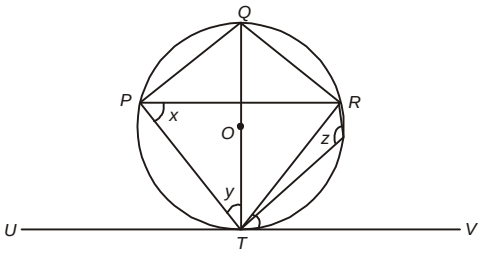
28. 

    In the figure *AB* is parallel to *CD* and *BE* is parallel to *FH*. What is ∠*FHE* equal to?
    (a) 110° (b) 120°
    (c) 125° (d) 130°

29. In the figure given above, ∠*AOB* = 46°; *AC* and *OB* intersect each other at right angles. What is the measure of ∠*OBC*? (*O* is the centre of the circle)



    (a) 44° (b) 46°
    (c) 67° (d) 78.5°

30. 

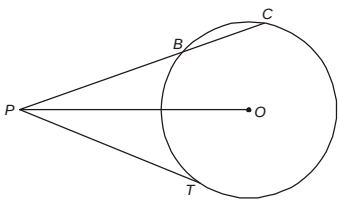
    In the figure given above, O is the centre of the circle. The line UTV is a tangent to the circle at T, ∠VTR = 52° and triangle PTR is an isosceles traingle such that TP = TR. What is ∠*x* + ∠*y* + ∠*s* equal to?
    (a) 175° (b) 208°
    (c) 218° (d) 250°

31. What is the number of points in the plane of a triangle *ABC* which are at equal distance from the vertices of the triangle?
    (a) 0 (b) 1
    (c) 2 (d) 3

32. The bisectors of the angles *ABC* and *BCA* of a triangle *ABC* meet in a point *O*. What is the angle at *O* facing the side *BC*?
    (a) 90° – (*A*/2) (b) 90° + (*A*/2)
    (c) 90° – *A* (d) 90° + *A*

33. *ABC* is a triangle in which *AB* = *AC*. Let *BC* be produced to *D*. From a piont E on the line *AC* let *EF* be a straight line such that *EF* is parallel to *AB*. Consider the quadrilateral *ECDF* thus formed. If ∠*ABC* = 65° and ∠*EFD* = 80°, then what is ∠*FDC* equal to
    (a) 43° (b) 41°
    (c) 37° (d) 35°

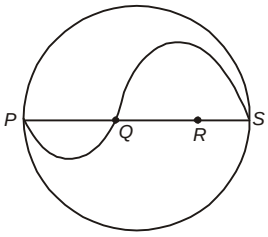
34. ABC is a triangle and the perpendicular drawn from A meets BC in D. If $AD^2 = BD.DC$, then which one of the following is correct?
    (a) ABC must be an obtuse angled triangle
    (b) ABC must be an acute angled triangle
    (c) Either $\angle B \geq 45°$ or $\angle C \geq 45°$
    (d) $AC^2 = AB^2 + BC^2$
35. Consider the following statements:
    I. Congruent triangles are similar.
    II. Similar trianlges are congruent.
    III. If the hypotenuse and a side of one right triangle are equal to the hypotenuse and a side of another right triangle respectively, then the two right triangles are congruent.
    Which of the statement given above is / are correct?
    (a) I only (b) II only
    (c) II and III (d) I and III
36. ABC is triangle. X is a piont outside the triangle ABC such that $CD = CX$, where D is the point of intersection of BC and AX; and $\angle BAX = \angle XAC$. Which one of the following is correct?
    (a) Triangles ABD, ACX are similar
    (b) $\angle ABD < \angle ACD$
    (c) $AC = CX$
    (d) $\angle ADB > \angle DXC$
37. A point P moves such that its distance from two given points A and B are equal. Then what is the focus of the point P?
    (a) A straight line which is the high right bisector of AB
    (b) A circle with centre at A
    (c) A circle with centre of B
    (d) A straight line passing through either A or B
38. In the figure given above, PT is a tangent to a circle of radius 6 cm. If P is at a distance of 10 cm from the centre O and PB = 5 cm, then what is the length of the chord BC?
    (a) 7.8 cm (b) 8.0 cm
    (c) 8.4 cm (d) 9.0 cm
39. A solid cone of height 8 cm and base radius 6 cm is melted and recast into identical cones, each of height 2 cm and radius 1 cm. What is the number of cones formed?
    (a) 36 (b) 72
    (c) 144 (d) 180
40. In the triangle ABC, AB = 2 cm, BC = 3cm, and AC = 4 cm, D is the middle point of AC. If a square is constructed on the side BD, what is the area of the square?
    (a) 4.5 cm^2 (b) 2.5 cm^2
    (c) 6.35 cm^2 (d) None of these
41. From a cylindrical log whose height is equal to its diameter, the greatest possible sphere has been taken out. What is the fraction of the original log which is cut away?
    (a) 1/2 (b) 1/3
    (c) 1/4 (d) 2/3
42. If a sphere of radius 10 cm is intersected by a plane at a distance 8 cm from its centre, what is the radius of the curve of intersection of the plane of the plane and the sphere?
    (a) 8 cm (b) 6 cm
    (c) 5 cm (d) 4 cm
43. A cylindrical can of internal diameter 24 cm contains was. A solid sphere of radius 6 cm is completely immersed to water in the cylinder. The water level increases by
    (a) 0.25 cm (b) 0.5 cm
    (c) 2 cm (d) 3 cm
44. A person rides a bicycle round a circular path of radius SC. The radius of the wheel of the bicycle is 50 cm. The cycle comes to the starting point for the first time in 1 h. What the number of revolutions of the wheel in 15 min?
    (a) 20 (b) 25
    (c) 30 (d) 35



45. *ABC* is a straight right-angled at *C*. If *p* is the length of perpendicular from *C* to *AB* and *a, b, c* are the sides, the which one of the following is correct?
(a) $pa = bc$ (b) $pb = ca$
(c) $pc = ab$ (d) $p^2 = ab$

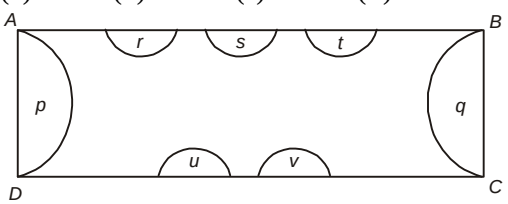
46. A bucket which is in the form of a frustum of a corner radii 3 and 5 unit and vertical height 6 unit. How may water can the bucket hold?
(a) 33 π cu unit (b) 45π cu unit
(c) 48π cu unit (d) None of these

47. How many litres of water (approximately) hemispherical container of radius 21 cm hold?
(a) 19.4 L (b) 38.8 L
(c) 194 L (d) 388 L

48. 

PQRS is a diameter of a circle of radius 6 cm as shown in the figure above. The lengths *PQ*, *QR* and *RS* are equal. Semi-circles are drawn on *PQ* and *QS* as diameters. What is the perimeter of the shaded region?
(a) 12π cm (b) 14π cm
(c) 16π cm (d) 18π cm

49. If the radius of the base and the height of a right circular cone are increased by 20% then what is the approximate percentage increase in volume?
(a) 60 (b) 68 (c) 73 (d) 75

50. 

