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Question Paper Code : P 1389

B.E./B.Tech. DEGREE EXAMINATION, NOVEMBER/DECEMBER 2009.

Fifth Semester

(Regulation 2004)

Computer Science and Engineering

MA 1256 — DISCRETE MATHEMATICS

(Common to B.E. (Part-Time) Fourth Semester – Regulation 2005)

Time : Three hours Maximum : 100 marks

Answer ALL questions.

PART A — ($10 \times 2 = 20$ marks)

1. Define atomic and compound statements.

2. When a set of formulae is consistent and inconsistent?

3. Symbolize the following statement with and without using the set of positive integers as the universe of discourse. "Give any positive integer, there is a greater positive integers".

4. What are free and bound variables in predicate logic?

5. Suppose that the sets A and B have m and n elements respectively. How many elements of $A \times B$? How many different relations are there from A to B?

6. Let $D(45)$ denote the set of all positive divisors of 12. Draw the Hasse diagram of $D(45)$.

7. If the function f is defined by $f(x) = x^2 + 1$ on a set $A = \{-2, -1, 0, 1, 2\}$, find the range of f.

8. Show that $f(x, y) = x^y$ is a primitive recursive function.

9. Show that the inverse of an element in a group $\langle G, * \rangle$ is unique.

10. Find the minimum distance between the code words $x = (1, 0, 0, 1)$, $y = (0, 1, 1, 0)$ and $z = (1, 0, 1, 0)$.

PART B — (5 × 16 = 80 marks)

11. (a) (i) Without constructing truth table verify whether $Q \vee (P \wedge \neg Q) \vee (\neg P \wedge \neg Q)$ is a contradiction or tautology. (8)

(ii) Show that $(\neg P \wedge (\neg Q \wedge R)) \vee (Q \wedge R) \vee (P \wedge R) \Leftrightarrow R$. (8)

Or

(b) (i) Show that $(R \wedge S)$ can be derived from the premises $P \to Q$, $Q \to \neg R, R, P \vee (R \wedge S)$. (6)

(ii) Without constructing truth table obtain PDNF of $(P \to (Q \wedge R)) \wedge (\neg P \to (\neg Q \wedge \neg R))$ and hence find its PCNF. (10)

12. (a) (i) Show that $(\forall x)\ (P(x) \to Q(x)) \wedge (\forall x)\ (Q(x) \to R(x)) \Rightarrow (\forall x)\ (P(x) \to R(x))$. (6)

(ii) Show that $(\exists x)\ P(x) \to (\forall x)\ Q(x) \Leftrightarrow (\forall x)\ (P(x) \to Q(x))$. (10)

Or

(b) (i) Use indirect method of proof to show that $(\exists z)\ Q(z)$ is not valid conclusion from the premises $(\forall x)\ (P(x) \to Q(x))$ and $(\exists y)\ Q(y)$. (8)

(ii) Using CP or otherwise obtain the following implication $(x)\ (P(x) \to Q(x)), (x)\ (R(x) \to \neg Q(x)) \Rightarrow (x)\ (R(x) \to \neg P(x))$. (8)

13. (a) (i) Let N be the set of all natural numbers with the relation R as follows : $a R b$ if and only if a divides b. Show that R is a partial order relation on N. (8)

(ii) Show that in a complemented distributive lattice, the De Morgan's laws hold. (8)

Or

(b) (i) Let R be the relation on $A = \{1, 2, 3\}$ such that (a, b) if and only if $a + b$ is even, find the relational matrix of R, R^{-1}, $\overline{R}$ and R^2. (8)

(ii) Show that every distributive lattice is modular. Is the converse true? Justify the claim. (8)

14. (a) (i) Let $A = \{1, 2, 3\}$. If f is a function of A into itself defined by $f(1) = 2, f(2) = 1, f(3) = 3$, find f^3 and f^{-1}. (8)

(ii) Test whether the function $f : Z \to Z$ defined by $f(x) = x^2 + 14x - 51$ is an injective and a surjective. (8)

Or

(b) (i) If $f, g : R \to R$, where $f(x) = ax + b, g(x) = 1 - x + x^2$ and $(g \circ f)(x) = 9x^2 - 9x + 3$ find the values of a and b. (8)

(ii) If A and B are any two subsets of a universal set U and f_A and f_B are the characteristic functions of A and B respectively. Show that $f_{A \cup B}(X) = f_A(X) + f_B(X) - f_{A \cap B}(X)$ for all $X \in U$. (8)

15. (a) (i) Show that the order of a subgroup of a finite group G divides the order of the group G. (8)

(ii) Find the code words generated by the parity check matrix $H = \begin{pmatrix} 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 \end{pmatrix}$, when the encoding function is $e : B^3 \to B^6$. (8)

Or

(b) (i) Let H be a nonempty subset of a group $\langle G, *\rangle$. Show that H is a subgroup of G if and only if $a * b^{-1} \in H$ for all $a, b \in H$. (8)

(ii) Show that the Kernel of a group homomorphism is a normal subgroup of a group. (8)