Reg. No. : | | | | | | | | | | | |

Question Paper Code : S 4696

B.E./B.Tech. DEGREE EXAMINATION, NOVEMBER/DECEMBER 2009.

Fourth Semester

Mechanical Engineering

MA 038 — NUMERICAL METHODS

(Common to Aeronautical Engineering, Civil Engineering, Instrumentation and Control Engineering, Automobile Engineering, Mechatronics Engineering and Production Engineering)

(Also common to Fifth Semester – Computer Science and Engineering, Metallurgical Engineering, Polymer Technology and Sixth Semester – Chemical Engineering, Electronics and Instrumentation Engineering, Textile Technology, Leather Technology)

(Regulation 2001)

Time : Three hours Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. Define diagonal dominance of a system of algebraic equations.

2. How do you find an initial approximation of a root of an equation while solving using numerical methods?

3. Define operators Δ, ∇, δ and μ.

4. Show that $E = e^{hD}$, where E, D are shifting and differential operators respectively and h is interval length.

5. Explain the geometrical meaning of Trapezoidal rule.

6. What is the error in Simpson's one-third rule?

7. Mention any two single step methods for solving an ordinary differential equation subject to initial conditions.

8. Using Euler's method compute $y(0.2)$ and $y(0.4)$, given $y' = x + y$, $y(0) = 1$.

9. Classify the following partial differential equation.

$$x\frac{\partial^2 u}{\partial x^2}-2x\frac{\partial^2 u}{\partial x\partial y}+(x-1)\frac{\partial^2 u}{\partial y^2}+2\frac{\partial u}{\partial x}-\frac{\partial u}{\partial y}=0.$$

10. Write finite differences of second order for 2D Laplace equation and obtain standard five point formula.

PART B — ($5 \times 16 = 80$ marks)

11. (a) (i) Find a real root of the equation $\cos x = 3x-1$ correct to 4 decimal places by iteration method.

(ii) Describe Gauss-Seidel method for solving linear system of equations.

Or

(b) (i) Solve the following system of equations using Gauss-elimination method. Use partial pivoting if necessary.

$4x_1+x_2+x_3=4$, $x_1+4x_2-2x_3=4$ and $3x_1+2x_2-4x_3=6$.

(ii) Find the dominant eigenvalue and the corresponding eigenvector of $A=\begin{pmatrix}1 & 6 & 1\\ 1 & 2 & 0\\ 0 & 0 & 3\end{pmatrix}$ using power method. Hence find the least eigen value of A^{-1}.

12. (a) (i) Using Stirling's formula and Bessel formula obtain the value of $y(45)$ given.

x :	40	44	48	52
y :	51.08	63.24	70.88	79.84

(ii) Derive Newton's forward and backward interpolation formulae.

Or

(b) (i) The following data are taken from the steam table.

Temp. (°C) :	140	150	160	170	180
Pressure (kgf / cm²) :	3.685	4.854	6.302	8.076	10.225

Find the pressure at temperatures 142° and 175°.

(ii) Find the equation $y=f(x)$ of least degree using Newton's divided difference formula which passes through the points $(-1,-21)$, $(1,15)$, $(2,12)$, $(3,3)$. Find also y at $x=0$.

13. (a) (i) Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ at $x = 0.96$ from the following data.

x :	0.96	0.98	1.00	1.02	1.04
y :	0.7825	0.7739	0.7651	0.7653	0.7473

(ii) Evaluate $\int_1^2 \frac{2x}{1+x^4}\,dx$, using the Gauss-Legendre 2-point and 3-point quadrature rules.

Or

(b) (i) The velocity v of a particle at distance s from a point on its path is given by the table :

s (ft) :	0	10	20	30	40	50	60
v (ft/sec.) :	47	58	64	65	61	52	38

Estimate the time taken to travel 60 ft. by using Simpson's 1/3 rule. Compare the result with Simpson's 3/8 rule.

(ii) Using Trapezoidal and Simpson's rules, evaluate $\int_2^{2.6}\int_4^{4.4} \frac{dxdy}{xy}$.

14. (a) (i) Find by Taylor's series method, the values of y at $x = 0.1$ and $x = 0.2$ to five places of decimals from $\frac{dy}{dx} = x^2y - 1$, $y(0) = 1$.

(ii) Using Runge-Kutta method of fourth order, find $y(0.8)$ correct to four decimal places if $y' = y - x^2$, $y(0.6) = 1.7379$.

Or

(b) (i) Using Milne's method find $y(4.4)$ given $5xy' + y^2 - 2 = 0$ given $y(4) = 1$, $y(4.1) = 1.0049$, $y(4.2) = 1.0097$ and $y(4.3) = 1.0143$.

(ii) Compute the value of $y(0.2)$ using Runge-Kutta method of fourth order given $y'' = -y$, $y(0) = 1$, $y'(0) = 0$.

15. (a) (i) Solve the boundary value problem :

$xy'' + y = 0$, $y(1) = 1$, $y(2) = 2$,

using finite difference method by taking 4 intervals.

(ii) Solve the equation $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$ subject to the conditions $u(x, 0) = \sin \pi x$, $0 \le x \le 1$; $u(0, t) = u(1, t) = 0$ using Crank-Nicolson method. Carryout computations for two levels, taking $h = 1/3$, $k = 1/36$.

Or

(b) (i) Evaluate the pivotal values of the equation $\frac{\partial^2 u}{\partial t^2} = 16 \frac{\partial^2 u}{\partial x^2}$, taking $h = 1$ upto $t = 1$. The boundary conditions are $u(0, t) = u(5, t) = 0$, $\left.\frac{\partial u}{\partial t}\right|_{(x,0)} = 0$ and $u(x, 0) = x^2(5 - x)$.

(ii) Solve the equation $\nabla^2 u = -10(x^2 + y^2 + 10)$ over the square mesh with sides $x = y = 0$, $x = y = 3$ with $u = 0$ on the boundary and mesh length is 1.

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