Seven semi-circular areas are removed from the rectangle *ABCD* as shown in the figure above. In which *AB* = 2 cm and *AD* = 0.5 cm. The radius of each semi-circle *r, s, t, u, v* is half of that of semi-circle p or q. What is the area of the remaining portion?
(a) $(128 - 13\pi)/128$ cm^2
(b) $(125 - 13\pi)/125$ cm^2
(c) $(128 - 15\pi)/128$ cm^2
(d) None of these

51. A wire is in the form of a circle of radius 42 cm. If it is bent into a square, then what is the side of the square?
(a) 66 cm (b) 42 cm
(c) 36 cm (d) 33 cm

52. How many litres of water flow out of a pipe having an area of cross-section of 5 cm^2 in one minute. If the speed of water in the pipe is 30 cm / s?
(a) 90 L (b) 15 L
(c) 9 L (d) 1.5 L

53. A right triangle *ABC* with sides 5 cm, 12 cm and 13 cm is revolved about the side 12 cm. What is the volume of the solid so obtained?
(a) 50π cm^3 (b) 100π cm^3
(c) 125π cm^3 (d) 150π cm^3

54. The ratio of volumes of two cones is 4 : 5 and the ratio of the radii of their bases is 2 : 3. What is the ratio of their vertical heights?
(a) 5 : 6 (b) 6 : 5
(c) 9 : 5 (d) 5 : 9

55. The paint in certain container is sufficient to paint an area equal to 5.875 m^2. How many bricks of dimensions 12.5 cm ×10 × 7.5 cm can be painted out of this container?
(a) 225 (b) 180 (c) 150 (d) 100

56. If *p* is an integer then every square integer is of the form
(a) $2p$ or $(4p - 1)$ (b) $4p$ or $(4p - 1)$
(c) $3p$ or $(3p + 1)$ (d) $4p$ or $(4p + 1)$

57. What least value must be given to ⊕ so that the number 84705 ⊕ 2 is divisible by 9?
(a) 0 (b) 1 (c) 2 (d) 3

58. If $x = (a+\sqrt{a^2+b^2})^{\frac{1}{3}} + (a-\sqrt{a^2+b^3})^{\frac{1}{3}}$ then what is the value of $x^3 + 3bx - 2a$?
(a) $2a^3$ (b) $-2a^3$
(c) 1 (d) 0

59. If the roots of the equation $\frac{x(x-1)-(m+1)}{(x-1)(m-1)} = \frac{x}{m}$ are equal, then what is the value of m?
(a) 1 (b) 1/2
(c) 0 (d) $-1/2$

60. What is $\frac{(x-y)^3+(y-z)^3+(z-x)^3}{4(x-y)(y-z)(z-x)}$ equal to?
(a) $-3/4$ (b) 1/4
(c) 3/4 (d) 0

61. For two natural numbers m and n, let g_{mn} denote the greatest common factor of m and n. Consider the following in respect of three natural numbers k, m and n.
I. $g_{m(nk)} = g_{(mn)k}$
II. $g_{mn}g_{nk} = g_{mk}$
Which of the above is / are correct?
(a) I only (b) II only
(c) Both I and II (d) Neither I nor II

62. What is the value of k for which the system of equations $x + 2y - 3 = 0$ and $5x + ky + 7 = 0$ has no solution?
(a) $-3/14$ (b) $-14/3$
(c) 1/10 (d) 10

63. If $x + y + z = 0$ then what is the value of $\frac{1}{x^2+y^2-z^2} + \frac{1}{y^2+z^2-z^2} + \frac{1}{z^2+x^2-y^2}$?
(a) $\frac{1}{x^2+y^2+z^2}$ (b) 1
(c) -1 (d) 0

64. The length of a line segement AB is 2 unit. It is divided into two parts at the point C such that $AC^2 = AB \times CB$. What is the length of CB?
(a) $3+\sqrt{5}$ unit (b) $3-\sqrt{5}$ unit
(c) $2-\sqrt{5}$ unit (d) $\sqrt{3}$ unit

65. If $pq + qr + rp = 0$, then what is the value of $\frac{p^2}{p^2-qr} + \frac{q^2}{q^2-rp} + \frac{r^2}{r^2-pq}$?
(a) 0 (b) 1
(c) -1 (d) 3

66. If $a = \frac{1+x}{2-x}$, the what is $\frac{1}{a+1} + \frac{2a+1}{a^2-1}$ equal to?
(a) $\frac{(1+x)(2+x)}{2x-1}$
(b) $\frac{(1-x)(2-x)}{x-2}$
(c) $\frac{(1+x)(2-x)}{2x-1}$
(d) $\frac{(1-x)(2-x)}{2x+1}$

67. The following table shows the percentage of male and female coffee drinkers and non-coffee drinkers in two Twons A and B.

Atributes	Town A		Town B	
	Male	Female	Male	Female
Coffee drinkers	40%	5%	25%	15%
Non-coffee drinkers	20%	35%	30%	30%

If the total population of the Twons A and B are 10000 and 20000 respectively, then what is the total number of female coffee drinkers in both towns?
(a) 8000 (b) 6000
(c) 3500 (d) 2500

68. If $(37)^x = (0.037)^y = 10000$, then what is the value of $\frac{1}{x} - \frac{1}{y}$?
(a) 1 (b) 2
(c) 1/2 (d) 1/4

69. If $ax = by = cz$ and $abc = 1$, then what is $xy + yz + zx$ equal
(a) xyz (b) $x + y + z$
(c) 0 (d) 1

70. When a polynomial is divided by alinear polynomial, then what is the remainder?
(a) Constant polynomial only
(b) Zero polynomial only
(c) Either constant o zero polynomial
(d) Liner polynomial

71. What is the ratio of sum of squares of roots to the product of the roots of the equation $7x^2 + 12x + 18 = 0$?
(a) 6 : 1 (b) –6 : 7
(c) – 6 : 1 (d) – 1 : 6

72. If $x = 2+\sqrt{3}$, then what is $(x^2 + x^{-2})$ equal to?
(a) 12 (b) 13
(c) 14 (d) 15

73. Which one of the following statements is correct?
(a) Remainder theorem is a special case of factor theorem.
(b) Factor theorem is a special case of Remainder theorem.
(c) Factor theorem and Remainder theorem are two indepdent results
(d) None of the above

74. When $x^{40} + 2$ is divided by $x^4 + 1$, what is the remainder?
(a) 1 (b) 2
(c) 3 (d) 4

75. If the HCF of $x^3 – 27$ and $x^3 + 4x^2 + 12x + k$ is quadratic polynomial, then what is the value of k?
(a) 27 (b) 9
(c) 3 (d) – 3

76. A number consists of two digits, whose sum is 10. If 18 is subtracted from the number, digits of the number are reversed. What is the product?
(a) 15 (b) 18
(c) 24 (d) 32

77. What is the LCM of 3 $(a^3 – b^3)$ and 11 $(a^4 – b^4)$?
(a) $33(a^3 – b^3)(a^2 + b^2)(a + b)$
(b) $(a^3 – b^3)(a^2 + b^2)(a – b)$
(c) $33 (a – b)(a^2 + b^2 – ab)(a^3 – b^3)$
(d) $33 (a^3 – b^3)(a^4 – b^4)$

78. Students of a class are made to stand in rows. If one student is extra in a row, there would be two rows less. If one student is less is n a row, there would be three rows more. What is is the number of students in the class?
(a) 65 (b) 55
(c) 60 (d) 50

79. A railway ticket for a child costs half the full fare but the reservation charge is the same on half ticket as much as on full ticket. One reserved first class ticket for a jouney between two stations is Rs. 362; one full and one half reserved first class tickets cost Rs. 554. What is reservations charge?
(a) Rs. 18 (b) Rs. 22
(c) Rs. 38 (d) Rs. 46

80. Out of a group of swans 7/2 times the square root of the number are swimming in the pool while the two remaining are playing outside the pool. What is the total number of swans?
(a) 4 (b) 8
(c) 12 (d) 16

