



Government of Tamilnadu

STANDARD EIGHT

TERM III

Volume 2

MATHEMATICS

SCIENCE

SOCIAL SCIENCE

NOT FOR SALE

Untouchability is Inhuman and a Crime

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MATHEMATICS

STANDARD EIGHT

TERM III

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1

Life Mathematics



Roger Bacon

[1214-1294]

English philosopher, Wonderful teacher emphasised on empirical methods. He became a master at Oxford.

He stated: “Neglect of mathematics works injury to all knowledge”.

He said, “The importance of mathematics for a common man to underpinned whenever he visits banks, shopping malls, railways, post offices, insurance companies, or deals with transport, business transaction, imports and exports, trade and commerce”.

- 1.1 Introduction
- 1.2 Revision - Profit, Loss and Simple Interest
- 1.3 Application of Percentage, Profit and Loss, Overhead Expenses, Discount and Tax
- 1.4 Compound Interest
- 1.5 Difference between S.I. and C.I.
- 1.6 Fixed Deposits and Recurring Deposits
- 1.7 Compound Variation
- 1.8 Time and Work

1.1 Introduction

Every human being wants to reach the height of ‘WIN-WIN’ situation throughout his life. To achieve it effectively, he allocates his time to work, to earn more wealth and fame.

From stone age to present world, from material exchange to money transaction, for his produce and land, man has been applying the idea of ratio and proportion. The monumental buildings like the Taj Mahal and the Tanjore Brihadisvara Temple, known for their aesthetic looks, also demonstrate our ancestors’ knowledge and skill of using right kind of ratio to keep them strong and wonderful.

Many of the existing things in the world are connected by cause and effect relationship as in rain and harvest, nutrition and health, income and expenditure, etc. and hence compound variation arises.

In our effort to survive and ambition to grow, we borrow or deposit money and compensate the process preferably by means of compound interest. The government bears the responsibility of the sectors like security, health, education and other amenities. To deliver these to all citizens, we pay various taxes from our income to the government.

This chapter covers the topics which are interwoven in our lives.

1.2 Revision: Profit, Loss and Simple Interest

We have already learnt about profit and loss and simple interest in our previous class. Let us recall the following results:

RESULTS ON PROFIT, LOSS AND SIMPLE INTEREST

- (i) Profit or Gain = Selling price – Cost price
- (ii) Loss = Cost price – Selling price
- (iii) Profit % = $\frac{\text{Profit}}{\text{C.P.}} \times 100$.
- (iv) Loss % = $\frac{\text{Loss}}{\text{C.P.}} \times 100$
- (v) Simple interest (I) = $\frac{\text{Principal} \times \text{Time} \times \text{Rate}}{100} = \frac{Pnr}{100}$
- (vi) Amount = Principal + Interest

1.3 Application of Percentage, Profit and Loss, Overhead Expenses, Discount and Tax

1.3.1. Application of Percentage

We have already learnt percentages in the previous classes. We present these ideas as follows:

- (i) Two percent = $2\% = \frac{2}{100}$
- (ii) 8% of 600 kg = $\frac{8}{100} \times 600 = 48$ kg
- (ii) 125% = $\frac{125}{100} = \frac{5}{4} = 1 \frac{1}{4}$

Now, we learn to apply percentages in some problems.

Example 1.1

What percent is 15 paise of 2 rupees 70 paise?

Solution

$$\begin{aligned}
 2 \text{ rupees } 70 \text{ paise} &= (2 \times 100 \text{ paise} + 70 \text{ paise}) \\
 &= 200 \text{ paise} + 70 \text{ paise} \\
 &= 270 \text{ paise} \\
 \text{Required percentage} &= \frac{15}{270} \times 100 = \frac{50}{9} = 5\frac{5}{9}\%.
 \end{aligned}$$



Do you know?

$$\text{A circle with a horizontal line through the center} = \frac{1}{2} = 50\%$$

$$\text{A circle with a vertical line through the center} = \frac{1}{4} = 25\%$$

$$\text{A circle with both horizontal and vertical lines through the center} = \frac{3}{4} = 75\%$$

Example 1.2

Find the total amount if 12% of it is ₹ 1080.

Solution

Let the total amount be x .

Given : 12% of the total amount = ₹ 1080

$$\frac{12}{100} \times x = 1080$$

$$x = \frac{1080 \times 100}{12} = ₹ 9000$$

∴ The total amount = ₹ 9000.

Example 1.3

72% of 25 students are good in Mathematics. How many are not good in Mathematics?

Solution

Percentage of students good in Mathematics = 72%

Number of students good in Mathematics = 72% of 25 students

$$= \frac{72}{100} \times 25 = 18 \text{ students}$$

Number of students not good in Mathematics = $25 - 18 = 7$.

Example 1.4

Find the number which is 15% less than 240.

Solution

$$15\% \text{ of } 240 = \frac{15}{100} \times 240 = 36$$

∴ The required number = $240 - 36 = 204$.

Example 1.5

The price of a house is decreased from Rupees Fifteen lakhs to Rupees Twelve lakhs. Find the percentage of decrease.

Solution

Original price = ₹ 15,00,000

Change in price = ₹ 12,00,000

Decrease in price = $15,00,000 - 12,00,000 = 3,00,000$

∴ Percentage of decrease = $\frac{300000}{1500000} \times 100 = 20\%$

Remember

Percentage of increase = $\frac{\text{Increase in amount}}{\text{Original amount}} \times 100$

Percentage of decrease = $\frac{\text{Decrease in amount}}{\text{Original amount}} \times 100$



Try these

15 sweets are divided between Sharath and Bharath, so that they get 20% and 80% of them respectively. Find the number of sweets got by each.

Activity



My Grandma says, in her childhood, gold was ₹ 100 per gram. Read a newspaper to know the price of gold and note down the price on the first of every month. Find the percentage of increase every month.



EXERCISE 1.1

- Choose the correct answer.
 - There are 5 oranges in a basket of 25 fruits. The percentage of oranges is ____
 (A) 5% (B) 25% (C) 10% (D) 20%
 - $\frac{2}{25} =$ _____ %.
 (A) 25 (B) 4 (C) 8 (D) 15
 - 15% of the total number of biscuits in a bottle is 30. The total number of biscuits is _____.
 (A) 100 (B) 200 (C) 150 (D) 300
 - The price of a scooter was ₹ 34,000 last year. It has increased by 25% this year. Then the increase in price is _____.
 (A) ₹ 6,500 (B) ₹ 8,500 (C) ₹ 8,000 (D) ₹ 7,000
 - A man saves ₹ 3,000 per month from his total salary of ₹ 20,000. The percentage of his savings is _____.
 (A) 15% (B) 5% (C) 10% (D) 20%
- 20% of the total quantity of oil is 40 litres. Find the total quantity of oil in litres.
 - 25% of a journey covers 5,000 km. How long is the whole journey?
 - 3.5% of an amount is ₹ 54.25. Find the amount.
 - 60% of the total time is 30 minutes. Find the total time.
 - 4% sales tax on the sale of an article is ₹ 2. What is the amount of sale?
- Meenu spends ₹ 2000 from her salary for recreation which is 5% of her salary. What is her salary?
- 25% of the total mangoes which are rotten is 1,250. Find the total number of mangoes in the basket. Also, find the number of good mangoes.

5. The marks obtained by Rani in her twelfth standard exams are tabulated below. Express these marks as percentages.

Subjects	Maximum marks	Marks obtained	Percentage of marks (out of 100)
(i) English	200	180	
(ii) Tamil	200	188	
(iii) Mathematics	200	195	
(iv) Physics	150	132	
(v) Chemistry	150	142	
(vi) Biology	150	140	

6. A school cricket team played 20 matches against another school. The first school won 25% of them. How many matches did the first school win?
7. Rahim deposited ₹ 10,000 in a company which pays 18% simple interest p.a. Find the interest he gets if the period is 5 years.
8. The marked price of a toy is ₹ 1,200. The shop keeper gave a discount of 15%. What is the selling price of the toy?
9. In an interview for a computer firm 1,500 applicants were interviewed. If 12% of them were selected, how many applicants were selected? Also find the number of applicants who were not selected.
10. An alloy consists of 30% copper and 40% zinc and the remaining is nickel. Find the amount of nickel in 20 kilograms of the alloy.
11. Pandian and Thamarai contested for the election to the Panchayat committee from their village. Pandian secured 11,484 votes which was 44% of the total votes. Thamarai secured 36% of the votes. Calculate (i) the number of votes cast in the village and (ii) the number of voters who did not vote for both the contestants.
12. A man spends 40% of his income for food, 15% for clothes and 20% for house rent and saves the rest. What is the percentage of his saving? If his income is ₹ 34,400, find the amount of his savings.
13. Jyothika secured 35 marks out of 50 in English and 27 marks out of 30 in Mathematics. In which subject did she get more marks and how much?
14. A worker receives ₹ 11,250 as bonus, which is 15% of his annual salary. What is his monthly salary?
15. The price of a suit is increased from ₹ 2,100 to ₹ 2,520. Find the percentage of increase.



