Reg. No. :

Question Paper Code : Z 8426

B.Sc. DEGREE EXAMINATION, NOVEMBER/DECEMBER 2009.

Third Semester

Computer Technology

BCS 231 — PARTIAL DIFFERENTIAL EQUATIONS AND INTEGRAL TRANSFORMS

(Common to B.Sc. Information Technology)

(Regulation 2003)

Time : Three hours — Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. Solve $pq = 1$.

2. Find the general solution of $px + qy = z$.

3. State Dirichlet's conditions.

4. Find a Fourier sine series for the function $f(x) = 1;\ 0 < x < \pi$.

5. State Fourier integral theorem.

6. If Fourier transform of $f(x)$ is $F(s)$, prove that the Fourier transform of $f(x)\cos ax$ is $\frac{1}{2}[F(s-a)+F(s+a)]$.

7. Find $L(\sin^2 5t)$.

8. Find $L^{-1}\left(\frac{1}{s(s+1)}\right)$.

9. Define Z-transform.

10. Find $Z(n)$.

PART B — (5 × 16 = 80 marks)

11. (a) (i) Form the partial differential equation by eliminating f from

$$f\left(x^2+y^2+z^2, x+y+z\right)=0.$$ (8)

(ii) Solve : $(mz-ny)p+(nx-lz)q=ly-mx$. (8)

Or

(b) (i) Solve : $\left(D^3+D^2D'-DD'^2-D'^3\right)z=e^x\cos 2y$. (8)

(ii) Solve : $z=px+qy+\sqrt{1+p^2+q^2}$. (8)

12. (a) Find the Fourier series of periodicity 2π for

$$f(x)=\begin{cases} x & in\ (0,\pi) \\ 2\pi-x & in\ (\pi,2\pi) \end{cases}$$

and hence deduce $\frac{1}{1^2}+\frac{1}{3^2}+\frac{1}{5^2}+\ldots=\frac{\pi^2}{8}$. (16)

Or

(b) (i) Find half range cosine series for $f(x)=x^2$ in $0\le x\le\pi$ and hence deduce the same $1-\frac{1}{2^2}+\frac{1}{3^2}-\frac{1}{4^2}+\ldots$ (8)

(ii) Find the Fourier series of periodicity 2π for $f(x)=x^2$ in $-\pi<x<\pi$. Hence show that $\frac{1}{1^4}+\frac{1}{2^4}+\frac{1}{3^4}+\ldots$to $\infty=\frac{\pi^4}{90}$. (8)

13. (a) (i) Find the Fourier transform of

$$f(x)=\begin{cases}1-x^2 & if\ |x|<1\\ 0 & if\ |x|>1\end{cases}$$

Hence evaluate $\int_0^\infty \left(\frac{x\cos x-\sin x}{x^3}\right)\cos\frac{x}{2}\,dx$. (8)

(ii) Find Fourier cosine and sine transforms of $e^{-ax}, a>0$. (8)

Or

(b) (i) Find Fourier cosine transform of $e^{-a^2x^2}$. (8)

(ii) Using Parseval's identify, evaluate

$$\int_0^\infty \frac{x^2}{\left(a^2+x^2\right)}dx \text{ if } a>0.$$ (8)

14. (a) (i) Find the laplace transform of a 'square wave' given by

$$f(t)=\begin{cases}E & for\ 0\le t<\frac{T}{2}\\ -E & for\ \frac{T}{2}\le t<T\end{cases}$$

and $f(t+T)=f(t)$. (8)

(ii) Find $L^{-1}\left(\log\frac{s^2+1}{(s-1)^2}\right)$. (8)

Or

(b) (i) Find $L^{-1}\left(\frac{s}{\left(s^2+a^2\right)^2}\right)$. (8)

(ii) Use convolution theorem to find the inverse laplace transform of $\frac{s}{\left(s^2+a^2\right)\left(s^2+b^2\right)}$. (8)

15. (a) (i) Find the Z-transform of $\frac{1}{n+1}$ and $\frac{1}{n!}$. (8)

(ii) If $F(z)=\frac{2z^2+5z+14}{(z-1)^4}$, evaluate f_2. (8)

Or

(b) (i) Use convolution theorem to find

$$Z^{-1}\left(\frac{z^2}{(z-a)^2}\right). \quad (8)$$

(ii) Find $Z^{-1}\left(\frac{z}{(z-2)(z+3)^2}\right)$. (8)