81. If the ratio of x to y is 25 times the ratio of y to x, then what is the ratio of x to y?
(a) 1 : 5 (b) 5 : 1
(c) 25 : 1 (d) 1 : 25

82. A milk vendor bought 28 L of milk at the cost of Rs. 8.50 per litre. After adding some water, he sold the mixture at the same price. If he gains 12.5%, how much water did he add?
(a) 5.5 L (b) 4.5 L
(c) 3.5 L (d) 2.5 L

83. If m and n are two integers such that $m = n^2 – n$, then $(m^2 – 2m)$ is always divisible by
(a) 9 (b) 16
(c) 24 (d) 48

84. Which one is the largest among the following?
(a) 0.725 (b) 0.725
(c) 0.725 (d) 0.725

85. 10% of the inhabitants of a certain city left that city. Later on 10% of the remaining inhabitants of that city again left the city. What is the remaining percentage of population of that city?
(a) 80% (b) 80.4%
(c) 80.6% (d) 81%

86. If the rate of interest is 10% per annum and is compounded half yearly, the principal of Rs. 400 in 3/2 yr will amount to
(a) Rs. 463.00 (b) Rs. 463.05
(c) Rs. 463.15 (d) Rs. 463.20

87. The number of workers in the employment guarantee scheme increased by 15 which resulted into an increase of 20%. What are the initial number of workers?
(a) 60 (b) 75
(c) 80 (d) 90

88. Two men *P* and *Q* start from a place walking at 5 km at 6.5 km/h respectively. What is the time they will take to ?? 92 km apart if they walk in opposite directions?
(a) 2 h (b) 4 h
(c) 6 h (d) 8 h

89. In an army camp ration is available for 100 soldier 10 days. After 2 days, 60 soldiers joined. Then for many more days will the remaining ration last?
(a) 7 days (b) 6 days
(c) 5 days (d) 4 days

90. Two trains travel in the same direction at 50 km/h and 32 km/h respectively. A man in the slower train observes that 15 s elapse before the faster train completely passes him. What is the length of the faster train?
(a) 75 m (b) 125 m
(c) 150 m (d) $\frac{625}{3}$ m

91. A person walks a distance in 114 days, when he rests 9 h a day. How long will he take to walk twice the distance if he walks twice as fast and rests twice as long each day as before?
(a) 57 days (b) 228 days
(c) 285 days (d) 324 days

92. If $x : y = 1 : 3$, $y : z = 5 : k$, $z : t = 2 : 5$ and $t : x + 3 : 4$, then what is the value of k?
(a) 1/2 (b) 1/3
(c) 2 (d) 3

93. The number $\sqrt{0.0001}$ is
(a) a rational number less than 0.01
(b) a rational number
(c) an irrational number
(d) neither a rational number nor an irrational number

94. A person P started a business with a capital of Rs. 2525 and another person Q joined P after some months with a capital of Rs. 1200. Out of the total annual profit of Rs. 1644, P's share was Rs. 1212. When did *Q* join as partners?
(a) After 2 months (b) After 9 months
(c) After 4 months (d) After 5 months

95. The mess charges for 35 students for 24 days in Rs. 6300. In how many days will the mess charges be Rs. 3375 for 25 students?
(a) 12 (b) 15
(c) 18 (d) 21

96. If $a + b = 2m^2$, $b + c = 6m$, $a + c = 2$, where m is a real number and $a \le b \le c$, then which one of the following is correct?
(a) $0 \le m \le 1/2$ (b) $-1 \le m \le 0$
(c) $1/3 \le m \le 1$ (d) $1 < m \le 2$

97. What is the least number which when divided by 42, 72 and 84 leaves the remainders 25, 55 and 67 respectively?
(a) 521 (b) 512
(c) 504 (d) 487

98. A trader sells two cycles at Rs. 1188 each and gains 10% on the first and loses 10% on the second. What is the profit or loss percent on the whole?
(a) 1% loss (b) 1% gain
(c) No loss no gain (d) 2% loss

99. A person borrowed Rs. 7500 at 16% compound interest. How much does he have to pay at the end of 2 yr to clear the loan?
(a) Rs. 9900 (b) Rs. 10092
(c) Rs. 11000 (d) Rs. 11052

100. *P* and *Q* can do a job in 2 days: *Q* and *R* can do it in 4 days; and *P* and *R* in 12/5 days. What is the number of days required for *P* alone to do the job?
(a) 5/2 (b) 3
(c) 14/5 (d) 6

ANSWERS

1. (a)	2. (b)	3. (c)	4. (c)	5. (c)	6. (d)	7. (b)	8. (b)	9. (b)	10. (d)
11. (c)	12. (a)	13. (a)	14. (c)	15. (c)	16. (b)	17. (c)	18. (b)	19. (b)	20. (b)
21. (c)	22. (c)	23. (b)	24. (c)	25. (d)	26. (c)	27. (a)	28. (a)	29. (c)	30. (c)
31. (b)	32. (b)	33. (d)	34. (d)	35. (d)	36. (a)	37. (a)	38. (a)	39. (c)	40. (b)
41. (d)	42. (b)	43. (c)	44. (b)	45. (c)	46. (d)	47. (a)	48. (a)	49. (c)	50. (a)
51. (a)	52. (c)	53. (b)	54. (c)	55. (d)	56. (c)	57. (b)	58. (d)	59. (d)	60. (c)
61. (d)	62. (a)	63. (d)	64. (b)	65. (b)	66. (c)	67. (c)	68. (c)	69. (c)	70. (c)
71. (b)	72. (c)	73. (b)	74. (c)	75. (b)	76. (c)	77. (a)	78. (c)	79. (b)	80. (d)
81. (b)	82. (c)	83. (c)	84. (d)	85. (d)	86. (b)	87. (b)	88. (d)	89. (c)	90. (a)
91. (c)	92. (a)	93. (b)	94. (b)	95. (c)	96. (c)	97. (d)	98. (a)	99. (b)	100. (b)

EXPLANATIONS

1. $GM = \sqrt[3]{125 \times 729 \times 1331}$
$= \sqrt[3]{5^3 \times 9^3 \times 11^3}$
$= 5 \times 9 \times 11$
$= 495$

2. Statement Ist and IIIrd are primary data.

3. Mode can be obtained from historgram.

4. Let three positive integers whose median is 5 are 1, 5, 6
$\therefore \quad AM = \frac{1+5+6}{3} = \frac{12}{3} = 4$

5. I. $HM = \frac{2 \times 8 \times 12}{8+12}$
$= \frac{2 \times 8 \times 12}{20}$
$= \frac{48}{5} = 9.6$

II. $HM = \frac{2 \times 9 \times 11}{9+11}$
$= \frac{2 \times 9 \times 11}{20} = 9.9$

III. $HM = \frac{2 \times 6 \times 24}{6+24}$
$= \frac{2 \times 6 \times 24}{30}$
$= \frac{48}{5} = 9.6$

Hence Ist & IIrd pirs have same harmonic means.