Try these

1. $40\% = 100\% - \text{_____}\%$
2. If 25% of students in a class come to school by walk, 65% of students come by bicycle and the remaining percentage by school bus, what percentage of students come by school bus?
3. In a particular class, 30% of the students take Hindi, 50% of them take Tamil and the remaining students take French as their second language. What is the percentage of students who take French as their second language?
4. In a city, 30% are females, 40% are males and the remaining are children. What is the percentage of the children?

Activity



Amutha buys silk sarees from two different merchants Ganesan and Govindan. Ganesan weaves 200 gm of silver thread with 100 gm of bronze thread whereas Govindan weaves 300 gm of silver thread with 200 gm of bronze thread for the sarees. Calculate the percentage of silver thread in each and find who gives a better quality. [Note: More the silver thread better the quality.]

1.3.2 Applications of Profit and Loss

In this section, we learn to solve problems on applications of profit and loss.

(i) Illustration of the formula for S.P.

Consider the following situation:

Rajesh buys a pen for ₹ 80 and sells it to his friend. If he wants to make a profit of 5%, can you say the price for which he would have sold?

Rajesh bought the pen for ₹ 80 which is the Cost Price (C.P.). When he sold, he makes a profit of 5% which is calculated on the C.P.

$$\therefore \text{Profit} = 5\% \text{ of C.P.} = \frac{5}{100} \times 80 = ₹ 4$$

Since there is a profit, $\text{S.P.} > \text{C.P.}$

$$\begin{aligned} \text{S.P.} &= \text{C.P.} + \text{Profit} \\ &= 80 + 4 = ₹ 84. \end{aligned}$$

\therefore The price for which Rajesh would have sold = ₹ 84.

The same problem can be done by formula method:

$$\begin{aligned} \text{Selling price (S.P.)} &= \frac{(100 + \text{Profit}\%)}{100} \times \text{C.P.} \\ &= \frac{(100 + 5)}{100} \times 80 = \frac{105}{100} \times 80 = ₹ 84. \end{aligned}$$



(ii) Illustration of the formula for C.P.

Consider the following situation:

Suppose a shopkeeper sells a wrist watch for ₹ 540 making a profit of 5 %, then what would have been the cost of the watch?

The shopkeeper had sold the watch at a profit of 5 % on the C.P. Since C.P. is not known, let us take it as ₹ 100.

Profit of 5% is made on the C.P.

$$\begin{aligned}\therefore \text{Profit} &= 5\% \text{ of C.P.} \\ &= \frac{5}{100} \times 100 \\ &= ₹ 5.\end{aligned}$$

We know, $\text{S.P.} = \text{C.P.} + \text{Profit}$

$$\begin{aligned}&= 100 + 5 \\ &= ₹ 105.\end{aligned}$$

Here, if S.P. is ₹ 105, then C.P. is ₹ 100.

When S.P. of the watch is ₹ 540, $\text{C.P.} = \frac{540 \times 100}{105} = ₹ 514.29$

\therefore The watch would have cost ₹ 514.29 for the shopkeeper.

The above problem can also be solved by using the formula method.

$$\begin{aligned}\text{C.P.} &= \left(\frac{100}{100 + \text{profit \%}} \right) \times \text{S.P.} \\ &= \frac{100}{100 + 5} \times 540 \\ &= \frac{100}{105} \times 540 \\ &= ₹ 514.29.\end{aligned}$$

We now summarize the formulae to calculate S.P. and C.P. as follows:

1. When there is a profit (i) $\text{C.P.} = \left(\frac{100}{100 + \text{profit \%}} \right) \times \text{S.P.}$	1. When there is a loss (ii) $\text{C.P.} = \left(\frac{100}{100 - \text{loss \%}} \right) \times \text{S.P.}$
2. When there is a profit (i) $\text{S.P.} = \left(\frac{100 + \text{profit \%}}{100} \right) \times \text{C.P.}$	2. When there is a loss, (ii) $\text{S.P.} = \left(\frac{100 - \text{loss \%}}{100} \right) \times \text{C.P.}$



Example 1.6

Hameed buys a colour TV set for ₹ 15,200 and sells it at a loss of 20%. What is the selling price of the TV set?

Solution

Raghul used this method:

Loss is 20% of the C.P.

$$= \frac{20}{100} \times 15200$$

$$= ₹ 3040$$

$$\text{S.P.} = \text{C.P.} - \text{Loss}$$

$$= 15,200 - 3,040$$

$$= ₹ 12,160$$

OR

Roshan used the formula method:

$$\text{C.P.} = ₹ 15,200$$

$$\text{Loss} = 20\%$$

$$\text{S.P.} = \frac{100 - \text{Loss}\%}{100} \times \text{C.P.}$$

$$= \frac{100 - 20}{100} \times 15200$$

$$= \frac{80}{100} \times 15200$$

$$= ₹ 12,160$$

Both Raghul and Roshan came out with the same answer that the selling price of the TV. set is ₹ 12,160.

Example 1.7

A scooty is sold for ₹ 13,600 and fetches a loss of 15%. Find the cost price of the scooty.

Devi used this method:

Loss of 15% means,

if C.P. is ₹ 100, Loss = ₹ 15

Therefore, S.P. would be

$$(100 - 15) = ₹ 85$$

If S.P. is ₹ 85, C.P. is ₹ 100

When S.P. is ₹ 13,600, then

$$\begin{aligned} \text{C.P.} &= \frac{100 \times 13600}{85} \\ &= ₹ 16,000 \end{aligned}$$

OR

Revathi used the formula method:

$$\text{Loss} = 15\%$$

$$\text{S.P.} = ₹ 13,600$$

$$\text{C.P.} = \frac{100}{100 - \text{Loss}\%} \times \text{S.P.}$$

$$= \frac{100}{100 - 15} \times 13600$$

$$= \frac{100}{85} \times 13600$$

$$= ₹ 16,000$$

Hence the cost price of the scooty is ₹ 16,000.



Try these

Items	Cost price in ₹	Profit/Loss	Selling price in ₹
Washing Machine	16,000	9% Profit	
Microwave Oven	13,500	12% Loss	
Wooden Shelf		13% Loss	6,786
Sofa set		12½% Profit	7,000
Air Conditioner	32,400	7% Profit	

Example 1.8

The cost price of 11 pens is equal to the selling price of 10 pens. Find the loss or gain percent.

Solution

Let S.P. of each pen be x .

$$\text{S.P. of 10 pens} = ₹ 10x$$

$$\text{S.P. of 11 pens} = ₹ 11x$$

Given : C.P. of 11 pens = S.P. of 10 pens = ₹ 10x

Here, S.P. > C.P.

$$\therefore \text{Profit} = \text{S.P.} - \text{C.P.}$$

$$= 11x - 10x = x$$

$$\text{Profit \%} = \frac{\text{Profit}}{\text{C.P.}} \times 100 = \frac{x}{10x} \times 100 = 10\%.$$

Example 1.9

A man sells two wrist watches at ₹ 594 each. On one he gains 10% and on the other he loses 10%. Find his gain or loss percent on the whole.

