

Reg. No. :

D 1610

Q.P. Code : [D 07 PMA 05]

(For the candidates admitted from 2007 onwards)

M.Sc. DEGREE EXAMINATION, DECEMBER 2010.

First Year

Mathematics

COMPLEX ANALYSIS

Time : Three hours

Maximum : 100 marks

Answer any FIVE questions.

All questions carry equal marks.

(5 × 20 = 100)

1. (a) Show that any analytic function satisfies Cauchy-Riemann equations.
(b) Show that an analytic function $f(z)$ in a region Ω is a constant if either the real part or imaginary part or the modulus or the argument is constant.
2. (a) (i) Prove that the reflection $z \rightarrow \bar{z}$ is not a linear transformation.
(ii) Find the linear transformation which carries $0, i, -i$ into $1, -1, 0$.

- (b) Given three distinct points z_2, z_3 and z_4 in the extended plane, define the cross ratio (z_1, z_2, z_3, z_4) . Prove that the cross ratio is invariant under linear transformation.
3. (a) State and Prove Cauchy's integral formula.
 (b) Find the poles and residues of the function $f(z) = \frac{1}{(z^2 - 1)^2}$.
4. (a) Show that $\int_{\gamma} p dx + q dy$, defined in Ω depends only on the end points of γ if and only if there exists a function $u(x, y)$ in Ω with $\frac{\partial u}{\partial x} = p, \frac{\partial u}{\partial y} = q$.
 (b) State and prove the maximum principle.
5. (a) Evaluate $\int_0^a \frac{x^2 dx}{x^4 + 5x^2 + 6}$.
 (b) State and prove the mean-value property of function u .
6. (a) State and prove Hurwitz theorem.
 (b) Prove that the Laurent development is not unique.
7. (a) Define convergence of an infinite product prove that $\prod_{n=2}^{\infty} \left(1 + \frac{1}{n^2}\right) = \frac{1}{2}$.
 (b) State and prove Riemann mapping theorem.
8. (a) Define Weierstrass P- function and prove that it is an elliptic function.
 (b) Derive the differential equation satisfied by the Weierstrass p-function.

Reg. No. :

D 1606

Q.P. Code : [D 07 PMA 01]

(For the candidates admitted from 2007 onwards)

M.Sc. DEGREE EXAMINATION, DECEMBER 2010.

First Year

Mathematics

ALGEBRA

Time : Three hours

Maximum : 100 marks

Answer any FIVE questions.

Each question carries 20 marks.

1. (a) If $O(G) = p^n$ where p is a prime, show that $z(a) \neq (e)$.
- (b) Show that S_{pk} has a p -Sylow subgroup.
2. (a) Show that the ideal $\mathcal{A} = (a_0)$ is a maximal ideal of the Euclidean ring R if and only if a_0 is prime of R .
- (b) Show that $\mathcal{J}[i]$ is a Euclidean ring.

3. (a) State and prove division algorithm for polynomials.
- (b) Show that the product of two primitive polynomials is also a primitive polynomial.
- (c) State and prove Eisenstein criterion.
4. (a) Show that the element $\alpha \in K$ is algebraic over F if and only if $F(\alpha)$ is a finite extension of F .
- (b) Show that any two splitting fields of the same polynomial over a given F are isomorphic.
5. (a) Show that the polynomial $f(x) \in F[x]$ has a multiple root if and only if $f(x)$ and $f'(x)$ have a non trivial common factor.
- (b) If $f(x) \in F[x]$ is irreducible, show that if the characteristic of F is $p \neq 0$, then $f(x)$ has a multiple root only if it is of the form $f(x) = g(x^p)$.
- (c) If F is of characteristic 0 and if a, b are algebraic over F show that there exists an element $c \in F(a, b)$ such that $F(a, b) = F(c)$.
6. (a) If F is a subfield of a field K , define $G(K, F)$. If K is a finite extension of F show that $G(K, F)$ is a finite group such that $O(G(K, F)) \leq [K : F]$.
- (b) Suppose that the field F has all n th roots of unity and $a \neq 0$ is in F . Let $x^n - a \in F[x]$ and let K be its splitting field over F . Show that
- (i) $K = F(u)$ where u is any root of $x^n - a$
- (ii) the Galois group of $x^n - a$ over F is abelian.
7. (a) If $T \in A(V)$ has all its characteristic roots in F show that there is a basis of V in which the matrix of T is triangular
- (b) Show that two nilpotent linear transformations are similar if and only if they have the same invariant.
8. (a) If F is of characteristic 0 and if S and T in $A_F(V)$ are such that $ST - TS$ commutes with S , show that $ST - TS$ is nilpotent.