6. Given $27 \times (81)^{2n+3} - 3m = 0$
$\Rightarrow \quad 33 \times (3)^{8n+12} = 3^m$
$\Rightarrow \quad 3^{8n+15} = 3^m$
$\Rightarrow \quad m = 8n + 15$

7. Let the GP series be
$a, ar, ar^2, ar^3, ar^4, \ldots$
According to the question
$ar^2 = a + 9$
$\Rightarrow \quad a(r^2 - 1) = 9 \quad \ldots$ (i)

and $ar = ar^3 + 18$

$\Rightarrow \quad ar(1 - r^2) = 18 \quad \ldots$ (ii)

$\therefore \quad \frac{a(r^2 - 1)}{-ar(r^2 - 1)} = \frac{9}{18}$

$\Rightarrow \quad -r = 2$

$\therefore$ From Eq. (i) $a(4 - 1) = 9$

$\Rightarrow \quad a = 3$

8. $\frac{\log_{13}(10)}{\log_{169}(10)} = \frac{\log_{13}(10)}{\log_{13^2}(10)}$

$= \frac{1}{1/2} = 2$

9. Ist case, $t = \frac{d}{s}$

$\Rightarrow \quad t = \frac{8}{4} = 2h$

IIn case, $t = \frac{6}{3} = 2h$

IIIrd case, $t = \frac{4}{2} = 2h$

Average speed $= \frac{\text{total distance}}{\text{total time}}$

$= \frac{8+6+4}{2+2+2} = \frac{18}{6}$

10. If $A \cap B = \phi$, then it is not necessary that either $A = \phi$ or $B = \phi$.

11. For $0 < \theta < \phi < 90°$

$0 < \sin^2\theta, \cos^2\theta < 1$

$\therefore \sin^2\theta + \cos^2\phi < 2$

12. Given, $(\sin\theta + \text{cosec}\,\theta) = 2.5$

$\Rightarrow \left(\sin\theta + \frac{1}{\sin\theta}\right) = \frac{5}{2}$

$\Rightarrow 2\sin^2\theta - 5\sin\theta + 2 = 0$

$\Rightarrow 2\sin^2\theta - 4\sin\theta - \sin\theta + 2 = 0$

$\Rightarrow 2\sin\theta(\sin\theta - 2) - 1(\sin\theta - 2) = 0$

$\Rightarrow (2\sin\theta - 1)(\sin\theta - 2) = 0$

$\Rightarrow \sin\theta = 1/2 \quad (\text{Q } \sin\theta \neq 2)$

$\Rightarrow \theta = 30°$

13. Let angles of a triangle in AP are $a, a + d, a + 2d$

Also, $a + 2d = 2a$

$a + a + d + a + 2d = 180°$

$\Rightarrow \quad 3a + 3d = 180°$

$\Rightarrow \quad 3a + 3\left(\frac{a}{2}\right) = 180°$

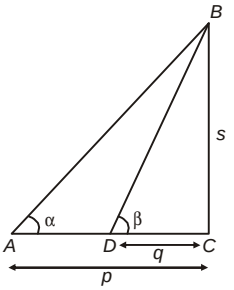
$\Rightarrow \quad 9a = 360°$

$\Rightarrow a = 49°$, d $= 20°$

$\therefore$ Ratio of angles $= 40° : 60° : 80°$

$= 2 : 3 : 4$

14. In ΔBAC,



$\tan\alpha = \frac{s}{P}$

$\Rightarrow \quad P = \frac{s}{\tan\alpha} \quad \ldots$(i)

In ΔBDC,

$\tan\beta = \frac{s}{q}$

$\Rightarrow \quad q = \frac{s}{\tan 3\alpha} \quad \ldots$(ii)

On subtracting Eqs. (ii) from (i), we get

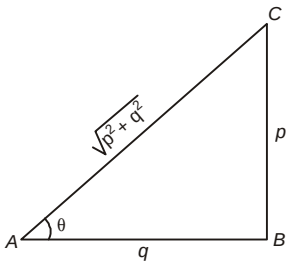
$p - q = \frac{s}{\tan\alpha} - \frac{s}{\tan 3\alpha}$

$= s\left[\frac{\tan 3\alpha - \tan\alpha}{\tan\alpha \tan 3\alpha}\right]$

16. $\text{cosec}^2\theta - 2 + \sin^2\theta = (\sin\theta - \text{cosec}\,\theta)^2$

Hence, it is always non-negative

17. Given $\tan\theta = \frac{p}{q}$



$$\Rightarrow \sec\theta = \frac{\sqrt{p^2+q^2}}{q} \text{ and } \operatorname{cosec}\theta = \frac{\sqrt{p^2+q^2}}{p}$$

$$\therefore \frac{p\sec\theta - q\operatorname{cosec}\theta}{p\sec\theta + q\operatorname{cosec}\theta}$$

$$= \frac{p\left(\frac{\sqrt{p^2+q^2}}{q}\right) - q\left(\frac{\sqrt{p^2+q^2}}{p}\right)}{p\left(\frac{\sqrt{p^2+q^2}}{q}\right) + q\left(\frac{\sqrt{p^2+q^2}}{p}\right)}$$

$$= \frac{\frac{p}{q} - \frac{q}{p}}{\frac{p}{q} + \frac{q}{p}} = \frac{p^2-q^2}{p^2+q^2}$$

18. $\sin^3 60° \cot 30° - 2\sec^2 45° + 3\cos 60° \tan 45° - \tan 2\ 60°$

$$= \left(\frac{\sqrt{3}}{2}\right)^3 (\sqrt{3}) - 3(\sqrt{2})^2 + 3\left(\frac{1}{2}\right)(1) - (\sqrt{3})^2$$

$$= \frac{9}{8} - 4 + \frac{3}{2} - 3 = \frac{-35}{8}$$

19. As we know sin x is increasing from 0 to 90°
$\therefore$ sin y > sin x

20. Let angles in circular measure are A and B,
then degree measures will be $\frac{\pi A}{180°}$ and $\frac{\pi B}{180°}$

According to the question

$$A + B = 1 \quad \text{... (i)}$$

and $$\frac{\pi A}{180°} - \frac{\pi B}{180°} = 1 \quad \text{...(ii)}$$

On solving Eqs. (i) and (ii), we get

$$A = \frac{90}{\pi}\left(\frac{\pi}{180} + 1\right)$$

$$\Rightarrow \quad A = \left(\frac{1}{2} + \frac{90}{\pi}\right)$$

From Eq. (i)

$$\frac{1}{2} + \frac{90}{\pi} + B = 1$$

$$\Rightarrow \quad B = \left(\frac{1}{2} - \frac{90}{\pi}\right)$$

21. Given $\frac{\cos\theta}{1-\sin\theta} - \frac{\cos\theta}{1+\sin\theta} = 2$

$$\Rightarrow \frac{\cos\theta + \sin\theta\cos\theta - \cos\theta + \cos\theta\sin\theta}{1-\sin^2\theta}$$

$\Rightarrow 2\sin\theta\cos\theta - 2\cos^2\theta$
$\Rightarrow \cos^2\theta(\tan\theta - 1) = 0$
$\Rightarrow \tan\theta - 1$ and $\cos\theta \neq 0$

$$\Rightarrow \theta = \frac{\pi}{4}$$

22. I. RHS = $\cos^2\theta(1+\tan\theta)(1-\tan\theta)$
$= \cos^2\theta(1-\tan^2\theta)$

$$= \cos^2\theta\left(\frac{\cos^2\theta - \sin^2\theta}{\cos^2\theta}\right)$$

$$= \frac{\cos^2\theta - \sin^2\theta}{\cos^2\theta + \sin^2\theta} = \text{LHS}$$

II. LHS $= \frac{1+\sin\theta}{1-\sin\theta} = \frac{(1+\sin\theta)^2}{1-\sin^2\theta}$

$$= \left(\frac{1+\sin\theta}{\cos\theta}\right)^2$$

$= (\sec\theta + \tan\theta)$

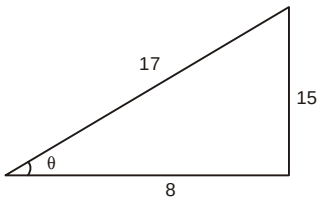
Hence, both statements are correct.

