Reg. No. : | | | | | | | | | | | |

Question Paper Code : R 3762

B.E./B.Tech. DEGREE EXAMINATION, NOVEMBER/DECEMBER 2009.

Third Semester

Civil Engineering

MA 231 — MATHEMATICS — III

(Common to all branches of B.E./B.Tech. except Civil Engineering and Computer Based Construction, Industrial Bio Technology, Bio Medical, Textile Chemistry)

(Regulation 2001)

Time : Three hours Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. Find the complete integral of $\sqrt{p}+\sqrt{q}=1$.

2. Solve : $(D-2D')(D-2D'+1)z=0$.

3. State the Parseval's formula for $f(x)$ defined in $(-l,l)$.

4. If $f(x)=\sin x$, $-\pi<x<\pi$, find a_n.

5. Classify the partial differential equation

   $(x+1)z_{xx}+\sqrt{2}(x+y+1)z_{xy}+(y+1)z_{yy}+yz_x-xz_y+2\sin x=0$.

6. A taut string of length 50 cm fastened at both ends, is disturbed from its position of equilibrium by imparting to each of its points an initial velocity of magnitude kx for $0<x<50$. Formulate the problem mathematically.

7. Find the Laplace transform of $f(t)=te^{-t}$.

8. State the initial and final value theorems of Laplace transform.

9. Find the Fourier transform of $f(x)$, defined as $f(x)=\begin{cases}1, & |x|<a\\ 0, & |x|>a\end{cases}$.

10. Find the Fourier cosine transform of $f(x)=e^{-ax}$, $a>0$.

PART B — (5 × 16 = 80 marks)

11. (a) (i) Solve the equation $x^2(y-z)p + y^2(z-x)q = z^2(x-y)$. (8)

(ii) Solve the equation $(D^2 + 4DD' + D'^2)\, z = e^{2x-y} + 2x$. (8)

Or

(b) (i) Solve the equation $pq + p + q = 0$. (4)

(ii) Solve $(D^2 - 2DD' + D'^2)z = \cos(x-3y) + 2 + xy + e^{x-y}$. (12)

12. (a) (i) Find the Fourier series of $f(x) = e^{-x}$ in $(-\pi, \pi)$. (8)

(ii) Find the Fourier sine series of $f(x) = x(\pi - x)$, $0 < x < \pi$. (8)

Or

(b) (i) Obtain the Fourier series of the function given by

$$f(x)=\begin{cases} 1+\dfrac{2x}{l}, & -l \le x \le 0 \\ 1-\dfrac{2x}{l}, & 0 \le x < l \end{cases}.$$ (8)

(ii) Compute the fundamental and first harmonics of the Fourier series of $f(x)$ in (0, 6) given by the table (8)

x:	0	1	2	3	4	5
$f(x)$:	4	8	15	7	6	2

13. (a) A tightly stretched string with end points $x = 0$ and $x = L$ is initially in a position given by $y(x,0) = kx(L-x)$. If it is released from this position, find the displacement $y(x, t)$ at any point of the string.

Or

(b) An insulated rod of length L has its ends A and B maintained at 0°C and 100°C respectively, until steady state conditions prevail. If B is suddenly reduced to 0°C and that at A is maintained at 0°C, find the temperature at a distance x from A at time t.

14. (a) (i) Find the Laplace transform of the periodic function

$$f(t)=\begin{cases} a, & 0 \le t < a \\ -a, & a \le x \le 2a \end{cases} \text{ and } f(t+2a)=f(t). \quad (8)$$

(ii) Solve the differential equation $y''-2y'-8y=0$, $y(0)=3$ and $y'(0)=6$ using Laplace transform. (8)

Or

(b) (i) Using convolution theorem, find $L^{-1}\left\{\frac{16}{(s-2)(s+4)}\right\}$. (8)

(ii) Solve $x'=2x-3y$; $y'=y-2x$, $x(0)=8$, and $y(0)=3$. (8)

15. (a) (i) Find the Fourier transform of $f(x)=\begin{cases} 1-x^2 & |x|<1 \\ 0 & |x|>1 \end{cases}$. Hence evaluate $\int_0^\infty \left(\frac{x\cos x-\sin x}{x^3}\right)\cos\frac{x}{2}\,dx$. (8)

(ii) Evaluate $\int_0^\infty \frac{dx}{(x^2+a^2)(x^2+b^2)}$ using transform methods. (8)

Or

(b) (i) Find the Fourier transform of $f(x)=\begin{cases} a-|x|, & |x|<a \\ 0, & |x|>a>0 \end{cases}$ and hence evaluate $\int_0^\infty \left(\frac{\sin x}{x}\right)^4 dx$. (8)

(ii) Find the Fourier sine transform of $e^{-ax}, a>0$ and hence show that $\int_0^\infty \frac{x\sin mx}{1+x^2}\,dx=\frac{\pi}{2}e^{-m}, m>0$. (8)

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