Solution

Given : S.P. of the first wrist watch = ₹ 594, Gain% = 10%

$$\begin{aligned} \therefore \text{C.P. of the first wrist watch} &= \frac{100}{100 + \text{profit\%}} \times \text{S.P.} \\ &= \frac{100}{(100 + 10)} \times 594 \\ &= \frac{100}{110} \times 594 = ₹ 540. \end{aligned}$$

Similarly, C.P. of the second watch on which he loses 10% is

$$\begin{aligned} &= \frac{100}{(100 - \text{Loss\%})} \times \text{S.P.} \\ &= \frac{100}{(100 - 10)} \times 594 = \frac{100}{90} \times 594 = ₹ 660. \end{aligned}$$

To say whether there was an overall Profit or Loss, we need to find the combined C.P. and S.P.

$$\text{Total C.P. of the two watches} = 540 + 660 = ₹ 1,200.$$

$$\text{Total S.P. of the two watches} = 594 + 594 = ₹ 1,188.$$

$$\text{Net Loss} = 1,200 - 1,188 = ₹ 12.$$

$$\begin{aligned} \text{Loss\%} &= \frac{\text{Loss}}{\text{C.P.}} \times 100 \\ &= \frac{12}{1200} \times 100 = 1\%. \end{aligned}$$

1.3.3. Application of Overhead Expenses

Maya went with her father to purchase an Air cooler. They bought it for ₹ 18,000. The shop wherein they bought was not nearer to their residence. So they have to arrange a vehicle to take the air cooler to their residence. They paid conveyance charges of ₹ 500. Hence the C.P. of the air cooler is not only ₹ 18,000 but it also includes the **Conveyance Charges** (Transportation charges) ₹ 500 which is called as **Overhead Expenses**.



Now,

$$\begin{aligned} \text{C.P. of the air cooler} &= \text{Real cost} + \text{Conveyance charges} \\ &= 18,000 + 500 = ₹ 18,500 \end{aligned}$$

Consider another situation, where Kishore's father buys an old Maruti car from a Chennai dealer for ₹ 2,75,000 and spends ₹ 25,000 for painting the car. And then he transports the car to his native village for which he spends again ₹ 2,000. Can you answer the following questions:

- (i) What is the overall cost price of the car?
- (ii) What is the real cost price of the car?
- (iii) What are the overhead expenses referred here?

In the above example the painting charges and the transportation charges are the overhead expenses.

$$\begin{aligned} \therefore \text{Cost price of the car} &= \text{Real cost price} + \text{Overhead expenses} \\ &= 2,75,000 + (25,000 + 2,000) \\ &= 2,75,000 + 27,000 = ₹ 3,02,000. \end{aligned}$$

Thus, we come to the conclusion that,

Sometimes when an article is bought or sold, some additional expenses are made while buying or before selling it. These expenses have to be included in the cost price. These expenses are referred to as **Overhead Expenses**. These may include expenses like amount spent on repairs, labour charges, transportation, etc.,

Example 1.10

Raju bought a motorcycle for ₹ 36,000 and then bought some extra fittings to make it perfect and good looking. He sold the bike at a profit of 10% and he got a selling price of ₹ 44,000. How much did he spend to buy the extra fittings made for the motorcycle?

Solution

Let the C.P. be ₹ 100.

$$\text{Profit} = 10\%, \text{ S.P.} = ₹ 110$$

If S.P. is ₹ 110, then C.P. is ₹ 100.

When S.P. is ₹ 44,000

$$\text{C.P.} = \frac{44000 \times 100}{110} = ₹ 40,000$$

$$\therefore \text{Amount spent on extra fittings} = 40,000 - 36,000 = ₹ 4,000.$$

EXERCISE 1.2

1. Find the Cost price / Selling price.

Cost price	Selling price	Profit	Loss
(i) ₹ 7,282		₹ 208	
(ii)	₹ 572	₹ 72	
(iii) ₹ 9,684			₹ 684
(iv)	₹ 1,973	₹ 273	
(v) ₹ 6,76,000			₹ 18,500

2. Fill up the appropriate boxes and leave the rest.

C.P.	S.P.	Profit & Profit %	Loss & Loss%
(i) ₹ 320	₹ 384		
(ii) ₹ 2,500	₹ 2,700		
(iii) ₹ 380	₹ 361		
(iv) ₹ 40			₹ 2 Loss
(v) ₹ 5,000		₹ 500 Profit.	

3. Find the S.P. if a profit of 5% is made on

- a bicycle of ₹ 700 with ₹ 50 as overhead charges.
- a computer table bought at ₹ 1,150 with ₹ 50 as transportation charges.
- a table-top wet grinder bought for ₹ 2,560 and an expense of ₹ 140 on repair charges.

4. By selling a table for ₹ 1,320, a trader gains 10%. Find the C.P. of the table.
5. The cost price of 16 note books is equal to the selling price of 12 note books. Find the gain percent.
6. A man sold two articles at ₹ 375 each. On the first article, he gains 25% and on the other, he loses 25%. How much does he gain or lose in the whole transaction? Also, find the gain or loss percent in the whole transaction.
7. Anbarasan purchased a house for ₹ 17,75,000 and spent ₹ 1,25,000 on its interior decoration. He sold the house to make a profit of 20%. Find the S.P. of the house.
8. After spending Rupees sixty thousand for remodelling a house, Amla sold a house at a profit of 20%. If the selling price was Rupees forty two lakhs, how much did she spend to buy the house?
9. Jaikumar bought a plot of land in the outskirts of the city for ₹ 21,00,000. He built a wall around it for which he spent ₹ 1,45,000. And then he wants to sell it at ₹ 25,00,000 by making an advertisement in the newspaper which costs him ₹ 5,000. Now, find his profit percent.
10. A man sold two varieties of his dog for ₹ 3,605 each. On one he made a gain of 15% and on the other a loss of 9%. Find his overall gain or loss.
[Hint: Find C.P. of each]

1.3.4 Application of Discounts

Yesterday Pooja went to a shop with her parents to purchase a dress for Pongal festival. She saw many banners in the shop. The content of which she didn't understand.

With an unclear mind, she entered the shop and purchased a frock.

The price labelled on the frock was ₹ 550. It is called as Marked Price (abbreviated as M.P.) and she gave the shopkeeper ₹ 550. But the shopkeeper returned the balance amount and informed her that there is a discount of 20%.

Here, 20% discount means, 20% discount on the Marked Price.

$$\text{Discount} = \frac{20}{100} \times 550 = ₹ 110.$$

Discount is the reduction on the **Marked Price** or **List Price of the article**.

The normal price attached to the article before the discount made is called as **Marked Price (M.P.)** or **List Price of the article**.



Amount paid by Pooja to the shop keeper is ₹ 440

$$= ₹ 550 - ₹ 110$$

$$= \text{Marked Price} - \text{Discount}$$

Hence we conclude the following:

$$\text{Discount} = \text{Marked Price} - \text{Selling Price}$$

$$\text{Selling Price} = \text{Marked Price} - \text{Discount}$$

$$\text{Marked Price} = \text{Selling Price} + \text{Discount}$$



During festival seasons and in the Tamil month of “Aadi”, discounts or rebates of 10%, 20%, 30%, etc., are offered to attract customers by Co-optex, Khadi and other shops for various items to promote sales.

Example 1.11

A bicycle marked at ₹ 1,500 is sold for ₹ 1,350. What is the percentage of discount?

Solution

Given : Marked Price = ₹ 1500, Selling Price = ₹ 1350

$$\text{Amount of discount} = \text{Marked Price} - \text{Selling Price}$$

$$= 1500 - 1350$$

$$= ₹ 150$$

$$\text{Discount for ₹ 1500} = ₹ 150$$

$$\text{Discount for ₹ 100} = \frac{150}{1500} \times 100$$

$$\text{Percentage of discount} = 10\%.$$

Since Discount is on Marked Price, we will have to use Marked Price as the base.

Example 1.12

The list price of a frock is ₹ 220. A discount of 20% on sales is announced. What is the amount of discount on it and its selling price?

Solution

Given : List (Marked) Price of the frock = ₹ 220, Rate of discount = 20%

$$\text{Amount of discount} = \frac{20}{100} \times 220$$

$$= ₹ 44$$

$$\therefore \text{Selling Price of the frock} = \text{Marked Price} - \text{Discount}$$

$$= 220 - 44$$

$$= ₹ 176.$$

Example 1.13

An almirah is sold at ₹ 5,225 after allowing a discount of 5%. Find its marked price.

Solution

Krishna used this method:

The discount is given in percentage.

Hence, the M.P. is taken as ₹ 100.

$$\text{Rate of discount} = 5\%$$

$$\text{Amount of discount} = \frac{5}{100} \times 100 \quad [\text{OR}]$$

$$= ₹ 5.$$

$$\begin{aligned} \text{Selling Price} &= \text{M.P.} - \text{Discount} \\ &= 100 - 5 = ₹ 95 \end{aligned}$$

If S.P. is ₹ 95, then M.P. is ₹ 100.

