Reg. No. : | | | | | | | | | | | | |

Question Paper Code : Z 8417

B.Sc. DEGREE EXAMINATION, NOVEMBER/DECEMBER 2009.

First Semester

Computer Technology

BCS 112 — TRIGONOMETRY, ALGEBRA AND CALCULUS

(Common to B.Sc Information Technology and B.Sc. Apparel and Fashion Technology)

(Regulation 2003)

Time : Three hours   Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. If $x+\frac{1}{x}=2\cos\theta$, find x.

2. Prove that $\cos i\theta=\cosh\theta$.

3. Give the matrix of the Quadratic form $Q(x,y,z)=x^2+y^2-z^2+2xy+4xz+3zy$.

4. Find the rank of matrix $A=\begin{bmatrix} 1 & 2 & 3 \\ 2 & \text{[illegible]} & 1 \\ \text{[illegible]} & -4 & 6 \end{bmatrix}$.

5. If $u=\sin^{-1}\frac{x}{y}+\tan^{-1}\frac{y}{x}$, find $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}$.

6. Give the Taylor's series expansion of e^{xy} in powers of $x-1$ and $y-1$ up to first degree terms.

7. Evaluate $\int_0^{\pi/2}\cos^3 x \; dx$.

8. Give the formula to find the length of a curve from one point to another point.

9. Find the particular integral of $(D^2-4D+4)\,y=e^{2x}$.

10. Convert the differential equation into linear differential equation with constant coefficients $x^2\dfrac{d^2y}{dx^2}+x\dfrac{dy}{dx}-3y=\sin(\log x)$.

PART B — (5 × 16 = 80 marks)

11. (a) (i) Prove that $(1+i)^n+(1-i)^n=2^{1+n/2}\cos\dfrac{n\pi}{4}$. (8)

(ii) Expand $\cos^3\theta\sin^6\theta$ in series of cosines of multiples of θ. (8)

Or

(b) (i) If $\tan(x+iy)=u+iv$ prove that $\dfrac{u}{v}=\dfrac{\sin 2x}{\sinh 2y}$. (8)

(ii) Separate in to real and imaginary part of $\tan^{-1}(x+iy)$. (8)

12. (a) (i) Test for constancy the following equations and hence solve $x+y+z=6, 3x+y+z=8, x-y+2z=5, 2x-2y+3z=7$. (8)

(ii) Find the eigenvalues and eigenvectors of $A=\begin{bmatrix}1&1&3\\1&5&1\\3&1&1\end{bmatrix}$. (8)

Or

(b) (i) Verify that the matrix $A=\begin{bmatrix}1&2&3\\2&-1&4\\3&1&-1\end{bmatrix}$ satisfies its own characteristic equation. (8)

(ii) Find the orthogonal matrix P to reduce the quadratic form $Q(x,y)=2x^2+2y^2+2xy$ into a canonical form. (8)

13. (a) (i) Discuss the maximum and minimum of $u=x^3+y^3-63x-63y+12xy$. (8)

(ii) If $u=(x^2+y^2+z^2)^{-1/2}$ prove that $\dfrac{\partial^2u}{\partial x^2}+\dfrac{\partial^2u}{\partial y^2}+\dfrac{\partial^2u}{\partial z^2}=0$. (8)

Or

(b) (i) If $u=f(x,y)$ where $x=r\cos\theta$ and $y=r\sin\theta$ prove that

$$\left(\frac{\partial u}{\partial x}\right)^2+\left(\frac{\partial u}{\partial y}\right)^2=\left(\frac{\partial x}{\partial r}\right)^2+\frac{1}{r^2}\left(\frac{\partial y}{\partial \theta}\right)^2. \quad (8)$$

(ii) Find the Jacobians of the transformation $x=r\sin\theta\cos\phi,\ y=r\sin\theta\sin\phi,\ z=r\cos\theta$. (8)

14. (a) (i) Evaluate $\int_0^{\pi}\frac{x\sin x}{1+\cos^2 x}dx$. (8)

(ii) Find the area cut off the parabola $4y=3x^2$ by the straight line $2y=3x+12$. (8)

Or

(b) (i) Find the reduction formula for $\int_0^{\pi/2}\sin^n x\,dx$.

(ii) Find the area enclosed by the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$.

15. (a) (i) Solve $(D^2+2D+2)y=e^{-x}\sin x$. (8)

(ii) Solve $\frac{dx}{dt}+y=\sin t,\ \frac{dy}{dt}+x=\cos t$. (8)

Or

(b) (i) Solve $x^2\frac{d^2y}{dx^2}-3x\frac{dy}{dx}+4y=x^2\cos(\log x)$. (8)

(ii) Solve $(D^2+9)y=\cos 3x\cos 5x$. (8)