23. Angle subtended $= \frac{\text{arc}}{\text{radius}}$

$$= \frac{4\pi}{8} = \frac{\pi}{2}$$

24. Given $\cot\theta = \frac{8}{15}$

$$\Rightarrow \quad \cos\theta = \frac{8}{17}$$

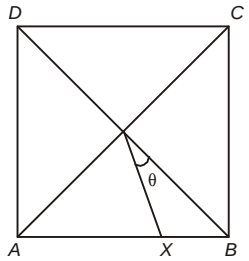


$$\therefore \quad \sqrt{\frac{1-\cos\theta}{1+\cos\theta}} = \sqrt{\frac{1-\frac{8}{17}}{1+\frac{8}{17}}}$$

$$= \sqrt{\frac{9}{25}} = \frac{3}{5}$$

26. The quadrilateral formed by joining the mid points of the sides is a parallelogram.

27. 74. Let $\angle XOB = 0$



In ΔOXB,

$\angle XOB + \angle OBX + \angle OXB = 180°$

$\Rightarrow \theta + 45° + \angle OXB = 180°$

$\Rightarrow \quad \angle OXB = 180° - 45° - \theta$

$\Rightarrow \quad \angle OXB = 135° - \theta$

Here, $\angle OXA + \angle OXB = 180°$

$\Rightarrow \angle OXA + 135° - \theta = 180°$

$$\angle OXA = 45° + \theta$$

In ΔOAX,

$AO = OX$ (given)

$\therefore \angle OXA = \angle AOX = 45° + \theta$

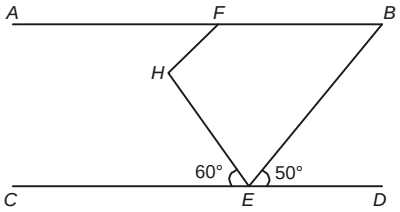
Since, $\angle AOX + \angle XOB = 90°$

$\Rightarrow \quad 45 + \theta + \theta = 90°$

$\Rightarrow \quad 20 = 45°$

$$\Rightarrow \quad \theta = \frac{45°}{2}$$

28. $\angle HEB = 180° - 60° - 50° = 70°$



Since, $HF \| BE$ and HE is transversal

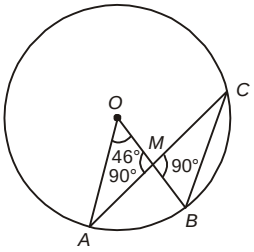
$\therefore \quad \angle FHE + \angle HEB = 180°$

(co-interior angle)

$\Rightarrow \quad \angle FHE + 70° = 180°$

$\Rightarrow \quad \angle FHE = 110°$

29. Since anlge subtend on the circumference is half of the angle subtend on centre.



$$\therefore \quad \angle ACB = \frac{1}{2}\angle AOB$$

$$= \frac{1}{2} \times 46° = 23°$$

In ΔMCB,

$\angle C + \angle B + \angle M = 180°$

$\Rightarrow \quad 23° + \angle B + 90° = 180°$

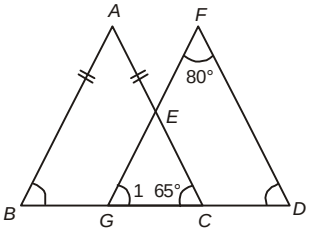
$\Rightarrow \quad \angle B = 67°$

31. Number of point is 1, because circumcentre is the only piont in the plane of a triangle, which is equidistant from the centre.

32. In a ΔABC, the bisector of $\angle B$ and $\angle C$ intersect each other at a point O.

Then $\angle BOC = 90° + \frac{1}{2}\angle A$

33. Here $\angle B = \angle C = 65°$



$\angle 1 = \angle B = 65°$ (corresponding angles)

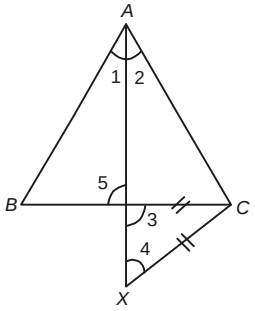
$\Rightarrow$ $65° + 80° + \angle D = 180°$

$\Rightarrow$ $\angle D = 35°$

34. Hence, option (d) is correct

35. Hence statement Ist and IIIrd are true.

36. In ΔDCX



$CD = CX$

$\angle 3 = \angle 4$

(opposite angle of same sides)

But $\angle 3 = \angle 5$

So, $\angle 4 = \angle 5$

In ΔABD and in ΔACX

$\angle 1 = \angle 2$ (given)

$\angle 4 = \angle 5$

$\angle B = \angle ACX$ (Rest angle)

$\therefore$ $\Delta ABD \sim \Delta ACX$

37. The focus of p is a straight line which is the right bisector of AB.

39. Volume of bigger cone $= \frac{1}{3}\pi(6)^2 \times 8$

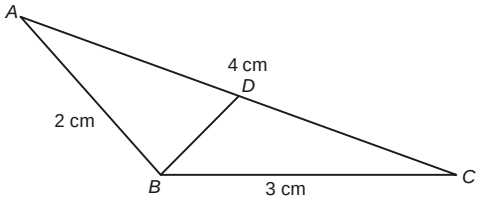
$= 96\pi$ cm^2

Volume of smaller cone $= \frac{1}{3}\pi(1)^2 \times 2$

$= \frac{2\pi}{3}$ cm^2

Required no. of cones $= \frac{96\pi}{\frac{2\pi}{3}} = 144$

40. In ΔABC,



$\cos A = \frac{b^2 + c^2 - a^2}{2bc} = \frac{4^2 + 2^2 - 3^2}{2 \times 4 \times 2}$

$= \frac{16 + 4 - 9}{16} = \frac{11}{16}$

In ΔABD,

BD $= a$, $b = 2$, $d = 2$

$\therefore$ $\cos A = \frac{b^2 + d^2 - a^2}{2bd}$

$\Rightarrow$ $\frac{11}{16} = \frac{2^2 + 2^2 - a^2}{2 \times 2 \times 2}$

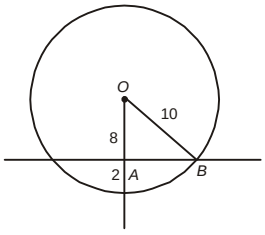
$\Rightarrow$ $11 = 16 - 2a^2$

$\Rightarrow$ $a^2 = \frac{5}{2}$

$\Rightarrow$ $a^2 = 2.5$

$\therefore$ Area of square $= (BD)^2 = a^2 = 2.5$ cm^2

42. In right ΔOAB



$AB^2 = 10^2 - 8^2 = 36$

$\Rightarrow \quad AB = 6\,\text{cm}$

43. Suppose water level increase by x cm.

$\therefore$ Volume of cylindrical can $= \pi (12)^2 \times x$

Volume of sphere $= \frac{4}{3}\pi(6)^3 = 288\pi\ \text{cm}^3$

$\therefore \quad \pi 144x = 288\pi$

$\Rightarrow \quad x = 2\,\text{cm}$

44. Circumsference of parth $= 2\pi \times 50$ m

$= 10000\pi$ cm

Circumference of wheel $= 2\pi \times 50 = 100\pi$ cm

Distance covered in 60 min $= 10000\pi$

Distance covered in 1 min $= \frac{10000}{60}\pi$

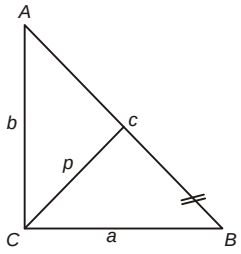
Distance covered in 15 min $= \frac{10000}{60}\pi \times 15$

