Reg. No. : | | | | | | | | | | | |

Question Paper Code : Y 8305

B.Sc. (Applied Science) DEGREE EXAMINATION, NOVEMBER/DECEMBER 2009.

Third Semester

Computer Technology

97 CT 301 — APPLIED MATHEMATICS — III

(Common to Information Technology – 99 UIT 08)

(Regulation 2002)

Time : Three hours — Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. If $L[F(t)]=F(s)$, then prove that $L[e^{-at} f(t)] = F(s+a)$.

2. Find $L[te^{-t}\cos(t)]$.

3. Find the Laplace transform of unit impulse function.

4. Prove that $L[f'(t)]=sF(s)-f(0)$, where $L[f(t)]=F(s)$.

5. Define the Euler's formula for the Fourier series expansion of the function $f(x)$ in the interval $(-L, L)$. Where L is any positive number.

6. Find the half range Fourier sine series for $f(x)=x$ in the interval $0<x<\pi$.

7. State the Cauchy-Riemann equations in polar coordinations.

8. State any two properties of analytic function.

9. State Cauchy's integral theorem.

10. Find the residue of $\frac{ze^z}{(z-a)^3}$ at $z=a$.

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PART B — (5 × 16 = 80 marks)

11. (a) (i) Find the Laplace transform of

$$f(t)=\begin{cases}1, 0<t<a\\ -1, a<t<2a\end{cases} \text{ and } f(t+2a)=f(t).$$ (8)

(ii) Find $L[t\sin^2 3t]$. (8)

Or

(b) (i) Find $L[f(t)]$, where

$$f(t)=\begin{cases}\cos t, & 0<t<\pi\\ \sin t, & t>\pi\end{cases}$$ (10)

(ii) Prove that

$$L[f''(t)]=S^2L[f(t)]-S\,f(0)-f'(0).$$ (6)

12. (a) (i) Find $L^{-1}\left[\dfrac{2s^2-4}{(s+1)(s-2)(s-3)}\right]$. (8)

(ii) Solve using Laplace transform, the differential equation $y''+2y'+5y=e^{-t}\sin t$, $y(0)=0$, $y'(0)=1$. (8)

Or

(b) (i) State and prove convolution theorem on Laplace transform. (8)

(ii) Solve the simultaneous equations $\dfrac{dx}{dt}-y=e^t$; $\dfrac{dy}{dt}+x=\sin t$, given $x(0)=1$, $y(0)=0$, using Laplace transform. (8)

13. (a) (i) Expand $f(x)=x\sin(x)$ as a cosine series in the interval $0<x<\pi$ and show that $1+2\left[\dfrac{1}{1.3}-\dfrac{1}{3.5}+\dfrac{1}{5.7}-....\infty\right]=\pi/2$. (8)

(ii) Find the Fourier series expansion for the function $f(x)=x^2$, $-\pi\le x\le\pi$. Hence, show that $\dfrac{1}{1^4}+\dfrac{1}{2^4}+\dfrac{1}{3^4}+....\infty=\dfrac{\pi^4}{90}$. (8)

Or

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(b) (i) Obtain the Fourier series for $f(x) = |\sin x|$ in the interval $(-\pi, \pi)$. (8)

(ii) Compute the first two harmonics of the Fourier series of $f(x)$ from the data given below : (8)

x :	0	$\pi/6$	$2\pi/6$	$3\pi/6$	$4\pi/6$	$5\pi/6$
$f(x)$:	5.35	3.5	6.09	7.82	8.58	8.43

x :	π	$7\pi/6$	$8\pi/6$	$9\pi/6$	$10\pi/6$	$11\pi/6$
$f(x)$:	7.73	6.98	6.19	6.04	5.55	5.01

14. (a) (i) Show that $u(x,y) = \exp(-x)[(x^2 - y^2)\cos(y) + 2xy\sin(y)]$ is harmonic and find the corresponding analytic function $w = f(z)$. (8)

(ii) Discuss the transformation $w = z^2 + k^2/z$, where k is a real and positive number, with suitable figures. (8)

Or

(b) (i) Derive the necessary conditions for $f(z)$ to be analytic in polar coordinates assuming the Cauchy-Riemann equations are true in Cartesian coordinates. (8)

(ii) Construct the bilinear transformation which maps the points $z = 0, 1, \alpha$ into the points $w = i, 1, -i$. (8)

15. (a) (i) Using cauchy's integral formula, find the value of $\int_C \frac{z+4}{z^2+2z+5}\,dz$ where C is the circle $|z+1-i| = 2$. (8)

(ii) Evaluate by contour integration $\int_{-\infty}^{\infty} \frac{x^2\,dx}{(x^2+a^2)(x^2+b^2)}$. (8)

Or

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(b) (i) **Find the Laurent expansion of the function** $f(z) = \frac{7z-2}{(z+1)z(z-2)}$ **in the region** $1 < |z+1| < 3$. (8)

(ii) **Show contour integration** $\int_0^{2\pi} \frac{d\theta}{13+5\sin\theta} = \frac{\pi}{12}$. (8)

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