Reg. No. : | | | | | | | | | | | |

Question Paper Code : Q 2722

B.E./B.Tech. DEGREE EXAMINATION, NOVEMBER/DECEMBER 2009.

First Year — Annual Pattern

Computer Science and Engineering

MA 1X01 — ENGINEERING MATHEMATICS — I

(Common to all branches)

(Regulation 2004)

Time : Three hours | Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. If 0.5, 1.5 are eigenvalues of $A = \begin{pmatrix} 1 & x \\ 1 & 1 \end{pmatrix}$, find the value of x.

2. Find the curvature of the curve $x^2 + y^2 + 4x - 1 = 0$.

3. Reduce $x^2 \dfrac{d^2y}{dx^2} + 3y = 0$ into a differential equation with constant coefficients.

4. Find the particular integral of $\dfrac{d^2y}{dx^2} - y = 2^x$.

5. Change the order of integration in $\displaystyle\int_0^1 \int_x^1 \frac{1}{x^2 + y^2}\, dx\, dy$.

6. Find the tangent plane at (6, 4, 3) to $xy + yz + zx = 54$.

7. Can $x^2 + y^2 - 2xy$ be the real part of an analytic function? Justify your answer.

8. Find the residue at $z=0$ of $f(z)=\dfrac{1}{z^2(z+1)}$.

9. Verify the initial value theorem given $f(t)$ = 't'.

10. What is the inverse Laplace transform of $\dfrac{1}{(s+2)^2}$?

PART B — (5 × 16 = 80 marks)

11. (a) (i) Show that the line joining any point θ on $x=a(\theta+\sin\theta)$, $y=a(1-\cos\theta)$ to its centre of curvature is bisected by the line $y=2a$. (10)

(ii) Find the inverse of $A=\begin{pmatrix}7 & 2 & -2\\ -6 & -1 & 2\\ 6 & 2 & -1\end{pmatrix}$ using Cayley Hamilton theorem. (6)

Or

(b) (i) Reduce the quadratic form $-x^2+y^2+4yz+4zx$ into canonical form. (8)

(ii) Find the maxima and minima of $f(x,y)=\sin x\sin y\sin(x+y)$ given $0<x,y<\pi$. (8)

12. (a) (i) Solve $\dfrac{d^2y}{dx^2}+121y=\tan 11x$ by the method of variation of parameters. (8)

(ii) Determine the bending curve $y(x)$ where $y(x)$ satisfies the differential equation $\dfrac{d^2y}{dx^2}=\dfrac{P(l-x)}{EI}$, given $y=0$ and $\dfrac{dy}{dx}=0$ at $x=0$. (8)

Or

(b) (i) Solve $(D^2+2D+1)y=2e^{-x}+\cos x+x^2+1$. (8)

(ii) A particle is executing a simple harmonic motion $\dfrac{d^2x}{dt^2}=-\mu x$. At $t=0$, $x=a$ and velocity $v=0$. Find the time taken to go from the position $x=\dfrac{a}{2}$ to $x=a$. Also prove that this time is $\dfrac{1}{6}$ of the period. (8)

13. (a) (i) Evaluate $\iiint dx\,dy\,dz$ over the volume cut off the sphere $x^2 + y^2 + z^2 = a^2$ by the cone $x^2 + y^2 = z^2$. (8)

(ii) A vector field is given by $F = (x^2 - y^2 + x)i - (2xy + y)j$. Show that the field is irrotational and find its scalar potential. Hence, evaluate the line integral from (1, 2) to (2, 1). (8)

Or

(b) (i) Find a and b so that the surfaces $ax^2 - byz = (a+2)x$ is orthogonal to $4x^2y + z^3 = 4$ at $(1, -1, 2)$. (6)

(ii) Verify Stoke's theorem for $F = (y - z)i + yz\,j - xz\,k$ where S is the surface bounded by the planes $x = 0$, $x = 1$, $y = 0$, $y = 1$, $z = 0$ and $z = 1$ above the xoy plane. (10)

14. (a) (i) If $u = x^2 - y^2$ and $v = -\dfrac{y}{x^2 + y^2}$, prove that both u and v satisfy Laplace equation, but that $u + iv$ is not a regular function of z. (8)

(ii) Using Cauchy's integral formula for derivatives, evaluate $\int_C \dfrac{e^{2z}}{(z+1)^4}\,dz$ where C is the circle $|z| = 2$. (8)

Or

(b) (i) Evaluate $\int_0^{2\pi} \dfrac{d\theta}{1 - 2a\cos\theta + a^2}$ when $|a| < 1$. (8)

(ii) Find the bilinear transformation if 1 and i are fixed points and the origin in z-plane goes to -1 in w-plane. (8)

15. (a) (i) Solve $t\,y' - y = 5$ by using Laplace Transform given $y(1) = 10$. (8)

(ii) Using convolution theorem find the inverse of $\dfrac{s}{(s^2 + 121)^2}$. (8)

Or

(b) (i) Find the Laplace transform of the square wave function

$$f(t)=\begin{cases} k \text{ in } 0 \le t \le a \\ -k \text{ in } a \le t \le 2a \end{cases} \text{ and } f(t+2a)=f(t) \text{ for all } t. \quad (6)$$

(ii) Solve the simultaneous equations :

$D^2x - Dy = \cos t$ and

$Dx + D^2y = -\sin t$ where $D = \frac{d}{dt}$

given that $x = 1$, $Dx = 0$, $y = 0$ and $Dy = 1$ at $t = 0$. (10)