$= 2500\,\pi$

No. of revolution $= \frac{2500\pi}{100\pi} = 25$

45. Area of $\Delta ABC = \frac{1}{2}\,ab$

Again, area of $\Delta\, ABC = \frac{1}{2}cp$



$\therefore \quad \frac{1}{2}\,ab = pc$

$\Rightarrow \quad pc = ab$

46. Volume of frustum of cone

$= \frac{\pi h}{3}[R^2 + r^2 + Rr]$

$= \frac{\pi \times 6}{3}[5^2 + 3^2 + 15]$

$= \pi \times 2 \times 49$

$= 98\pi$ cu unit

47. Volume of hemisphere $= \frac{2}{3}\pi r^3$

$= \frac{2}{3} \times \frac{22}{7} \times 21 \times 21 \times 21$

$= 44 \times 441$

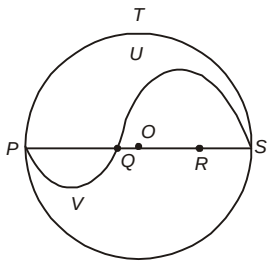
$= 19404\ \text{cm}^3$

$= \frac{19404}{1000}L$

$= 19.4\,\text{L}$

48. Given $OS = 6$ cm

$\therefore PQ = QR = RS = 4$ cm



$\therefore$ Perimeter of shaded region

= Perimeter of semicircle PTS + Perimeter of semicircle QUS + perimeter of semicricle PVQ

$= \pi(6) + \pi(4) + \pi(2)$

$= 12\pi$ cm

49. Since, volume of cone $V_1 = \frac{4}{3}\pi r^2 h$

After increasing by 20% of radius and height then volume of cone

$V_2 = \frac{4}{3}\pi\left(1\frac{1}{5}\right)^2 r\left(1+\frac{1}{5}\right)h$

$= \frac{4}{3}\pi \times \frac{216}{125}r^2 h$

$\therefore$ Required percentage $= \frac{V_2 - V_1}{V_1} \times 100$

$= \frac{\frac{4}{3}\pi r^2 h\left(\frac{216}{125} - 1\right)}{\frac{4}{3}\pi r^2 h} \times 100$

$= \frac{91}{125} \times 100 = 72.8\%$

$= 73\%$ (approx.)

50. Area of 2 bigger semicircle $= 2 \times \frac{\pi r^2}{2}$

$= 2\pi\left(\frac{0.5}{2}\right) \times \frac{1}{2}$

$= \frac{0.25\pi}{4}$ cm^2

Area of 5 smaller semicircles $= \frac{5\pi r^2}{2}$

$= 5 \times \pi \times \frac{1}{2} \times \left(\frac{0.5}{4}\right)^2$

$= \frac{5\pi}{2} \times \frac{0.25}{16}$

$= \frac{1.25\pi}{32}$ cm^2

Area of rectangle $ABCD = 2 \times 0.5 = 1$ cm^2

Area of remaining portion

$= 1 - \frac{0.25}{4}\pi - \frac{1.25\pi}{32}$

$= 1 - \frac{\pi}{16} - \frac{5\pi}{128}$

$= \frac{128 - 8\pi - 5\pi}{128}$

$= \frac{128 - 13\pi}{128}$ cm^2

51. Circumference of circle $= 2\pi \times 42$

$= 2 \times \frac{22}{7} \times 42 = 264$ cm

Perimeter of square $= 4x$

$\Rightarrow \quad 264 = 4x$

$\Rightarrow \quad x = 66$ cm

52. Height of water in a second = 30 cm

Height of water in a 60 second = 30 × 60

i.e., $h = 1800$ cm

Area of cross section $\pi r^2 = 5$ cm^2

Volume of water flow in one minute $= \pi r^2 h$

$= 5 \times 1800$

$= 9000$ cm^3

$= \frac{9000}{1000} L$

$= 9$ L

54. Given, $\frac{\text{volume of Ist cone}}{\text{volume of IIn cone}} = \frac{4}{5}$

$\therefore \quad \frac{\frac{1}{3}\pi(2)^2 h_1}{\frac{1}{3}\pi(3)^2 h_2}$

$\Rightarrow \quad \frac{4h_1}{9h_2} = \frac{4}{5}$

$\Rightarrow \quad \frac{h_1}{h_2} = \frac{9}{5}$

55. Let $l = 12.5$ cm,

$b = 10$ cm,

$h = 7.5$ cm

$\therefore$ Area of a brick $= 2(lb + bh + hl)$

$= 2[12.5 \times 10 + 10 \times 7.5 + 12.5 \times 7.5]$

$= 2[125 + 75 + 93.75]$

$= 2 \times 293.75 = 587.50$ cm^2

Area to be painted $= 5.875$ m^2

$= 5.875 \times 10^4$ cm^2

$= 58750$ cm^2

No. of bricks $= \frac{58750}{587.50} = 100$

56. In option (c).

When we put $p = 1, 2, 3, 4, \ldots$ we get

3 or 4; 6 or 7, 9 or 11, 12 or 13, 15 or 16 ....

Hence, we get all square integers.

57. The given number is divisible by 9 if last two digits is divisible by 3.

$\therefore$ We take $\otimes = 1$

58. Given

$x = (a + \sqrt{a^2 + b^3})^{1/3} + (a - \sqrt{a^2 + b^3})^{1/3}$

On cubing both sides, we get

$$x^3 = (a+\sqrt{a^2+b^3})+(a-\sqrt{a^2+b^3}) + 3(a+\sqrt{a^2+b^3})^{1/3}(a-\sqrt{a^2+b^3})^{1/3} \{(a+\sqrt{a^2+b^3})^{1/3}+(a+\sqrt{a^2+b^3})^{1/3}\}$$

$$\Rightarrow \quad x^3 = 2a - 3b\,(x)$$

$$\Rightarrow \quad x^3 + 3bx - 2a = 0$$

59. Given, $\frac{x(x-1)-(m+1)}{(x-1)(m-1)} = \frac{x}{m}$

$\Rightarrow m\,(x^2 - x - m - 1)\; x\;(mx - x - m + 1)$

$\Rightarrow mx^2 - mx - m\,(m+1)$

$= mx^2 - x^2 - mx + x$

$\Rightarrow x^2 - x - m\,(m+1) = 0$

Let roots be α and α

$\therefore \alpha + \alpha = 1,\ \alpha.\ \alpha = -\ m\,(m+1)$

$$\Rightarrow \alpha = \frac{1}{2}\left(\frac{1}{2}\right)^2 = -\,m\,(m+1)$$

$$\Rightarrow \quad 4m^2 + 4m + 1 = 0$$

$$\Rightarrow \quad (2m+1)^2 = 0$$

$$\Rightarrow \quad m = -\frac{1}{2}$$

60. We know, if $a + b + c = 0$

$\therefore a^3 + b^3 + c^3 = 3abc$

Here $x - y + y - z + z - x = 0$

$$\therefore \quad \frac{(x-u)^3+(y-z)^3+(z-x)^3}{4(x-y)(y-z)(z-x)}$$

$$= \frac{3(x-y)(y-z)(z-x)}{4(x-y)(y-z)(z-x)} = \frac{3}{4}$$

61. I. Let 3 natural no, are $m = 16$, $n = 15$, $k = 20$

$\therefore g_{m\,(nk)} = g_{16(300)} g_{(mn)k} = g_{240(20)}$

$= 4 = 20$

$\therefore \quad g_{m(nk)} \neq g_{mn(k)}$

II. $\quad g_{mn}\, g_{nk} = g_{16\times 15}\, g_{15\times 20}$

$= 1 \times 5 = 5$

and $\quad g_{mk} = g_{16\times 20} = 4$

$\therefore g_{mn}\, g_{nk} \neq g_{mk}$

So neither I nor II is true

62. Since given system of equations

$x + 2y - 3 = 0$ and $5x + ky + 7 = 0$

has no solution.