- (b) If $\{v_1, \dots, v_n\}$ is an orthonormal basis of V and if the matrix $T \in A(V)$ in this basis (α_{ij}) , show that the matrix of T^* in this basis is (β_{ij}) where $\beta_{ij} = \overline{\alpha_{ji}}$.
- (c) Show that the Hermitian linear transformation T is non negative if and only if its characteristic roots are non negative.

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Reg. No. :

D 1607

Q.P. Code : [D 07 PMA 02]

(For the candidates admitted from 2007 onwards)

M.Sc. DEGREE EXAMINATION, DECEMBER 2010.

First Year

Mathematics

REAL ANALYSIS

Time : Three hours

Maximum : 100 marks

Answer any FIVE questions.

Each question carries 20 marks.

1. (a) Show that $f \in R(\alpha)$ if and only if for $\epsilon > 0$ there exists a partition P on $[a, b]$ such that $U(p, f, \alpha) - L(p, f, \alpha) < \epsilon$.
- (b) Suppose f is bounded on $[a, b]$, f has only finitely many points of discontinuities on $[a, b]$ and α is continuous at every point at which f is discontinuous show that $f \in R(\alpha)$.
- (c) State and prove change of variable theorem.

2. (a) If γ' is continuous on $[a, b]$, show that γ is rectifiable and $\Lambda(f) = \int_a^b |\gamma'(f)| dt$.

(b) Let $f \in R$ on $[a, b]$ and $F(x) = \int_a^x f(t) dt$ for $x \in [a, b]$. Show that F is continuous and if f is continuous at x_0 show that $F'(x_0) = f(x_0)$.

(c) State and prove fundamental theorem of calculus.

3. (a) If $\{f_n\}$ is a sequence of continuous functions on E and if $f_n \rightarrow f$ uniformly on E , show that f is continuous on E . Is this true for pointwise convergence? Justify your answer.

(b) Suppose $\{f_n\}$ is a sequence of functions differentiable on $[a, b]$ and $\{f_n(x_0)\}$ converges for some $x_0 \in [a, b]$. If $\{f'_n\}$ converges uniformly on $[a, b]$ show that $\{f_n\}$ converges uniformly on $[a, b]$ to a function f and $f'(x) = \lim_{n \rightarrow \infty} f'_n(x)$.

4. Let \mathcal{V}^A be an algebra of real continuous functions on a compact set A . If \mathcal{V}^A separate points on K and if \mathcal{V}^A vanishes at no point of K , show that the uniform closure B of \mathcal{V}^A consists of all real continuous functions on K .

5. (a) Suppose f maps an open set $E \subset R^n$ into R^m . Show that $f \in \zeta'(E)$ if and only if the partial derivatives $D_j f_j$ exist and are continuous on E for $1 \leq i \leq m, 1 \leq j \leq n$.

(b) Suppose f is defined in an open set $E \subset R^2$, $D_1 f, D_{21} f, D_2 f$ exist at every point of E and $D_{21} f$ is continuous at some point $(a, b) \in E$. Show that $D_{12} f$ exists at (a, b) and $(D_{12} f)(a, b) = (D_{21} f)(a, b)$.

6. State and prove implicit function theorem.

7. (a) Construct a non measurable subset of $[0, 1]$.
 (b) Let f be defined and bounded on a measurable set E with mE finite. Show that f is measurable if and only if

$$\inf_{f \leq \psi} \int_E \psi(x) dx = \sup_{f \geq \varphi} \int_E \varphi(x) dx \text{ for all simple function } \varphi \text{ and } \psi.$$

8. (a) If f is bounded and measurable on $[a, b]$

and $F(x) = \int_a^x f(t) dt + F(a)$, show that

$F'(x) = f(x)$ for almost $x \in [a, b]$.