When S.P. is ₹ 5225,

$$\text{M.P.} = \frac{100}{95} \times 5225$$

∴ M.P. of the almirah = ₹ 5,500.

Vignesh used the formula method:

$$\text{S.P.} = \text{Rs } 5225$$

$$\text{Discount} = 5\%$$

$$\text{M.P.} = ?$$

$$\text{M.P.} = \left(\frac{100}{100 - \text{Discount}\%} \right) \times \text{S.P.}$$

$$= \left(\frac{100}{100 - 5} \right) \times 5225$$

$$= \frac{100}{95} \times 5225$$

$$= ₹ 5,500.$$

Example 1.14

A shopkeeper allows a discount of 10% to his customers and still gains 20%. Find the marked price of an article which costs ₹ 450 to the shopkeeper.

Solution

Vanitha used this method:

Let M.P. be ₹ 100.

Discount = 10% of M. P.

$$\begin{aligned} &= \frac{10}{100} \text{ of M.P.} = \frac{10}{100} \times 100 \\ &= ₹ 10 \end{aligned}$$

$$\begin{aligned} \text{S.P.} &= \text{M.P.} - \text{Discount} \\ &= 100 - 10 = ₹ 90 \end{aligned}$$

Gain = 20% of C.P.

$$= \frac{20}{100} \times 450 = ₹ 90$$

$$\begin{aligned} \text{S.P.} &= \text{C.P.} + \text{Gain} \\ &= 450 + 90 = ₹ 540. \end{aligned}$$

If S.P. is ₹ 90, then M.P. is ₹ 100.

When S.P. is ₹ 540,

$$\text{M.P.} = \frac{540 \times 100}{90} = ₹ 600$$

∴ The M.P. of an article = ₹ 600

Vimal used the formula method:

Discount = 10%, Gain = 20%,
C.P. = ₹ 450, M.P. = ?

$$\text{M.P.} = \frac{100 + \text{Gain}\%}{100 - \text{Discount}\%} \times \text{C.P.}$$

$$= \frac{(100 + 20)}{(100 - 10)} \times 450$$

$$= \frac{120}{90} \times 450$$

$$= ₹ 600$$

Example 1.15

A dealer allows a discount of 10% and still gains 10%. What is the cost price of the book which is marked at ₹ 220?

Solution

Sugandan used this method:

$$\begin{aligned} \text{M.P.} &= ₹ 220. \\ \text{Discount} &= 10\% \text{ of M.P.} \\ &= \frac{10}{100} \times 220 = ₹ 22 \end{aligned}$$

$$\begin{aligned} \text{S.P.} &= \text{M.P.} - \text{Discount} \\ &= 220 - 22 = ₹ 198 \end{aligned}$$

Let C.P. be ₹ 100.

$$\begin{aligned} \text{Gain} &= 10\% \text{ of C. P.} \\ &= \frac{10}{100} \times 100 = ₹ 10 \end{aligned}$$

$$\begin{aligned} \text{S.P.} &= \text{C.P.} + \text{Gain} \\ &= 100 + 10 \\ &= ₹ 110. \end{aligned}$$

If S.P. is ₹ 110, then C.P. is ₹ 100.

$$\begin{aligned} \text{When S.P. is ₹ 198,} \\ \text{C.P.} &= \frac{198 \times 100}{110} \\ &= ₹ 180. \end{aligned}$$

Mukundan used the formula method:

$$\text{Discount} = 10\%$$

$$\text{Gain} = 10\%$$

$$\text{M.P.} = ₹ 220$$

$$\begin{aligned} \text{C.P.} &= \frac{100 - \text{Discount}\%}{100 + \text{Gain}\%} \times \text{M.P.} \\ &= \frac{100 - 10}{100 + 10} \times 220 \\ &= \frac{90}{110} \times 220 = ₹ 180. \end{aligned}$$

[OR]

Example 1.16

A television set was sold for ₹ 14,400 after giving successive discounts of 10% and 20% respectively. What was the marked price?

Solution

$$\text{Selling Price} = ₹ 14,400$$

Let the M.P. be ₹ 100.

$$\text{First discount} = 10\% = \frac{10}{100} \times 100 = ₹ 10$$

$$\text{S.P. after the first discount} = 100 - 10 = ₹ 90$$

$$\text{Second discount} = 20\% = \frac{20}{100} \times 90 = ₹ 18$$

$$\text{Selling Price after the second discount} = 90 - 18 = ₹ 72$$

If S.P. is ₹ 72, then M.P. is ₹ 100.

When S.P. is ₹ 14,400,

$$\begin{aligned} \text{M.P.} &= \frac{14400 \times 100}{72} = ₹ 20,000 \\ \text{M.P.} &= ₹ 20,000. \end{aligned}$$

Example 1.17

A trader buys an article for ₹ 1,200 and marks it 30% above the C.P. He then sells it after allowing a discount of 20%. Find the S.P. and profit percent.

Solution:

Let C.P. of the article be ₹ 100

M.P. = 30% above C.P. = ₹ 130

If C.P. is ₹ 100, then M.P. is ₹ 130.

When C.P. is ₹ 1200, $M.P. = \frac{1200 \times 130}{100} = ₹ 1560$

Discount = 20% of 1560 = $\frac{20}{100} \times 1560 = ₹ 312$

S.P. = M.P. – Discount

= 1560 – 312 = ₹ 1248

Profit = S.P. – C.P.

= 1248 – 1200 = ₹ 48.

$$\begin{aligned} \therefore \text{Profit \%} &= \frac{\text{Profit}}{\text{C.P.}} \times 100 \\ &= \frac{48}{1200} \times 100 = 4\% \end{aligned}$$

**Try these**

A shop gives 20% discount. What will be the S.P. of the following?

- (i) A dress marked at ₹ 120
- (ii) A bag marked at ₹ 250
- (iii) A pair of shoes marked at ₹ 750.

1.3.5 Application of Tax

Children! Very often we find advertisements in newspapers and on television requesting people to pay their taxes in time. What is this tax? Why does the Government collect the tax from the common people?

Tax is paid to the Government to create better infrastructure facilities like roads, railways, irrigation facility, water supply, electricity, etc., to build a better nation. For all these, the money needed by the Government is collected in the form of tax from the people.



Chapter 1

Taxes are of two types:

1. Direct Tax

Tax which is collected in the form of Income Tax, Property Tax, Professional Tax, Water Tax, etc., is called as Direct Tax. These are paid directly to the government by the public.

2. Indirect Tax

Some of the taxes which are not paid directly to the government are Indirect Taxes and are explained below.

Excise Tax

This tax is charged on some items which are manufactured in the country. This is collected by the Government of India.

Service Tax

Tax which is charged in Hotels, Cinema theatres, for service of Chartered Accountants, Telephone Bills, etc., come under Service Tax. This tax is collected by the service provider from the user and deposited to the Government.

Income Tax

This is the most important source of revenue for the Government which is collected from every citizen who is earning more than a minimum stipulated income annually. As true citizens of our country, we should be aware of our duty and pay the tax on time.

Sales Tax / Value Added Tax

Sales Tax

Sales Tax is the tax levied on the sales made by a seller at the time of selling the product. When the buyer buys the commodity the sales tax is paid by him together with the price of the commodity.

This sales tax is charged by the Government on the selling price of an item and is added to the value of the bill.

These days, however, the prices include the tax known as **Value Added Tax (VAT)**. This means that the price we pay for an item is added with VAT.

Sales Tax is charged by the Government on the Sales of an Item.

TIN No. 33571280275/DL 1-4-95 Area Code No. 0 6 5 DL 1-4-95		CASH BILL		7777 7777	
M.H.T. WATCH & RADIO CENTRE					
Authorised Dealers in:					
TITAN SINATA TIMEX HUMIT Classic					
CITIZEN SARTIME Ajanta SAMAY ORPAT					
368, HIGH ROAD, CHENNAI - 600 001.					
Mrs. <i>Shree Ponnani</i>					
Chennai.					
No.	50476	Date	29/8/10		
Qty.	DESCRIPTION OF ARTICLES	RATE	Rs.	P.	P.
1	Scissors (Rate 4/03) ST 863-1		780		00
	Broken watch 4/2		780		00
	V.A.T. 10%				
TOTAL			858		00
Thank You!					
ALL MAJOR CREDIT & DEBIT CARDS ACCEPTED					
Please show bill cannot be taken back or exchanged.					
for M.H.T. Watch & Radio Centre					

Activity



Can you find the prevailing rate of Sales Tax for some commodities in the year 2011.

