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Question Paper Code : S 4699

B.E./B.Tech. DEGREE EXAMINATION, NOVEMBER/DECEMBER 2009.

Second Semester

Civil Engineering

MA 132 — MATHEMATICS – II

(Common to all branches of B.E./B.Tech. except Information Technology)

Time : Three hours Maximum : 100 marks

Use of statistical tables permitted.

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. Evaluate $\int_1^a \int_1^b \frac{dxdy}{xy}$.

2. Change the order of integration in $\int_{x^2}^{x} f(x,y)dydx$.

3. Find a unit normal to the surface $x^2+y^2=z$ at (1, 2, 5).

4. Prove that $curl(grad\phi)=0$.

5. Test whether the function $f(z)=z^2$ is analytic or not.

6. Find the invariant points of the transformation $W=\frac{3z-5}{z+1}$.

7. Classify the singularity of the function $f(z)=e^{1/z}$.

8. State Cauchy's residue theorem.

9. Prove that the first moment about mean is always zero.

10. Define *Correlation Coefficient.*

PART B — (5 × 16 = 80 marks)

11. (a) (i) Find the volume bounded by the cylinder $x^2+y^2=4$ and the planes $y+z=4$ and $z=0$. (8)

(ii) Change the order integration $\int_0^a \int_y^a \frac{x}{x^2+y^2}\,dx\,dy$ and then evaluate. (8)

Or

(b) (i) Prove that $\beta(m,n)=\frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$. (8)

(ii) Find $\int_S \int r\,dr\,d\theta$ over the are S which is the region between the circles $r=2\sin\theta$ and $r=4\sin\theta$. (8)

12. (a) (i) Show that $\vec{F}=(6xy+z^3)\vec{i}+(3x^2-z)\vec{j}+(3z^2-y)\vec{k}$ is irrotational. Find ϕ such that $\vec{F}=\nabla\phi$. (8)

(ii) Verify Gauss's divergence theorem for $\vec{F}=x^2\,\vec{i}+y^2\vec{j}+z^2\vec{k}$ taken over the cube bounded by planes $x=0, x=1, y=1, z=0$ and $z=1$. (8)

Or

(b) (i) Prove that the area bounded by a simple closed curve C is given by $\frac{1}{2}\int_C (x\,dy-y\,dx)$. Hence find the area of the ellipse. (8)

(ii) Verify Green's theorem for $\int_C [(xy+y^2)\,dx+x^2\,dy]$ where C is the boundary of the common area between $y=x^2$ and $y=x$. (8)

13. (a) (i) Verify that $v=x^2-y^2+\frac{x}{x^2+y^2}$ is harmonic and find u such that $w=u+iv$ is analytic. Express w as a function of z. (8)

(ii) If $f(z)$ is a regular function of z, prove that

$$\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)|f(z)|^2=4|f'(z)|^2. \quad (8)$$

Or

(b) (i) Under the transformation $w=z^2$, obtain the map in the w – plane of the square with vertices (0, 0), (2, 0) (2, 2) and (0, 2) in z – plane. (8)

(ii) Find the bilinear transformation that maps $(-1, i, 1)$ onto $(1, i, -1)$. (8)

14. (a) (i) Find the Laurent's series of $f(z)=\frac{1}{z(1-z)}$ valid in the region

(1) $|z+1|<1$,

(2) $1<|z+1|<2$ and

(3) $|z+1|>2$. (8)

(ii) Evaluate $\int_0^{2\pi}\frac{d\theta}{2+\cos\theta}$ using contour integration. (8)

Or

(b) (i) Evaluate $\int_{-\infty}^{\infty}\frac{x^2-x+2}{x^4+10x^2+9}dx$ using contour integration. (8)

(ii) Use Cauchy's integral formula to evaluate $\int_C\frac{\sin\pi z^2+\cos\pi z^2}{(z-2)(z-3)}dz$, where C is the circle $|z|=4$. (8)

15. (a) (i) Samples of two types of electric bulbs were tested for length of life and the following data were obtained.

	Size	Mean	S.D.
Sample I	8	1234 hours	36 hours
Sample II	7	1036 hours	40 hours

Is the difference in the means sufficient to warrant that type I bulbs are superior to type II bulbs? (8)

(ii) In a large city A, 20% of a random sample of 900 school boys had a slight physical defect. In another large city B, 18.5% of a random sample of 1600 school boys had the same defect. Is the difference between the proportions significant? (8)

Or

(b) (i) Calculate the Coefficient of Correlation and the lines of regression for the following data : (8)

x :	4	7	5	6	8	5	6	6	4	9
y :	2.5	6	4.5	5	4.5	2	3	4.5	3	5.5

(ii) A set of five similar coins is tossed 320 times and the result is

No. of Heads :	0	1	2	3	4	5
Frequency :	6	27	72	112	71	32

Test the hypothesis that the data follow a binomial distribution. (8)