(b) Show that a function F is an indefinite integral if and only if it is absolutely continuous.

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Reg. No. :

D 1608

Q.P. Code : [D 07 PMA 03]

(For the candidates admitted from 2007 onwards)

M.Sc. DEGREE EXAMINATION, DECEMBER 2010.

First Year

Mathematics

DIFFERENTIAL EQUATIONS

Time : Three hours

Maximum : 100 marks

Answer any FIVE questions.

(5 × 20 = 100)

1. (a) Let $\vec{f}(t)$ be periodic with period w . Prove that a solution $\vec{x}(t)$ of $\vec{x}' = A\vec{x} + \vec{f}(t)$ is periodic of period w if and only if $\vec{x}(0) = \vec{x}(w)$.
- (b) Prove that the general solution of the system $\vec{x}' = A\vec{x}$, $t \in I$ is $\vec{x} = e^{tA}\vec{c}$.

Further show that the solution of the system with initial condition $\vec{x}(t_0) = \vec{x}_0$, $t_0 \in I$ is $\vec{x}(t) = e^{(t-t_0)A}\vec{x}_0$, $t \in I$.

2. (a) Discuss Picard's theorem for the existence of unique solution for a class of nonlinear initial value problems.

(b) Derive all the solutions of the IVP : $x' = 4x^{3/4}$, $x(0) = 0$.

3. (a) Determine the solution of the wave equation $\frac{\partial^2 u}{\partial t^2} = C^2 \frac{\partial^2 u}{\partial x^2}$ with initial conditions

$u(x, 0) = f(x)$, $\frac{\partial u}{\partial t}(x, 0) = g(x)$. Investigate the physical significance of the solution.

(b) Discuss the solution of the problem $\frac{\partial^2 u}{\partial t^2} = C^2 \frac{\partial^2 u}{\partial x^2}$, $0 < x < l$, $t > 0$ with $u(x, 0) = f(x)$, $u_t(x, 0) = g(x)$, $0 \leq x \leq l$, $u(0, t) = 0$, $u(l, t) = 0$, $t \geq 0$.

4. (a) Derive the solution of Laplace equation $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$, $0 < x < a$, $0 < y < b$, $u(x, 0) = f(x)$, $u(x, b) = 0$, $u_x(0, y) = 0$, $u_x(a, y) = 0$.

(b) Derive the solution of the heat problem $\frac{\partial u}{\partial t} = K \frac{\partial^2 u}{\partial x^2}$, $0 < x < l$, $t > 0$ with $u(0, t) = 0$, $u(l, t) = 0$, $t \geq 0$, $u(x, 0) = f(x)$, $0 \leq x \leq l$.

5. (a) (i) State and prove the uniqueness theorem for Dirichlet problem.

(ii) State and prove continuity theorem.

(b) Solve the problem

$$\nabla^2 u = 0, \quad 0 < x < 1, \quad 0 < y < 1, \quad u(x, 0) = 0, \\ u(x, 1) = 0, \quad u(0, y) = 0, \quad u(1, y) = \sin \pi y \cdot \cos \pi y, \\ 0 \leq y \leq 1$$

6. (a) Determine e^{At} and a fundamental matrix for the system $\bar{x}' = A\bar{x}$ where

$$A = \begin{bmatrix} -1 & 2 & 3 \\ 0 & -2 & 1 \\ 0 & 3 & 0 \end{bmatrix}$$

(b) Prove that $x(t)$ is a solution of IVP $x' = f(t, x)$, $x(t_0) = x_0$ on some interval I if and only if $x(t)$ is a solution of integral equation

$$x(t) = x_0 + \int_{t_0}^t f(s, x(s)) ds.$$

7. (a) Find the solution of the initial boundary value problem $\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2}$, $0 < x < \infty$, $t > 0$,

$$u(x, 0) = \cos\left(\frac{\pi x}{2}\right), \quad 0 \leq x < \infty, \quad \frac{\partial u}{\partial t}(x, 0) = 0,$$

$$\frac{\partial u}{\partial x}(0, t) = 0, \quad t \geq 0.$$

- (b) Prove that there exists at most one solution of the wave equation $\frac{\partial^2 u}{\partial t^2} = C^2 \frac{\partial^2 u}{\partial x^2}$, $0 < x < l$, $t > 0$, under certain initial conditions to be stated.