1. Electrical instruments _____ %
2. Petrol _____ % 3. Diesel _____ %
4. Domestic appliances _____ %
5. Chemicals _____ %



Do you know?

The Government gives exemption of Sales Tax for some commodities like rice, sugar, milk, salt, pen, pencils and books.

Calculation of Sales Tax

$$\text{Amount of Sales tax} = \frac{\text{Rate of Sales tax}}{100} \times \text{Cost of the item}$$

$$\text{Rate of Sales tax} = \frac{\text{Amount of Sales tax}}{\text{Cost of the item}} \times 100$$

$$\text{Bill amount} = \text{Cost of the item} + \text{Amount of Sales tax}$$

Example 1.18

Vinodh purchased musical instruments for ₹ 12,000. If the rate of sales tax is 8%, find the sales tax and the total amount paid by him.

Solution

$$\text{Value of the musical instruments} = ₹ 12,000$$

$$\text{Rate of Sales Tax} = 8\%$$

$$\begin{aligned} \text{Amount of Sales Tax} &= \frac{8}{100} \times 12000 \\ &= ₹ 960 \end{aligned}$$

$$\begin{aligned} \text{Total amount paid by Vinodh including Sales Tax} &= 12,000 + 960 \\ &= ₹ 12,960 \end{aligned}$$

Example 1.19

A refrigerator is purchased for ₹ 14,355, including sales tax. If the actual cost price of the refrigerator is ₹ 13,050, find the rate of sales tax.

Solution

Given: For the refrigerator, bill amount = ₹ 14,355, Cost price = ₹ 13,050.

$$\begin{aligned} \text{Sales tax} &= \text{Bill amount} - \text{Cost of the item} \\ &= 14,355 - 13,050 = ₹ 1,305 \end{aligned}$$

$$\begin{aligned} \text{Rate of Sales Tax} &= \frac{\text{Amount of Sales Tax}}{\text{Cost of the item}} \times 100 \\ &= \frac{1305}{13050} \times 100 = 10\% \end{aligned}$$

Example 1.20

Priya bought a suitcase for ₹ 2,730. The VAT for this item is 5%. What was the price of the suitcase before VAT was added? Also state how much is the VAT.

Solution

Given : VAT is 5%.

If the price without VAT is ₹ 100, then the price including VAT is ₹ 105.

Now, when price including VAT is ₹ 105, original price is ₹ 100.

When price including VAT is ₹ 2,730, the original price of the suitcase

$$= \frac{100}{105} \times 2730 = ₹ 2,600$$

The original price of the suitcase = ₹ 2,600

$$\therefore \text{VAT} = 2,730 - 2,600 = ₹ 130$$

**Try these**

- Find the buying price of each of the following when 5% Sales Tax is added on the purchase of:
 - A pillow for ₹ 60
 - Two bars of soap at ₹ 25 each.
- If 8% VAT is included in the prices, find the original price of :
 - An electric water heater bought for ₹ 14,500
 - A crockery set bought for ₹ 200.

EXERCISE 1.3

- Choose the correct answer:
 - The discount is always on the _____.
(A) Marked Price (B) Cost Price (C) Selling Price (D) Interest
 - If M.P. = ₹ 140, S.P. = ₹ 105, then Discount = _____.
(A) ₹ 245 (B) ₹ 25 (C) ₹ 30 (D) ₹ 35
 - _____ = Marked Price – Discount.
(A) Cost Price (B) Selling Price (C) List Price (D) Market price
 - The tax added to the value of the product is called _____ Tax.
(A) Sales Tax (B) VAT (C) Excise Tax (D) Service Tax
 - If the S.P. of an article is ₹ 240 and the discount given on it is ₹ 28, then the M.P. is _____.
(A) ₹ 212 (B) ₹ 228 (C) ₹ 268 (D) ₹ 258
- The price marked on a book is ₹ 450. The shopkeeper gives 20% discount on it at a book exhibition. What is the Selling Price?
- A television set was sold for ₹ 5,760 after giving successive discounts of 10% and 20% respectively. What was the Marked Price?

4. Sekar bought a computer for ₹ 38,000 and a printer for ₹ 8,000. If the rate of sales tax is 7% for these items, find the price he has to pay to buy these two items.
5. The selling price with VAT, on a cooking range is ₹ 19,610. If the VAT is 6%, what is the original price of the cooking range?
6. Richard got a discount of 10% on the suit he bought. The marked price was ₹ 5,000 for the suit. If he had to pay sales tax of 10% on the price at which he bought, how much did he pay?
7. The sales tax on a refrigerator at the rate of 9% is ₹ 1,170. Find the actual sale price.
8. A trader marks his goods 40% above the cost price. He sells them at a discount of 5%. What is his loss or gain percentage?
9. A T.V. with marked price ₹ 11,500 is sold at 10% discount. Due to festival season, the shop keeper allows a further discount of 5%. Find the net selling price of the T.V.
10. A person pays ₹ 2,800 for a cooler listed at ₹ 3,500. Find the discount percent offered.
11. Deepa purchased 15 shirts at the rate of ₹ 1,200 each and sold them at a profit of 5%. If the customer has to pay sales tax at the rate of 4%, how much will one shirt cost to the customer?
12. Find the discount, discount percent, selling price and the marked price.

Sl. No	Items	M. P	Rate of Discount	Amount of Discount	S. P
(i)	Saree	₹ 2,300	20%		
(ii)	Pen set	₹ 140			₹ 105
(iii)	Dining table		20%		₹ 16,000
(iv)	Washing Machine	₹ 14,500			₹ 13,775
(v)	Crockery set	₹ 3,224	12½%		



Try these

Which is a better offer? Two successive discounts of 20% and 5% or a single discount of 25%. Give appropriate reasons.

1.4. Compound Interest

In class VII, we have learnt about Simple Interest and the formula for calculating Simple Interest and Amount. In this chapter, we shall discuss the concept of Compound Interest and the method of calculating Compound Interest and Amount at the end of a certain specified period.

Vinay borrowed ₹ 50,000 from a bank for a fixed time period of 2 years. at the rate of 4% per annum.

Vinay has to pay for the first year,

$$\begin{aligned}\text{Simple interest} &= \frac{P \times n \times r}{100} \\ &= \frac{50000 \times 1 \times 4}{100} = ₹ 2,000\end{aligned}$$

Suppose he fails to pay the simple interest ₹ 2,000 at the end of first year, then the interest ₹ 2,000 is added to the old Principal ₹ 50,000 and now the sum = $P + I = ₹ 52,000$ becomes the new Principal for the second year for which the interest is calculated.

Now in the second year he will have to pay an interest of

$$\begin{aligned}\text{S.I.} &= \frac{P \times n \times r}{100} \\ &= \frac{52000 \times 1 \times 4}{100} = ₹ 2,080\end{aligned}$$

Therefore Vinay will have to pay more interest for the second year.

This way of calculating interest is called **Compound Interest**.

Generally in banks, insurance companies, post offices and in other companies which lend money and accept deposits, compound interest is followed to find the interest.

Example 1.21

Ramlal deposited ₹ 8,000 with a finance company for 3 years at an interest of 15% per annum. What is the compound interest that Ramlal gets after 3 years?

Solution

Step 1: Principal for the first year = ₹ 8,000

$$\begin{aligned}\text{Interest for the first year} &= \frac{P \times n \times r}{100} \\ &= \frac{8000 \times 1 \times 15}{100} = ₹ 1,200\end{aligned}$$

Amount at the end of first year = $P + I = 8,000 + 1,200 = ₹ 9,200$



Do you know?

When the interest is paid on the Principal only, it is called **Simple Interest**. But if the interest is paid on the Principal as well as on the accrued interest, it is called **Compound Interest**.