$$\therefore \quad \begin{vmatrix} 1 & 2 \\ 5 & k \end{vmatrix} = 0$$

$$\Rightarrow \quad k - 10 = 0$$

$$\Rightarrow \quad k = 10$$

63. Given $x + y + z = 0$...(i)

Now $\quad \frac{1}{x^2+y^2-z^2} = \frac{1}{z^2-2xy-z^2}$

$$= -\frac{1}{2xy}$$

$$\therefore \quad \frac{1}{x^2+y^2-z^2}+\frac{1}{y^2+z^2-x^2}+\frac{1}{z^2+x^2-y^2}$$

$$= \frac{1}{-2xy}+\frac{1}{-2yz}+\frac{1}{-2zx}$$

$$= -\frac{1}{2}\left[\frac{z+x+y}{xyz}\right]$$

$= 0$ [from Eq. (i)]

64. Given $AC^2 = AB \times CB$

$$\Rightarrow \quad x^2 = 2 \times (2 - x)$$

$$\Rightarrow \quad x^2 = 4 - 2x$$

$$\Rightarrow \quad x^2 + 2x - 4 = 0$$

$$\Rightarrow \quad x = \frac{-2 \pm \sqrt{4+16}}{2\times 1}$$

$$\Rightarrow \quad x = -1 \pm \sqrt{5}$$

Now, $BC = 2 - (-1 + \sqrt{5})$

$= 3 - \sqrt{5}$ (neglect $3 + \sqrt{5}$)

65. Given $pq + qr + rp = 0$

$$\therefore \quad \frac{p^2}{p^2-qr}+\frac{q^2}{q^2-rp}+\frac{r^2}{r^2-pq}$$

$$= \frac{p^2}{p^2+rp+pq}+\frac{q^2}{q^2+pq+qr}+\frac{r^2}{r^2+qr+rp}$$

$$= \frac{p}{p+r+q} + \frac{q}{q+p+r} + \frac{r}{r+q+p}$$

$$= \frac{p+q+r}{p+q+r} = 1$$

66. Given, $a = \frac{1+x}{2-x}$

$$\therefore \frac{1}{a+1} + \frac{2a+1}{a^2-1} = \frac{3a}{a^2-1}$$

$$= \frac{3\left(\frac{1+x}{2-x}\right)}{\left(\frac{1+x}{2-x}\right)-1}$$

$$= \frac{3(1+x)(2-x)}{1+x^2+2x-(4+x^2-4x)}$$

$$= \frac{3(1+x)(2-x)}{6x-3} = \frac{(1+x)(2-x)}{(2x-1)}$$

67. Total number of female coffee dirnkers
= 5% of 10000 + 15% of 20000
= 500 + 3000
= Rs. 3500

68. Given $(3.7)^x = (0.037)^y = 10000$
$\Rightarrow (37)^x = 10^4$ and $(0.037)^y = 10^4$

$$\Rightarrow 37 = 10^{\frac{4}{x}+1} \text{ and } 37 = 10^{\frac{4}{y}+3}$$

$$\Rightarrow 10^{\frac{4}{x}+1} = 10^{\frac{4}{y}+3}$$

$$\Rightarrow \frac{4}{x}+1 \quad \frac{4}{y}+3$$

$$\Rightarrow \text{d}\frac{4}{x} - \frac{4}{y} = 3-1$$

$$\Rightarrow \frac{1}{x} - \frac{1}{y} = \frac{1}{2}$$

69. Given $a^x = b^y = c^z = k$ (say)

$$\Rightarrow \quad a = k^{1/x},$$

$$b = \text{k}^{1/y},$$

and $\quad c = k^{1/z}$

$$\therefore \quad abc = k^{\frac{1}{x}+\frac{1}{y}+\frac{1}{z}}$$

$$\Rightarrow \quad 1 = k^{\frac{1}{x}+\frac{1}{y}+\frac{1}{z}}$$

$$\Rightarrow \quad \frac{1}{x}+\frac{1}{y}+\frac{1}{z} = 0$$

$$\Rightarrow \quad xy + yz + zx = 0$$

70. When a polynomial is divided by a linear polynomial, then the remainder is either constant or zero polynomial.

71. Let roots, be α and β

$$\therefore \alpha + \beta = \frac{-12}{7} \text{ and } \alpha\beta = \frac{18}{7}$$

$$\Rightarrow \alpha^2 + \beta^2 + 2\alpha\beta = \frac{144}{49}$$

$$\Rightarrow \alpha^2 + \beta^2 = \frac{144}{49} - \frac{36}{7} = \frac{108}{49}$$

$$\therefore \frac{\alpha^2+\beta^2}{\alpha\beta} = \frac{\frac{49}{18}}{7} = -\frac{6}{7}$$

72. $\therefore x^2 + \frac{1}{x^2} = \left(x\frac{1}{x}\right)^2 - 2$

$$= \left(2+\sqrt{3}+\frac{1}{2+\sqrt{3}}\right)^2 - 2$$

$$= \left(2+\sqrt{3}+\frac{2-\sqrt{3}}{1}\right)^2 - 2 \quad 16 - 2 = 14$$

73. Factor theorem is a special case of remainder thorem.

74. Let $\quad f(x) = x^{40} + 2$

Put $x^4 = -1$

$\therefore f(x) = (-1)^{10} + 2 = 3$

76. Let the two digits number be $10y + x$.
According to the given condition,

$$x + y = 10,$$

and $\quad 10y + x - 18 = 10x + y$

$\Rightarrow \quad 9x - 9y = -18$

$\Rightarrow \quad x - y = -2$

On solving, we get

$x = 4$ and $y = 6$

$\therefore$ Product $= xy = 4 \times 6 = 24$

77. Now, $3(a^3 - b^3) = 3(a - b)(a^2 + b^2 + ab)$

and $11(a^4 - b^4) = 11(a - b)(a + b)(a^2 + b^2)$

$\therefore$ Required LCM $= 33(a^3 - b^3)(a + b)(a^2 + b^2)$

78. Let total no. of students $= x$

and total no. of rows $= y$

Students in each row $= \frac{x}{y}$

According to given condition

$$\frac{x}{y} - \frac{x}{y+1} = 2 \qquad ...(i)$$

and $\frac{x}{y-1} - \frac{x}{y} = 3$

$$\Rightarrow \quad -\frac{x}{y} + \frac{x}{y-1} = 3 \qquad ...(ii)$$

From Eqs. (i) and (ii), we get

$$-\frac{x}{y+1} + \frac{x}{y-1} = 5$$

$$\Rightarrow \quad \frac{2x}{y^2 - 1} = 5$$

$$\Rightarrow \quad x = \frac{5y^2 - 5}{2}$$

On putting value of x in Eq. (ii), we get

$$\frac{5 - 5y^2}{2y} + \frac{5y^2 - 5}{2(y-1)} = 3$$

$$\Rightarrow \frac{5(1 - y^2)}{2y} + \frac{5(y^2 - 1)}{2(y-1)} = 3$$

$$\Rightarrow \quad \frac{5(y^2 - 1)}{2y^2 - 2y} = 3$$

$\Rightarrow \quad 6y^2 - 6y = 5y^2 - 5$

$\Rightarrow \quad y^2 - 6y + 5 = 0$

$\Rightarrow \quad (y - 5)(y - 1) = 0$

$\Rightarrow \quad y = 5 \qquad (\text{Q } y \neq -1)$

$\therefore$ From Eq. (i),

$$\frac{x}{5} - \frac{x}{6} = 2$$

$$\Rightarrow \quad \frac{x}{30} = 2$$

$\Rightarrow \quad x = 60$

79. Let full fare = Rs. x

and reservation charges = Rs. y

$\therefore \; x + y = 362 \qquad ...(i)$

and $1\frac{1}{2}x + 2y = 554$

$\Rightarrow 3x + 4y = 1108$

On solving Eqs. (i) and (ii) we get

$x = 340, y = 22$

Hence, reservation charge = Rs. 22.