8. (a) Find the solution of the problem

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0, \quad 0 < x < a, \quad 0 < y < b,$$

$$u(x, 0) = f(x), \quad 0 \leq x \leq a, \quad u(x, b) = 0,$$

$$u(0, y) = 0, \quad u(a, y) = 0.$$

- (b) Let $\Phi(t)$, $t \in I$, denote a fundamental matrix of the system $\bar{x}' = A\bar{x}$ such that $\Phi(0) = E$ where A is a constant matrix. Prove that Φ satisfies

$$\Phi(t+s) = \Phi(t)\Phi(s)$$

for all values of t and $s \in I$. Here E denotes the identity matrix.

Reg. No. :

D 1609

Q.P. Code : [D 07 PMA 04]

(For the candidates admitted from 2007 onwards)

M.Sc. DEGREE EXAMINATION, DECEMBER 2010.

First Year

Mathematics

Numerical Methods

Time : Three hours

Maximum : 100 marks

Answer any FIVE questions.

(5 × 20 = 100)

1. (a) Solve $x^3 = 2x + 5$ for the positive root by iteration method.
- (b) Find the root of the equation
$$x^4 - 1.1x^3 + 2.3x^2 + 0.5x + 3.3 = 0$$
by Bairstow method using starting factor $x^2 + x + 1$.

2. (a) From the following table, obtain $\frac{dy}{dx}$ and

$$\frac{d^2y}{dx^2} \text{ at } x = 1.2.$$

x:	1.0	1.2	1.4	1.6	1.8	2.0	2.2
y:	2.7183	3.3201	4.0552	4.9530	6.0496	7.3891	9.0250

- (b) Use Romberg's method to compute

$$I = \int_0^1 \frac{dx}{1+x^2} \text{ with } h = 0.5, 0.25, 0.125 \text{ correct}$$

to three decimal places.

3. (a) Solve by Gauss elimination method :

$$2x + y + 4z = 12.$$

$$8x - 3y + 2z = 20.$$

$$4x + 11y - z = 33.$$

- (b) Solve the system of non-linear equations :

$$4 - x^2 - y^2 = 0$$

$$1 - e^x - y = 0.$$

4. (a) Solve by LU decomposition method :

$$2x + y + 4z = 12.$$

$$8x - 3y + 2z = 20.$$

$$4x + 11y - z = 33.$$

- (b) Solve by Gauss-Seidel method :

$$27x + 6y - z = 85,$$

$$6x + 15y + 2z = 72,$$

$$x + y + 54z = 110.$$

5. (a) If $\frac{dy}{dx} = x + y$ and $y(1) = 0$, find $y(1.1)$ and $y(1.2)$ by Taylor's series method.

- (b) Using modified Euler method, find $y(0.1)$

given that $\frac{dy}{dx} = x^2 - y$, $y(0) = 1$.

6. (a) Compute the values of $y(0.1)$ and $y(0.2)$ using Runge-Kutta method of fourth order $\frac{dy}{dx} = -y$, $y(0) = 1$.

- (b) Given $\frac{dy}{dx} = \frac{1}{2}(1+x^2)y^2$ and $y(0) = 1$, $y(0.1) = 1.06$, $y(0.2) = 1.12$, $y(0.3) = 1.21$, evaluate $y(0.4)$ by Milne's predictor - corrector method.