Step 2: The amount at the end of the first year becomes the Principal for the second year.

$$\text{Principal for the second year} = ₹ 9,200$$

$$\begin{aligned} \text{Interest for the second year} &= \frac{P \times n \times r}{100} \\ &= \frac{9200 \times 1 \times 15}{100} = ₹ 1,380 \end{aligned}$$

$$\text{Amount at the end of **second year**} = P + I = 9,200 + 1,380 = ₹ 10,580$$

Step 3: The amount at the end of the second year becomes the Principal for the third year.

$$\text{Principal for the third year} = ₹ 10,580$$

$$\begin{aligned} \text{Interest for the third year} &= \frac{P \times n \times r}{100} \\ &= \frac{10580 \times 1 \times 15}{100} = ₹ 1,587 \end{aligned}$$

$$\begin{aligned} \text{Amount at the end of **third year**} &= P + I \\ &= 10,580 + 1,587 = ₹ 12,167 \end{aligned}$$

Hence, the Compound Interest that Ramlal gets after three years is

$$A - P = 12,167 - 8,000 = ₹ 4,167.$$

Deduction of formula for Compound Interest

The above method which we have used for the calculation of Compound Interest is quite lengthy and cumbersome, especially when the period of time is very large. Hence we shall obtain a formula for the computation of Amount and Compound Interest.

If the Principal is P , Rate of interest per annum is $r\%$ and the period of time or the number of years is n , then we deduce the compound interest formula as follows:

Step 1 : Principal for the first year = P

$$\begin{aligned} \text{Interest for the first year} &= \frac{P \times n \times r}{100} \\ &= \frac{P \times 1 \times r}{100} = \frac{Pr}{100} \end{aligned}$$

$$\begin{aligned} \text{Amount at the end of **first year**} &= P + I \\ &= P + \frac{Pr}{100} \\ &= P \left(1 + \frac{r}{100} \right) \end{aligned}$$

$$\begin{aligned} \text{Step 2 :} \quad \text{Principal for the second year} &= P\left(1 + \frac{r}{100}\right) \\ \text{Interest for the second year} &= \frac{P\left(1 + \frac{r}{100}\right) \times 1 \times r}{100} \\ &\quad \text{(using the S.I. formula)} \\ &= P\left(1 + \frac{r}{100}\right) \times \frac{r}{100} \end{aligned}$$

$$\begin{aligned} \text{Amount at the end of second year} &= P + I \\ &= P\left(1 + \frac{r}{100}\right) + P\left(1 + \frac{r}{100}\right) \times \frac{r}{100} \\ &= P\left(1 + \frac{r}{100}\right)\left(1 + \frac{r}{100}\right) \\ &= \mathbf{P\left(1 + \frac{r}{100}\right)^2} \end{aligned}$$

$$\begin{aligned} \text{Step 3 :} \quad \text{Principal for the third year} &= P\left(1 + \frac{r}{100}\right)^2 \\ \text{Interest for the third year} &= \frac{P\left(1 + \frac{r}{100}\right)^2 \times 1 \times r}{100} \\ &\quad \text{(using the S.I. formula)} \\ &= P\left(1 + \frac{r}{100}\right)^2 \times \frac{r}{100} \end{aligned}$$

$$\begin{aligned} \text{Amount at the end of third year} &= P + I \\ &= P\left(1 + \frac{r}{100}\right)^2 + P\left(1 + \frac{r}{100}\right)^2 \times \frac{r}{100} \\ &= P\left(1 + \frac{r}{100}\right)^2\left(1 + \frac{r}{100}\right) \\ &= \mathbf{P\left(1 + \frac{r}{100}\right)^3} \end{aligned}$$

Similarly, Amount at the end of n^{th} year is $A = P\left(1 + \frac{r}{100}\right)^n$

and C. I. at the end of ' n ' years is given by $A - P$

$$\text{(i. e.)} \quad \text{C. I.} = P\left(1 + \frac{r}{100}\right)^n - P$$

To Compute Compound Interest

Case 1: Compounded Annually

When the interest is added to the Principal at the end of each year, we say that the interest is compounded annually.

$$\text{Here} \quad A = P\left(1 + \frac{r}{100}\right)^n \text{ and C.I.} = A - P$$

Case 2: Compounded Half - Yearly (Semi - Annually)

When the interest is compounded Half - Yearly, there are two conversion periods in a year each after 6 months. In such situations, the Half - Yearly rate will be half of the annual rate, that is $\left(\frac{r}{2}\right)$.

$$\text{In this case, } A = P\left[1 + \frac{1}{2}\left(\frac{r}{100}\right)\right]^{2n} \text{ and C.I.} = A - P$$

Case 3: Compounded Quarterly

When the interest is compounded quarterly, there are four conversion periods in a year and the quarterly rate will be one-fourth of the annual rate, that is $\left(\frac{r}{4}\right)$.

$$\text{In this case, } A = P\left[1 + \frac{1}{4}\left(\frac{r}{100}\right)\right]^{4n} \text{ and C.I.} = A - P$$

Case 4: Compounded when time being fraction of a year

When interest is compounded annually but time being a fraction.

In this case, when interest is compounded annually but time being a fraction of a year, say $5\frac{1}{4}$ years, then amount A is given by

$$A = P\left(1 + \frac{r}{100}\right)^5 \left[1 + \frac{1}{4}\left(\frac{r}{100}\right)\right] \text{ and C.I.} = A - P$$

\downarrow \downarrow
 for 5 years for $\frac{1}{4}$ of year

Example 1.22

Find the C.I. on ₹ 15,625 at 8% p.a. for 3 years compounded annually.

Solution

We know,

$$\begin{aligned} \text{Amount after 3 years} &= P\left(1 + \frac{r}{100}\right)^3 \\ &= 15625\left(1 + \frac{8}{100}\right)^3 \\ &= 15625\left(1 + \frac{2}{25}\right)^3 \\ &= 15625\left(\frac{27}{25}\right)^3 \\ &= 15625 \times \frac{27}{25} \times \frac{27}{25} \times \frac{27}{25} \\ &= ₹ 19,683 \end{aligned}$$

$$\begin{aligned} \text{Now, Compound interest} &= A - P = 19,683 - 15,625 \\ &= ₹ 4,058 \end{aligned}$$

To find the C.I. when the interest is compounded annually or half-yearly

Let us see what happens to ₹ 100 over a period of one year if an interest is compounded annually or half-yearly.

S.No	Annually	Half-yearly
1	P = ₹ 100 at 10% per annum compounded annually	P = ₹ 100 at 10% per annum compounded half-yearly
2	The time period taken is 1 year	The time period is 6 months or $\frac{1}{2}$ year.
3	$I = \frac{100 \times 10 \times 1}{100} = ₹ 10$	$I = \frac{100 \times 10 \times \frac{1}{2}}{100} = ₹ 5$
4	A = 100 + 10 = ₹ 110	A = 100 + 5 = ₹ 105 For the next 6 months, P = ₹ 105
		So, $I = \frac{105 \times 10 \times \frac{1}{2}}{100} = ₹ 5.25$ and A = 105 + 5.25 = ₹ 110.25
5	A = ₹ 110	A = ₹ 110.25

Thus, if interest is compounded half - yearly, we compute the interest two times and rate is taken as half of the annual rate.

Example 1.23

Find the compound interest on ₹ 1000 at the rate of 10% per annum for 18 months when interest is compounded half-yearly.

Solution

Here, P = ₹ 1000, $r = 10\%$ per annum

and $n = 18$ months = $\frac{18}{12}$ years = $\frac{3}{2}$ years = $1\frac{1}{2}$ years

$$\therefore \text{Amount after 18 months} = P \left[1 + \frac{1}{2} \left(\frac{r}{100} \right) \right]^{2n}$$

$$= 1000 \left[1 + \frac{1}{2} \left(\frac{10}{100} \right) \right]^{2 \times \frac{3}{2}}$$

$$= 1000 \left(1 + \frac{10}{200} \right)^3$$

$$= 1000 \left(\frac{21}{20} \right)^3$$

$$= 1000 \times \frac{21}{20} \times \frac{21}{20} \times \frac{21}{20}$$

$$= ₹ 1157.625$$

$$= ₹ 1157.63$$

$$\text{C. I.} = A - P$$

$$= 1157.63 - 1000 = ₹ 157.63$$



Try these

A sum is taken for one year at 8% p. a. If interest is compounded after every three months, how many times will interest be charged in one year?

Example 1.24

Find the compound interest on ₹ 20,000 at 15% per annum for $2\frac{1}{3}$ years.