80. Let total no. of swans $= x$

No. of swans swimming in the pool $= \frac{7}{2}\sqrt{x}$

Remaining swans = 2

$$\therefore \quad \frac{7}{2}\sqrt{x} + 2 = x$$

$$\Rightarrow \quad \frac{7}{2}\sqrt{x} = x - 2$$

On squaring both sides, we get

$$\frac{49}{4}x = x^2 + 4 - 4x$$

$\Rightarrow \quad 4x^2 - 65x + 16 = 0$

$\Rightarrow \quad (4x - 1)(x - 16) = 0$

$$\Rightarrow \quad x = 16 \left(\text{Q } x \neq \frac{1}{4}\right)$$

81. Given $\frac{x}{y} = 25\left(\frac{y}{x}\right)$

$$\Rightarrow \quad \frac{x^2}{y^2} = \frac{25}{1}$$

$\Rightarrow \quad \frac{x}{y} = \frac{5}{1}$

82. Total CP of milk = 28 × 8.50
= Rs. 238
$\therefore$ Profit = 12.5% of 238
$= \frac{12.5}{100} \times 238 = 29.75$
Let he added × litre of water.
$\therefore$ Profit = $x \times 8.5$
$\Rightarrow$ $29.75 = x \times 8.5$
$\Rightarrow$ $x = 3.5$ L

83. Given, $m = n^2 - n$
$\therefore$ $m^2 - 2m = (n^2 - n)^2 - 2(n^2 - n)$
$= n(n-1)[n^2 - n - 2]$
$= (n+1)\, n\, (n-1)\, (n-2)$
It is a product of consecutive number.
Hence, it is divisible by 24(41).

85. Let initial no. of inhabitants = 100
Firstly after 10% left, no. of inhabitants
= 100 – 10 = 90
Secondly again 10% left, no. of inhabitants
= 90 – 10% of 90 = 81
$\therefore$ Remaining % of population = 81%

86. Given, $R = \frac{10}{2} = 5\%$
$T = 3$, P = Rs. 400
$\therefore \quad A = P\left(1 + \frac{R}{100}\right)$
$= 400\left(1 + \frac{5}{100}\right)^3$
$= 400\left(\frac{21}{20}\right)^3$
= Rs. 463.05

87. Let initial no. of workers = x
$\therefore$ 20% of x = 15
$\Rightarrow \quad \frac{20}{100} \times x = 15$
$\Rightarrow \quad x = 75$

88. Since, time = $\frac{\text{distance}}{\text{speed}}$
$= \frac{92}{6.5+5} = \frac{92}{11.5} = 8$ h

89.

Soilders	Days
100	8
160	x

$\Rightarrow \quad \frac{160}{100} = \frac{8}{x}$
$\Rightarrow \quad x = \frac{8 \times 100}{160} = 5$ days

90. Let x is the length of fastest train.
Since, time = $\frac{\text{distance}}{\text{speed}}$
$\therefore \quad \frac{15}{3600} = \frac{x}{50 - 32}$
$\Rightarrow \quad x = \frac{15 \times 18}{3600}$
$\Rightarrow x = \frac{3}{40}\text{km} = \frac{3}{40} \times 1000 = 75$ m

90.

Distance	Speed	hour per day	days
1	1	15	114
2	2	6	x

$1:2$
$2:1$ } $114 : x$
$6:15$

$\therefore 1 \times 2 \times 6 \times x = 2 \times 1 \times 15 \times 114$
$\Rightarrow \quad x = \frac{2 \times 15 \times 114}{2 \times 6}$
$\Rightarrow \quad x = 285$ days

92. Given $x : y = 1 : 3$, $y : z = 5 : k$, $z : t = 2 : 5$
and $t : x = 3 : 4$
$\Rightarrow x : y = 1 : 3$, $y : z = 3 : \frac{3k}{5}$, $z : t = \frac{3}{10} : \frac{3}{4}$
and $t : x = \frac{3}{4} : 1$

$\Rightarrow \quad k = \frac{1}{2}$

93. $\sqrt{0.0001} = \sqrt{\frac{00001}{10000}}$

$= \frac{1}{100} = 0.01$

Hence, it is a rational number.

94. Let Q join x month.

$\therefore$ Ratio of capital $= 2525 \times 12 : 1200 \times x$

$= 2525 : 100x$

$= 101 : 4x$

$\therefore \quad$ P's profit $= \frac{101}{101+4x} \times 1644$

$\Rightarrow \quad 1212 = \frac{101 \times 1644}{101+4x}$

$\Rightarrow \quad \frac{1}{137} = \frac{1}{101+4x}$

$\Rightarrow \quad 4x = 36$

$\Rightarrow \quad x = 9$

95. According to the question,

$35 \times 24 : 25 \times x :: 6300 : 3375$

$\Rightarrow \quad 35 \times 24 \times 3375 = 25 \times x \times 6300$

$\Rightarrow \quad x = \frac{35 \times 24 \times 3375}{25 \times 6300}$

$= 18$

96. Given, $a + b = 2m^2$ ...(i)

$b + c = 6m$ ...(ii)

and $a + c = 2$ ... (iii)

On adding Eqs. (i), (ii) and (iii), we get

$2(a + b + c) = 2m^2 + 6m + 2$

$\Rightarrow \quad a + b + c = m^2 + 3m + 1$ ...(iv)

From Eqs. (iv) – Eq. (ii),

$a = m^2 - 3m + 1$

From Eq. (iv) – Eq. (iii)

$b = m^2 + 3m - 1$

From Eq. (iv) – Eq. (i)

$c = -m^2 + 3m + 1$

As $a \le b$ and $b \le c$

$\Rightarrow \quad m^2 - 3m + 1 \le m^2 + 3m - 1$ and

$m^2 + 3m - 1 \le -m^2 + 3m + 1$

$\Rightarrow \quad 6m \ge 2$ and $2m^2 \le 2$

$\Rightarrow \quad m \ge \frac{1}{3}$ and $-1 \le 1$

$\therefore \quad \frac{1}{3} \le m \le 1$

97. Here difference $= (42 - 25) = (72 - 55) = (84 - 67) = 17$

Now, LCM $\frac{1}{2}(42, 72, 84) = 504$

$\therefore$ The required number $= 504 - 17 = 487$

98. By using the rule,

When there is a profit of x% and loses of y% then the resultant

$\text{Profit / loss} = \left(x - y - \frac{xy}{100}\right)\%$

Here $x = y = 10\%$

$\text{Profit / loss} = \left(10 - 10 - \frac{10 \times 10}{100}\right)\% = -1\%$

Negative sign represent, there is a loss of 1%.

99. Given, P = Rs. 7500, $R = 16\%$ $n = 2$yr

$\therefore \quad A = P\left(1 + \frac{R}{100}\right)^n$

$= 75000\left(1 + \frac{16}{100}\right)^2$

$= 7500 \times \frac{29}{25} \times \frac{29}{25}$

$\therefore \quad A =$ Rs. 10092