7. (a) Solve $\frac{d^2x}{dt^2} - \left(1 - \frac{t}{5}\right)x = t$, $x(1) = 2$, $x(3) = -1$ using Central-difference approximations.
- (b) Find the dominant eigen value and the corresponding eigen vector of $A = \begin{pmatrix} 1 & 6 & 1 \\ 1 & 2 & 0 \\ 0 & 0 & 3 \end{pmatrix}$.
8. (a) Solve $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$ subject to $u(0,t) = 0$, $u(1,t) = 0$ and $u(x,0) = \sin \pi x$, $0 < x < 1$
- (b) Solve by Crank-Nicolson method $\frac{\partial^2 u}{\partial x^2} = \frac{\partial u}{\partial t}$ subject to $u(x,0) = 0$, $u(0,t) = 0$ and $u(1,t) = t$ for two time steps.

Reg. No. :

D 1611

Q.P. Code : [D 07 PMA 06]

(For the candidates admitted from 2007 onwards)

M.Sc. DEGREE EXAMINATION, DECEMBER 2010.

Second Year

Mathematics

MECHANICS

Time : Three hours

Maximum : 100 marks

Answer any FIVE questions.

All questions carry equal marks.

(5 × 20 = 100)

1. (a) Obtain the Lagrange's equation from the Newton's law of motion.
(b) Derive Lagrange's equation of motion for holonomic system.
2. (a) Discuss the motion of a loop rolling without slipping down an inclined plane.
(b) Define canonical momentum. Show that the generalised momentum conjugate to a cyclic coordinate is conserved.

3. (a) Explain Legendre transformations with example.
 (b) Discuss canonical equation of Hamilton.
4. (a) Discuss the principle of least action.
 (b) Explain Routh's procedure.
5. (a) Solve the simple harmonic oscillator problem in one dimension using a canonical transformation with reference to the Hamiltonian concept.
 (b) Define poisson bracket of two functions. Show that all poisson brackets are canonical invariants.
6. (a) Explain canonical transformation with an example.
 (b) Derive the Hamilton-Jacobi's partial differential equation in (n +1) variables for Hamilton's principle function.
7. (a) Explain integral invariants of poincare.
 (b) Derive the equation:

$$\frac{d}{dt} \left(\frac{\partial L}{\partial \dot{q}_j} \right) - \frac{\partial L}{\partial q_j} + \frac{\partial \mathcal{F}}{\partial q_j} = 0 \quad \text{Where } L \text{ is the}$$

Lagrangian and \mathcal{F} is the Rayleigh's dissipation function.

8. (a) Explain Harmonic Oscillator Problem.
 (b) Discuss the solution of the central force problem in terms of the polar coordinates (r, ψ) in the plane of the orbit.

Reg. No. :

D 1612

Q.P. Code : [D 07 PMA 07]

(For the candidates admitted from 2007 onwards)

M.Sc. DEGREE EXAMINATION, DECEMBER 2010.

Second Year

Mathematics

OPERATIONS RESEARCH

Time : Three hours

Maximum : 100 marks

Answer any FIVE questions.

Each question carries 20 marks.

1. (a) Obtain all the basic solution to the following system of linear equation.

$$2x_1 + 6x_2 + 2x_3 + x_4 = 3$$

$$6x_1 + 4x_2 + 4x_3 + 6x_4 = 4$$

Which of them are basic feasible solutions and which are non-degenerate basic solutions? Is the non-degenerate solution feasible?

- (b) Use simplex method to solve the LPP

$$\text{Maximize } z = 5x_1 + 2x_2 + 3x_3 - x_4 + x_5$$

Subject to the constraints :

$$x_1 + 2x_2 + 2x_3 + x_4 = 8$$

$$3x_1 + 4x_2 + x_3 + x_5 = 7$$

$$x_1, x_2, x_3, x_4, x_5 \geq 0.$$

2. (a) What are the indications obtain in phase-I to proceed phase II in solving LPP by two-phase method? Solve the following LPP by Two-phase method