Solution

Here, $P = ₹ 20,000$, $r = 15\%$ p. a. and $n = 2\frac{1}{3}$ years.

$$\begin{aligned} \text{Amount after } 2\frac{1}{3} \text{ years} = A &= P\left(1 + \frac{r}{100}\right)^2 \left[1 + \frac{1}{3}\left(\frac{r}{100}\right)\right] \\ &= 20000\left(1 + \frac{15}{100}\right)^2 \left[1 + \frac{1}{3}\left(\frac{15}{100}\right)\right] \\ &= 20000 \left(1 + \frac{3}{20}\right)^2 \left(1 + \frac{1}{20}\right) \\ &= 20000 \left(\frac{23}{20}\right)^2 \left(\frac{21}{20}\right) \\ &= 20000 \times \frac{23}{20} \times \frac{23}{20} \times \frac{21}{20} \\ &= ₹ 27,772.50 \end{aligned}$$

$$\begin{aligned} \text{C.I.} &= A - P \\ &= 27,772.50 - 20,000 \\ &= ₹ 7,772.50 \end{aligned}$$

Inverse Problems on Compound Interest

We have already learnt the formula, $A = P\left(1 + \frac{r}{100}\right)^n$,

where four variables A , P , r and n are involved. Out of these four variables, if any three variables are known, then we can calculate the fourth variable.

Example 1.25

At what rate per annum will ₹ 640 amount to ₹ 774.40 in 2 years, when interest is being compounded annually?

Solution:

Given: $P = ₹ 640$, $A = ₹ 774.40$, $n = 2$ years, $r = ?$

$$\begin{aligned} \text{We know,} \quad A &= P\left(1 + \frac{r}{100}\right)^n \\ 774.40 &= 640\left(1 + \frac{r}{100}\right)^2 \\ \frac{774.40}{640} &= \left(1 + \frac{r}{100}\right)^2 \\ \frac{77440}{64000} &= \left(1 + \frac{r}{100}\right)^2 \\ \frac{121}{100} &= \left(1 + \frac{r}{100}\right)^2 \end{aligned}$$

$$\left(\frac{11}{10}\right)^2 = \left(1 + \frac{r}{100}\right)^2$$

$$\frac{11}{10} = 1 + \frac{r}{100}$$

$$\frac{r}{100} = \frac{11}{10} - 1$$

$$\frac{r}{100} = \frac{11 - 10}{10}$$

$$\frac{r}{100} = \frac{1}{10}$$

$$r = \frac{100}{10}$$

Rate $r = 10\%$ per annum.

Example 1.26

In how much time will a sum of ₹ 1600 amount to ₹ 1852.20 at 5% per annum compound interest.

Solution

Given: $P = ₹ 1600$, $A = ₹ 1852.20$, $r = 5\%$ per annum, $n = ?$

We know,

$$A = P\left(1 + \frac{r}{100}\right)^n$$

$$1852.20 = 1600\left(1 + \frac{5}{100}\right)^n$$

$$\frac{1852.20}{1600} = \left(\frac{105}{100}\right)^n$$

$$\frac{185220}{160000} = \left(\frac{21}{20}\right)^n$$

$$\frac{9261}{8000} = \left(\frac{21}{20}\right)^n$$

$$\left(\frac{21}{20}\right)^3 = \left(\frac{21}{20}\right)^n$$

$$\therefore n = 3 \text{ years}$$



Try these

Find the time period and rate for each of the cases given below :

1. A sum taken for 2 years at 8% p. a. compounded half - yearly.
2. A sum taken for $1\frac{1}{2}$ years at 4% p. a. compounded half - yearly.

1.5 Difference between Simple Interest and Compound Interest

When P is the Principal, $n = 2$ years and r is the Rate of interest,

$$\text{Difference between C. I. and S. I. for 2 years} = P\left(\frac{r}{100}\right)^2$$

Example 1.27

Find the difference between Simple Interest and Compound Interest for a sum of ₹ 8,000 lent at 10% p. a. in 2 years.

Solution

Here, $P = ₹ 8000$, $n = 2$ years, $r = 10\%$ p. a.

$$\begin{aligned}
 \text{Difference between Compound Interest and Simple Interest for 2 years} &= P\left(\frac{r}{100}\right)^2 \\
 &= 8000 \left(\frac{10}{100}\right)^2 \\
 &= 8000 \left(\frac{1}{10}\right)^2 \\
 &= 8000 \times \frac{1}{10} \times \frac{1}{10} = ₹ 80
 \end{aligned}$$

EXERCISE 1.4

1. Find the Amount and Compound Interest in the following cases:

Sl. No.	Principal in Rupees	Rate% per annum	Time in years
(i)	1000	5%	3
(ii)	4000	10%	2
(iii)	18,000	10%	$2\frac{1}{2}$

2. Sangeetha borrowed ₹ 8,000 from Alex for 2 years at $12\frac{1}{2}\%$ per annum. What interest did Sangeetha pay to Alex if the interest is compounded annually?
3. Maria invested ₹ 80,000 in a business. She would be paid interest at 5% per annum compounded annually. Find (i) the amount standing to her credit at the end of second year and (ii) the interest for the third year.
4. Find the compound interest on ₹ 24,000 compounded half - yearly for $1\frac{1}{2}$ years at the rate of 10% per annum.
5. Find the amount that Dravid would receive if he invests ₹ 8,192 for 18 months at $12\frac{1}{2}\%$ per annum, the interest being compounded half - yearly.
6. Find the compound interest on ₹ 15,625 for 9 months, at 16% per annum compounded quarterly.
7. Find the Principal that will yield a compound interest of ₹ 1,632 in 2 years at 4% rate of interest per annum.
8. Vicky borrowed ₹ 26,400 from a bank to buy a scooter at the rate of 15% p.a. compounded yearly. What amount will he pay at the end of 2 years and 4 months to clear the loan?
9. Arif took a loan of ₹ 80,000 from a bank. If the rate of interest is 10% p. a., find the difference in amounts he would be paying after $1\frac{1}{2}$ years if the interest is (i) compounded annually and (ii) compounded half - yearly.
10. Find the difference between simple interest and compound interest on ₹ 2,400 at 2 years at 5% per annum compounded annually.
11. Find the difference between simple interest and compound interest on ₹ 6,400 for 2 years at $6\frac{1}{4}\%$ p. a. compounded annually.

12. The difference between C. I. and S. I. for 2 years on a sum of money lent at 5% p.a. is ₹ 5. Find the sum of money lent.
13. Sujatha borrows ₹ 12,500 at 12% per annum for 3 years at simple interest and Radhika borrows the same amount for the same period at 10% per annum compounded annually. Who pays more interest and by how much?
14. What sum is invested for $1\frac{1}{2}$ years at the rate of 4% p.a. compounded half-yearly which amounts to ₹ 1,32,651?
15. Gayathri invested a sum of ₹ 12,000 at 5% p.a. at compound interest. She received an amount of ₹ 13,230 after ' n ' years. Find the value of ' n '.
16. At what rate percent compound interest per annum will ₹ 640 amount to ₹ 774.40 in 2 years?
17. Find the rate percent per annum, if ₹ 2,000 amount to ₹ 2,315.25 in an year and a half, interest being compounded half-yearly.

1.5.1 Appreciation and Depreciation

a) Appreciation

In situations like growth of population, growth of bacteria, increase in the value of an asset, increase in price of certain valuable articles, etc., the following formula is used.

$$A = P\left(1 + \frac{r}{100}\right)^n$$

b) Depreciation

In certain cases where the cost of machines, vehicles, value of some articles, buildings, etc., decreases, the following formula can be used.

$$A = P\left(1 - \frac{r}{100}\right)^n$$

Example 1.28

The population of a village increases at the rate of 7% every year. If the present population is 90,000, what will be the population after 2 years?

Solution

Present population $P = 90,000$, Rate of increase $r = 7\%$,

Number of years $n = 2$.

$$\text{The population after 'n' years} = P\left(1 + \frac{r}{100}\right)^n$$

$$\therefore \text{The population after two years} = 90000 \left(1 + \frac{7}{100}\right)^2$$



Do you know?

World Population

Year	Population
1700	600,000,000
1800	900,000,000
1900	1,500,000,000
2000	6,000,000,000

In 3 centuries, population has multiplied 10 fold.

$$\begin{aligned}
 &= 90000 \left(\frac{107}{100}\right)^2 \\
 &= 90000 \times \frac{107}{100} \times \frac{107}{100} \\
 &= 103041
 \end{aligned}$$

The population after two years = 1,03,041

Example 1.29

The value of a machine depreciates by 5% each year. A man pays ₹ 30,000 for the machine. Find its value after three years.

