Reg. No. :

Question Paper Code : R 3761

B.E./B.Tech. DEGREE EXAMINATION, NOVEMBER/DECEMBER 2009.

Fourth Semester

Computer Science and Engineering

MA 040 — PROBABILITY AND QUEUEING THEORY

(Regulation 2001)

Time : Three hours Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. The cumulative distribution function of a random variable is given by

$$F(x)=\begin{cases}0, & x\le 1\\ c(x-1)^4, & 1<x\le 3\\ 1, & x>3\end{cases}$$ Find the value of c and probability density function.

2. One card is drawn from a well shuffled pack of 52 playing cards. What is the probability that it is a red numeric card bearing a multiple of 2?

3. State Central limit theorem.

4. The joint probability density function is given by $f(x,y)=x+y, 0\le x, y\le 1$, find $E(XY)$.

5. Check whether the random process $X(t)=A\cos 2\pi t$ is first order stationary, where A is a constant.

6. Define Markov process.

7. An equipment is to be designed to have a minimum reliability of 0.8 and minimum availability of 0.98 over a period of 2×10^3 hr. Determine the mean repair time.

8. If the transition probability matrix of a Markov chain is $\begin{pmatrix}0 & 1\\ \frac{1}{2} & \frac{1}{2}\end{pmatrix}$, find the steady-state distribution of the chain.

9. Define traffic intensity.

10. For an M/M/1 queuing system, if $\lambda = 6$ and $\mu = 8$, find the probability of at least 10 customers in the system.

PART B — (5 × 16 = 80 marks)

11. (a) (i) The number of monthly break downs of a bike is a random variable having Poisson distribution with mean 1.8, find that this bike will function for a month

(1) without a break down

(2) with only one break down

(3) with atleast one break down. (12)

(ii) If X is a uniformly distributed over $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$, find the p.d.f. of $y = \tan x$. (4)

Or

(b) (i) A random variable X has following probability distribution.

x :	-2	-1	0	1	2	3
$p(x)$:	0.1	K	0.2	$2K$	0.3	$3K$

Find K, $P(X<2)$, cumulative distribution function of X and the mean of X. (8)

(ii) Two defective tubes get mixed up with 2 good ones. The tubes are tested, one by one, until both defectives are found. What is the probability that the last defective tube is obtained on (1) the second test, (2) the third test and (3) the fourth test? (8)

12. (a) The joint probability density function of a bivariate random variable is given by

$$f_{XY}(x,y) = \begin{cases} kxy, & 0 \le x \le 1, \ 0 \le y \le 1 \\ & 0, \text{ otherwise.} \end{cases}$$

(i) Find the value of k, (ii) Find $P(X+Y<1)$ and (iii) Are X and Y independent? (16)

Or

(b) (i) If X and Y are independent random variables each normally distributed with mean zero and variance σ^2, find the density function of $R = \sqrt{X^2+Y^2}$ and $\theta = \tan^{-1}\left(\frac{Y}{X}\right)$. (8)

(ii) The pdf of X and Y is given by $f(x,y) = kxye^{-(x^2+y^2)}, x>0, y>0$. Prove that X and Y are independent. (8)

13. (a) (i) If $\{N_1(t)\}$ and $\{N_2(t)\}$ are two independent Poisson process with parameters λ_1 and λ_2 respectively, show that

$$P[N_1(t)= k / \{N_1(t)+N_2(t) = n\}] = \binom{n}{k} p^k q^{n-k},$$

where $p = \dfrac{\lambda_1}{\lambda_1+\lambda_2}$ and $q = \dfrac{\lambda_2}{\lambda_1+\lambda_2}$. (8)

(ii) For a random process $X(t) = Y \sin \omega t$, Y is a uniform random variable in the interval $(-1,1)$. Check whether the process is WSS or not. (8)

Or

(b) (i) Verify the sine wave process defined by $X(t) = Y \cos \omega t$, where Y is uniformly distributed over (0,1) is a SSS. (8)

(ii) A machine goes out of order whenever a component fails. A failure of this part follows a Poisson process with a mean rate of one per week. Find the probability that the two weeks have elapsed since last failure. If there are 5 spare parts of this component in an inventory and that the next supply is not due in 10 weeks, find the probability the machine will not be out of order in the next 10 weeks. (8)

14. (a) (i) Given that $R(t) = e^{-\sqrt{0.001t}}, t \geq 0$. (1) Compute the reliability for a 50 hour mission, (2) Show that the hazard rate is decreasing, (3) Given a 10 hours wear-in period, compute the reliability for a 50 hour mission, (4) What is the design life for a reliability of 0.95, given a 10 hour wear-in period? (8)

(ii) A series system is composed of 10 independent components. If the desired value of system reliability is 0.99, how good must the components be from the reliability point of view? (8)

Or

(b) (i) An electric bulb has a failure rate of 0.0002/hr when glowing and that of 0.00002/hr when not glowing. At the instant of switching ON, the failure rate is estimated to be 0.0005/switching. What is the average failure rate of the bulb if on the average it is switched 6 times every day and it remains ON for a total of 8 hrs in the day on the average? (8)

(ii) A man either drives a car (or) catches a train to go to office each day. He never goes 2 days in a row by train but if he drives one day, then the next day he is just as likely to drive again as he is to travel by train. Now suppose that on the first day of the week, the man tossed a fair die and drove to work if and only if a 6 appeared. Find (1) the probability that he takes a train on the third day and (2) the probability that he drives to work in the long run. (8)

15. (a) (i) Customers arrive at a watch repair shop according to a Poisson process at a rate of one per every 10 minutes, and the service time is an exponential random variable with mean 8 minutes.

(1) Find the average number of customers in the shop.

(2) Find the average time a customer's spends in the shop.

(3) Find the average number of customers in the queue.

(4) What is the probability that the server is idle? (8)

(ii) A one-man barber shop takes exactly 25 minutes to complete one-hair cut. If customers arrive at the barber shop in a Poisson fashion at an average rate of one every 40 minutes, how long on the average a customer spends in the shop? Also find the average time a customer must wait for service. (8)

Or

(b) Find the system size probabilities for an M/M/C queueing system under steady-state conditions. Also obtain the expression for mean number of customers in the system and waiting time of a customer in the system. (16)