$$\text{Minimize } z = x_1 + x_2 + x_3$$

Subject to

$$x_1 - 3x_2 + 4x_3 = 5$$

$$x_1 - 2x_2 \leq 3$$

$$2x_2 - x_3 \geq 4$$

$$x_1 \geq 0, x_2 \geq 0 \text{ and } x_3 \text{ is unrestricted.}$$

- (b) Use two-phase simplex method to

$$\text{Maximize } z = 3x_1 + 2x_2 + x_3 + 4x_4$$

Subject to

$$4x_1 + 5x_2 + x_3 - 3x_4 = 5$$

$$2x_1 - 3x_2 - 4x_3 + 5x_4 = 7$$

$$x_1 + 4x_2 + 25x_3 - 4x_4 = 6$$

$$x_1, x_2, x_3, x_4 \geq 0.$$

3. (a) Solve the following LPP by dual simplex method

$$\text{Minimize } z = x_1 + 2x_2 + 3x_3$$

Subject to

$$x_1 - x_2 + x_3 \geq 4$$

$$x_1 + x_2 + x_3 \leq 8$$

$$x_2 - x_3 \geq 2$$

$$x_1, x_2, x_3 \geq 0.$$

- (b) Use principle of duality to solve the following LP problem

$$\text{Minimize } z = 2x_1 + 2x_2$$

Subject to

$$2x_1 + 4x_2 \geq 1$$

$$x_1 + 2x_2 \geq 1$$

$$2x_1 + x_2 \geq 1$$

$$x_1, x_2 \geq 0.$$

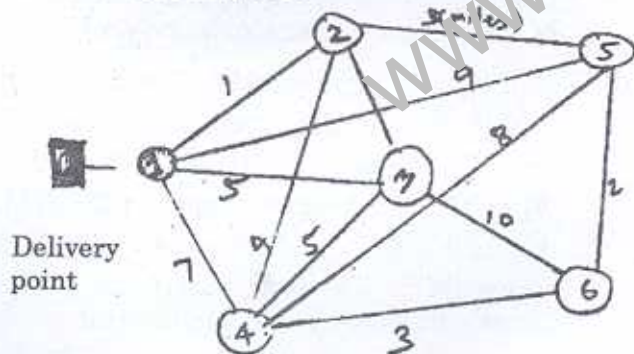
4. (a) Indicate how a transshipment problem can be solved as a transportation problem.

- (b) A firm having two sources S_1 and S_2 wishes to ship its product to two destination D_1 and D_2 . The number of units available at S_1 and S_2 are 5 and 25 respectively and the product demanded at D_1 and D_2 are 20 and 10 units respectively. The firm, instead of shipping directly from sources to destinations, decides

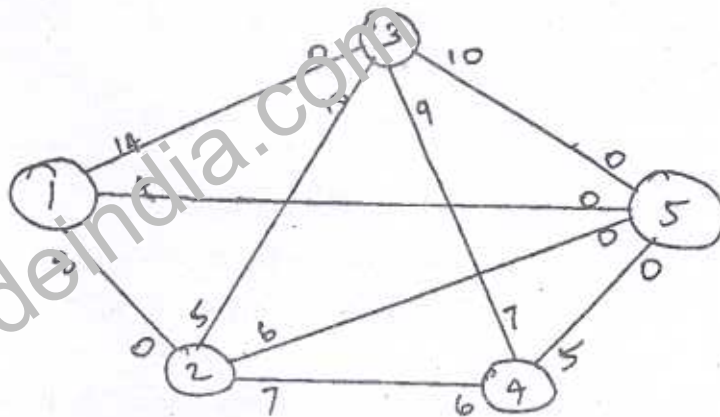
to investigate the possibility of transshipment. The unit transportation costs (in rupees) are given in the following table. Find the optimal shipping schedule.

	Source		Destination		Available	
	S1	S2	D1	D2		
Sources	S1	0	2	3	4	5
	S2	2	0	2	4	25
Destination	D1	3	2	0	1	—
	D2	4	4	1	0	—
Demand	—	—	20	10		

5. (a) The Midwest TV cable company is in the process of providing cable service to five new housing development areas. The following network depicts the potential TV linkages among the five areas. The cable miles are shown on each branch. Find the minimal spanning tree.



- (b) Determine the maximal flow and the optimal flow in each arc for the network in the figure.



