



8. State Parseval's identity in Fourier transforms.
9. Find the Z-transform of  $3^n$ .
10. Does the Z-transform of  $n!$  exist? Justify your answer.

PART B — (5 × 16 = 80 marks)

11. (a) (i) Form the partial differential equation by eliminating the arbitrary function  $g$  from the relation  $g(x^2 + y^2 + z^2, xyz) = 0$ . (8)
- (ii) Solve:  $(3z - 4y)p + (4x - 2z)q = 2y - 3x$ . (8)

Or

- (b) (i) Solve:  $z = px + qy + \sqrt{1 + p^2 + q^2}$ . (8)
- (ii) Solve:  $(D^3 - 7DD^2 - 6D^3)Z = e^{2x+3y} + \sin(x+2y)$ . (8)

12. (a) Expand  $f(x) = x - x^2$  as a Fourier series in  $-\pi < x < \pi$  and using this series find the R.M.S. value of  $f(x)$  in the interval  $(-\pi, \pi)$ . (16)

Or

- (b) Find the first three harmonics of Fourier series of  $y = f(x)$  from the following data: (16)

$x$ :	$0^\circ$	$30^\circ$	$60^\circ$	$90^\circ$	$120^\circ$	$150^\circ$	$180^\circ$	$210^\circ$	$240^\circ$	$270^\circ$	$300^\circ$	$330^\circ$
$y$ :	298	356	378	337	254	152	80	51	60	93	147	221

13. (a) A tightly stretched string with fixed end points  $x = 0$  and  $x = 10$  is initially at rest in its equilibrium position. If it is set vibrating giving each point a velocity  $3x(10 - x)$ , find the displacement  $y$  at any time and at any distance from end  $x = 0$ . (16)

Or

- (b) A rectangular plate with insulated surfaces is 8 cm wide and so long compared to its width that it may be considered as an infinite plate. If the temperature along short edge  $y = 0$  is  $u(x, 0) = 100 \sin\left(\frac{\pi x}{8}\right)$ ,  $0 < x < 8$ , while two long edges  $x = 0$  and  $x = 8$  as well as the other short edge are kept at  $0^\circ\text{C}$ . Find the steady-state temperature at any point of the plate. (16)

14. (a) Find the Fourier sine and cosine transform of  $x^{n-1}$ ,  $0 < n < 1$ . Hence show that  $\frac{1}{\sqrt{x}}$  is self reciprocal under both the transforms. (16)

Or

- (b) Find the Fourier transform of  $f(x) = \begin{cases} 1, & |x| < a \\ 0, & |x| > a > 0 \end{cases}$  and deduce the value of

(i)  $\int_0^{\infty} \frac{\sin t}{t} dt$  and

(ii)  $\int_0^{\infty} \left(\frac{\sin t}{t}\right)^2 dt$ . (16)

15. (a) (i) Form the difference equation whose solution is  $y_n = (A + Bn)2^n$ . (8)

- (ii) Evaluate the inverse Z transform of  $\frac{z^2}{(z-5)(z-4)}$ , using convolution theorem. (8)

Or

- (b) (i) Find the value of Z transform of  $\sin\left(\frac{n\pi}{4}\right)$ . (8)

- (ii) Solve the difference equation  $y_{k+2} + 2y_{k+1} + y_k = k$ , given  $y_0 = y_1 = 0$  using Z transform. (8)