Reg. No. :

Question Paper Code : Z 8422

B.Sc. DEGREE EXAMINATION, NOVEMBER/DECEMBER 2009.

Second Semester

Computer Technology

BCS 122 — ANALYTICAL GEOMETRY AND REAL AND COMPLEX ANALYSIS

(Common to B.Sc. Apparel and Fashion Technology)

(Regulation 2003)

Time : Three hours　　　　Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. Sketch the area of integration for $\iint_R f(x,y)\,dx\,dy$ where R is the region enclosed by $x = 2a$ and $x^3 = 4ay$.

2. Evaluate $\int_0^4\int_0^x e^{y/x}\,dydx$.

3. State Greens Theorem.

4. Find the gradient of $x^3 + y^3 + z^3 - 3xyz$ at (1,1,1).

5. Find the centre and radius of the sphere $x^2 + y^2 + z^2 - 2x - 2y + 4z = 10$.

6. Find the angle between the lines whose direction cosines are (1,1,1) and (2,0,1).

7. Check for analyticity of $\cos(x + iy)$.

8. Show that an analytic function with constant real part is constant.

9. Evaluate $\int_C \frac{e^{-z}}{z+1} dz$ where C is $|z| = \frac{1}{2}$ using Cauchy's integral formula.

10. Show that the function $u = x^3 + y^3 - 3xy^2 + 2xy - y^2$ is harmonic.

PART B — (5 × 16 = 80 marks)

11. (a) (i) Change the order of integration and then evaluate $\int_0^1 \int_{x^2}^{2-x} x^2 y^2 \, dy\, dx$.

(ii) Evaluate $\iiint_V (x+y+z) \, dx\, dy\, dz$ where V is the region inside the cylinder $x^2 + y^2 = a^2$ bounded by the planes $z = 0$ and $z = 3$.

Or

(b) (i) Find the area enclosed by the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.

(ii) Change the order of integration and the evaluate $\int_{-a}^{a} \int_0^{\sqrt{a^2-x^2}} x^2 y^2 \, dy\, dx$.

12. (a) (i) Evaluate $\iint_S \vec{F} \cdot \hat{n} \, dS$ where $\vec{F} = x^2 i + y^2 j + z^2 k$ and S is the portion of the plane $x + y + z = 1$ in the first octant.

(ii) Using Green's Theorem evaluate $\int_C (2x^2 - y^2)dx + (x^2 + y^2)dy$ where C is the boundary of the arc enclosed by the circle $x^2 + y^2 = 1$ in the upper half plane.

Or

(b) Verify Gauss divergence theorem for $\vec{F} = 4xi - y^2 j + z^2 k$ over the cube bounded by $x = 0, x = 2, y = 0, y = 2, z = 0, z = 2$.

13. (a) (i) Find the equation of the plane passing through (1,2,–3) and (2,–4,2) and perpendicular to the plane $2x - 3y + 5z = 6$.

(ii) Find the shortest distance between the lines $\frac{x-2}{2}=\frac{y+3}{-3}=\frac{z-1}{1}$ and $\frac{x-2}{3}=\frac{y-2}{-5}=\frac{z+2}{2}$.

Or

(b) (i) Show that the lines $\frac{x-5}{4}=\frac{y-7}{4}=\frac{z+3}{-5}$ and $\frac{x-8}{7}=\frac{y-4}{1}=\frac{z-5}{3}$ are coplanar.

(ii) Find the equation of the sphere having the circle $x^2+y^2+z^2+10y-4z-8=0$ as a great circle when intercepted by the plane $x+y+z=3$.

14. (a) (i) Find an analytic function whose imaginary part is $e^{-x}\left[2xy\cos y+(y^2-x^2)\sin y\right]$.

(ii) Show that an analytic function with constant modules is constant.

Or

(b) (i) If $f(z)=u+iv$ is analytic show that $\left[\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right]|f(z)|^2=4|f'(z)|^2$.

(ii) Find an analytic function whose imaginary part is $3x^2y-y^3$.

15. (a) (i) Obtain Laurent's series for $f(z)=\frac{1}{(z+2)(1+z^2)}$ in $|z|<1$ and $1<|z|<2$.

(ii) Prove that $\int_{-\infty}^{\infty}\frac{dx}{(x^2+1)^3}=\frac{3\pi}{8}$ using contour integration.

Or

(b) (i) Obtain Taylor's series expansion of $\frac{z^2-1}{(z+2)(z+3)}$ in $|z|<2$.

(ii) Evaluate $\int_{0}^{2\pi}\frac{d\theta}{(5+4\cos\theta)}$ using contour integration.