6. (a) Use revised simplex method to solve the LPP

$$\text{Minimize } z = -4x_1 + x_2 + 2x_3$$

Subject to

$$2x_1 - 3x_2 + 2x_3 \leq 12$$

$$-5x_1 + 2x_2 + 3x_3 \geq 4$$

$$3x_1 - 2x_3 = -1$$

$$x_1, x_2, x_3 \geq 0.$$

(b) Consider the following LP.

$$\text{Maximize } z = 5x_1 + 12x_2 + 4x_3$$

Subject to

$$2x_1 - x_2 + 3x_3 = 2$$

$$x_1 + 2x_2 + x_3 + x_4 = 5$$

$$x_1, x_2, x_3, x_4 \geq 0.$$

(i) Write the dual

(ii) Verify that $B = (P_1, P_2)$ is optimal by computing $z_j - c_j$ for all non-basic p_j .

7. (a) Explain why is simulation used? Write a short note on Monte Carlo simulation.
- (b) Identify the discrete events needed to simulate the following situation. Metalco job shop receives two types of jobs : regular and rush. All jobs are processed on two consecutive machines with ample buffer areas-rush jobs always assume non pre-emptive priority over regular jobs.

8. (a) The inter arrival time of customers at Hairkare babershop is exponential with mean 12 minutes. The shop is operated by only one barber who gives two types of haircut : a crew cut that takes about 10 minutes and a regular cut that takes about 15 minutes. One out of every five customers, on the average, requires a crew cut, but the process is random. Compute the resulting average queue length, average facility utilization and average waiting time in the queue.

- (b) On January 1 (this year), Bakery A had 40% of its local market share while the other two bakeries B and C had 40% and 20%, respectively of the market share. Based upon a study by a marketing research firm, the following facts were compiled. Bakery A retains 90% of its customers while gaining 5% of competitor B's customers and 10% of C's customers. Bakery B retains 85% of its customers while gaining 5% of A's customers and 7% of C's customers. Bakery C retains

83% of its customers and gains 5% of A's customers as well as 10% of B's customers. What will each firm's share be on January 1, next year and what will each firm's market share be at equilibrium?

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Reg. No. :

D 1613

Q.P. Code : [D 07 PMA 08]

(For the candidates admitted from 2007 onwards)

M.Sc. DEGREE EXAMINATION, DECEMBER 2010.

Second Year

Mathematics

Paper VIII — TOPOLOGY

Time : Three hours

Maximum : 100 marks

Answer any FIVE questions.

Each question carries 20 marks.

(5 × 20 = 100)

1. (a) Show that every non-empty finite ordered set has the order type of a section $\{1, 2, \dots, n\}$ of Z_+ , so it is necessarily well ordered.
- (b) Let X and Y be topological spaces let $f: X \rightarrow Y$ prove that the following condition are equivalent.
 - (i) f is continuous

- (ii) for every subset A of X , one has $f(\overline{A}) \subset \overline{f(A)}$
- (iii) for every closed set B in Y , the set $f^{-1}(B)$ is closed in X .

2. (a) Let $f: A \rightarrow \prod_{\alpha \in J} X_\alpha$ be given by the equation $f(a) = (f_\alpha(a))_{\alpha \in J}$. Where $f_\alpha: A \rightarrow X_\alpha$ for each α . Let $\prod X_\alpha$ have the product topology. Prove that the function f is continuous if and only if each function f_α is continuous.
- (b) Let $\overline{d}(a, b) = \min \{ [a, b], 1 \}$ be the standard bounded metric on R . If x and y are two points of R^w define $D(x, y) = \text{lub} \left\{ \frac{\overline{d}(x_i, y_i)}{i} \right\}$. Prove that D is metric that induces the product topology on R^w .
3. (a) If L is a linear continuum in the order topology. Prove that L is connected and so is every interval and ray in L .
- (b) Prove that the product of finitely many compact spaces is compact.

4. (a) State and prove uniform continuity theorem.
- (b) Let X be a metrizable space. Prove that the following statements are equivalent.
- (i) X is compact
 - (ii) X is limit point compact
 - (iii) X is sequentially compact.
5. (a) Prove the following statement
- (i) A subspace of a Hausdorff space is Hausdorff and a product of Hausdorff space is Hausdorff.
 - (ii) A subspace of a regular space is regular and product of regular spaces is regular.
- (b) Prove that every metrizable space is normal.
6. (a) Prove that a subspace of a completely regular space is completely regular and a product of completely regular spaces is completely regular.
- (b) Let (X, d) be a metric space. prove that there is an isometric imbedding of X into a complete metric space.

7. (a) Prove that the fundamental group of the circle is infinite cyclic.
- (b) Let $x_0 \in S^1$ prove that inclusion mapping $j: (S^1, X_0) \rightarrow (R^2 - 0, X_0)$ induces an isomorphism of fundamental groups.
8. (a) State and prove special Van Kampen theorem.
- (b) Prove that for $n \geq 2$, the n th sphere S^n is simply connected.
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Reg. No. :

D 1614

Q.P. Code : [D 07 PMA 09]

(For the candidates admitted from 2007 onwards)

M.Sc. DEGREE EXAMINATION, DECEMBER 2010.

Second Year

Mathematics

COMPUTER PROGRAMMING (C++ THEORY)

Time : Three hours

Maximum : 100 marks

Answer any FIVE questions

All questions carry equal marks

(5 × 20 = 100)

1. (a) Explain the basic concepts of object oriented programming.
(b) Describe object oriented language.
2. (a) Explain the structure of C++ program.
(b) List a few areas of application of oop technology.

3. (a) What is meant by conditional and unconditional statement? Explain any two conditional statements with suitable example.
(b) Write a program to find the prime numbers from 1.... 100.
4. (a) Write short notes on virtual function.
(b) Discuss the binary mode of I/O functions.
5. (a) Write a program to illustrate a function friendly to two classes.
(b) Explain getline() and write() functions.
6. (a) What is constructors? Explain the uses of constructors.
(b) List out the advantages of inheritance.
7. (a) What is friend function? What are the merits and demerits of using friend functions?
(b) Discuss operator overloading with an example.
8. (a) Describe the syntax of the single inheritance in C++.
(b) Explain virtual base classes.

Reg. No. :

D 1615

Q.P. Code : [D 07 PMA 10]

(For the candidates admitted from 2007 onwards)

M.Sc. DEGREE EXAMINATION, DECEMBER 2010.

Second Year

Mathematics

FUNCTIONAL ANALYSIS

Time : Three hours

Maximum : 100 marks

Answer any FIVE questions.

All questions carry equal marks.

(5 × 20 = 100)

1. (a) If M is a closed linear subspace of a normed linear space N and $x_0 \notin M$ then prove that there exists a functional f_0 in N^* such that $f_0(M) = 0$ and $f_0(x_0) \neq 0$.
- (b) State and prove open mapping theorem.
2. (a) State and prove uniform boundedness theorem.
- (b) Prove that a non-empty subset X of a normed linear space N is bounded iff $f(X)$ is a bounded set of real numbers for each f in N^* .

3. (a) State and prove Bessel's inequality.
(b) An operator T on H is unitary iff it is an isometric isomorphism of H onto itself.
4. (a) Prove that a closed convex subset C of a Hilbert space H contains a unique vector of smallest norm.
(b) If M is a closed linear subspace of a Hilbert space H then prove that $H = M \oplus M^\perp$.
5. (a) If M is a closed linear subspace of H then it is invariant under an operator T iff M^\perp is invariant under T^* .
(b) Two matrices in A_n are similar iff they are the matrices of a single operator on H relative to different bases.
6. (a) Prove that the mapping $x \rightarrow x^{-1}$ of G into G is continuous and also a homeomorphism of G onto itself.
(b) Prove that $\sigma(x)$ is non-empty.
7. (a) If r is an element of R then prove that $1-r$ is regular.
(b) If I is a proper closed two-sided ideal in A then prove that the quotient algebra A/I is a Banach algebra.
8. (a) Prove that the maximal ideal space M is a compact Hausdorff space.
(b) If x is a normal element in a B^* algebra then prove that $\|x^2\| = \|x\|^2$.