Solution

Present value of the machine $P = ₹ 30,000$, Rate of depreciation $r = 5\%$,

Number of years $n = 3$

The value of the machine after ' n ' years = $P\left(1 - \frac{r}{100}\right)^n$

$$\begin{aligned}
 \therefore \text{The value of the machine after three years} &= 30000\left(1 - \frac{5}{100}\right)^3 \\
 &= 30000\left(\frac{95}{100}\right)^3 \\
 &= 30000 \times \frac{95}{100} \times \frac{95}{100} \times \frac{95}{100} \\
 &= 25721.25
 \end{aligned}$$

The value of the machine after three years = ₹ 25,721.25

Example 1.30

The population of a village has a constant growth of 5% every year. If its present population is 1,04,832, what was the population two years ago?

Solution

Let P be the population two years ago.

$$\therefore P\left(1 + \frac{5}{100}\right)^2 = 104832$$

$$P\left(\frac{105}{100}\right)^2 = 104832$$

$$P \times \frac{105}{100} \times \frac{105}{100} = 104832$$

$$P = \frac{104832 \times 100 \times 100}{105 \times 105}$$

$$= 95085.71$$

$$= 95,086 \text{ (rounding off to the nearest whole number)}$$

\therefore Two years ago the population was 95,086.

EXERCISE 1.5

1. The number of students enrolled in a school is 2000. If the enrollment increases by 5% every year, how many students will be there after two years?
2. A car which costs ₹ 3,50,000 depreciates by 10% every year. What will be the worth of the car after three years?
3. A motorcycle was bought at ₹ 50,000. The value depreciated at the rate of 8% per annum. Find the value after one year.
4. In a Laboratory, the count of bacteria in a certain experiment was increasing at the rate of 2.5% per hour. Find the bacteria at the end of 2 hours if the count was initially 5,06,000.
5. From a village people started migrating to nearby cities due to unemployment problem. The population of the village two years ago was 6,000. The migration is taking place at the rate of 5% per annum. Find the present population.
6. The present value of an oil engine is ₹ 14,580. What was the worth of the engine 3 years before if the value depreciates at the rate of 10% every year?
7. The population of a village increases by 9% every year which is due to the job opportunities available in that village. If the present population of the village is 11,881, what was the population two years ago?

1.6 Fixed Deposits and Recurring Deposits

Banks, post offices and many other financial institutions accept deposits from public at varying rates of interest. People save in these institutions to get regular periodical income.

Different saving schemes are offered by these financial institutions. Few of those schemes are

- (i) Fixed Deposit and (ii) Recurring Deposit

(i) Fixed Deposit

In this type of deposit, people invest a quantum of money for specific periods. Such a deposit is called Fixed Deposit (in short form, F.D.)

Note: Deposits can either be for a short term or long term. Depending on the period of deposits, they offer a higher rate of interest.

(ii) Recurring Deposit

Recurring Deposit (in short form, R.D.) is entirely different from Fixed Deposit.

In this scheme, the depositor has the freedom to choose an amount according to his saving capacity, to be deposited regularly every month over a period of years in the bank or in the post office.



The bank or post office repays the total amount deposited together with the interest at the end of the period. This type of Deposit is known as Recurring Deposit.



Do you know?

The monthly instalments can be paid on any day within the month for R.D.

Note: The interest on Recurring Deposit is calculated using simple interest method.

To find the formula for calculating interest and the maturity amount for R.D :

Let $r\%$ be the rate of interest paid and 'P' be the monthly instalment paid for 'n' months.

$$\text{Interest} = \frac{PNr}{100}, \text{ where } N = \frac{1}{12} \left[\frac{n(n+1)}{2} \right] \text{ years}$$

$$\text{Total Amount due at maturity is } A = Pn + \frac{PNr}{100}$$

Example 1.31

Tharun makes a deposit of Rupees two lakhs in a bank for 5 years. If the rate of interest is 8% per annum, find the maturity value.

Solution

Principal deposited $P = ₹ 2,00,000$, $n = 5$ years, $r = 8\%$ p. a.

$$\begin{aligned} \text{Interest} &= \frac{PNr}{100} = 200000 \times 5 \times \frac{8}{100} \\ &= ₹ 80,000 \end{aligned}$$

$$\therefore \text{Maturity value after 5 years} = 2,00,000 + 80,000 = ₹ 2,80,000.$$

Example 1.32

Vaideesh deposits ₹ 500 at the beginning of every month for 5 years in a post office. If the rate of interest is 7.5%, find the amount he will receive at the end of 5 years.

Solution

Amount deposited every month, $P = ₹ 500$

Number of months, $n = 5 \times 12 = 60$ months

Rate of interest, $r = 7\frac{1}{2}\% = \frac{15}{2}\%$

Total deposit made $= Pn = 500 \times 60$

$= ₹ 30,000$

Period for recurring deposit, $N = \frac{1}{12} \left[\frac{n(n+1)}{2} \right]$ years

$$= \frac{1}{24} \times 60 \times 61 = \frac{305}{2} \text{ years}$$

$$\begin{aligned}\text{Interest, } I &= \frac{PNr}{100} \\ &= 500 \times \frac{305}{2} \times \frac{15}{2 \times 100} \\ &= ₹ 5,718.75\end{aligned}$$

$$\begin{aligned}\text{Total amount due} &= Pn + \frac{PNr}{100} \\ &= 30,000 + 5,718.75 \\ &= ₹ 35,718.75\end{aligned}$$

Example 1.33

Vishal deposited ₹ 200 per month for 5 years in a recurring deposit account in a post office. If he received ₹ 13,830 find the rate of interest.

Solution

Maturity Amount, $A = ₹ 13,830$, $P = ₹ 200$, $n = 5 \times 12 = 60$ months

$$\begin{aligned}\text{Period, } N &= \frac{1}{12} \left[\frac{n(n+1)}{2} \right] \text{ years} \\ &= \frac{1}{12} \times 60 \times \frac{61}{2} = \frac{305}{2} \text{ years}\end{aligned}$$

$$\text{Amount Deposited} = Pn = 200 \times 60 = ₹ 12,000$$

$$\text{Maturity Amount} = Pn + \frac{PNr}{100}$$

$$13830 = 12000 + 200 \times \frac{305}{2} \times \frac{r}{100}$$

$$13830 - 12000 = 305 \times r$$

$$1830 = 305 \times r$$

$$\therefore r = \frac{1830}{305} = 6\%$$

1.6.1 Hire Purchase and Instalments

Banks and financial institutions have introduced a scheme called hire purchase and instalment to satisfy the needs of today's consumers.

Hire purchase: Under this scheme, the article will not be owned by the buyer for a certain period of time. Only when the buyer has paid the complete price of the article purchased, he/she will become its owner.

Instalment: The cost of the article along with interest and certain other charges is divided by the number of months of the loan period. The amount thus got is known as the instalment.

Equated Monthly Instalment (E.M.I.)

Equated Monthly Instalment is also as equivalent as the instalment scheme but with a diminishing concept. We have to repay the cost of things with the interest along with certain charges. The total amount received should be divided by the period of months. The amount thus arrived is known as Equated Monthly Instalment.

$$\text{E.M.I} = \frac{\text{Principal} + \text{Interest}}{\text{Number of months}}$$

Different schemes of Hire purchase and Instalment scheme

1. 0% interest scheme: Companies take processing charge and 4 or 5 months instalments in advance.

2. 100% Finance: Companies add interest and the processing charges to the cost price.

3. Discount Sale: To promote sales, discount is given in the instalment schemes.

4. Initial Payment: A certain part of the price of the article is paid towards the purchase in advance. It is also known as Cash down payment.

Example 1.34

The cost price of a washing machine is ₹ 18,940. The table given below illustrates various schemes to purchase the washing machine through instalments. Choose the best scheme to purchase.

Sl. No	Different schemes	S. P. in ₹	Initial payments	Rate of interest	Processing fee	Period
(i)	75% Finance	18,940	25%	12%	1%	24 months
(ii)	100% Finance	18,940	Nil	16%	2%	24 months
(iii)	0% Finance	18,940	4 E. M. I. in advance	Nil	2%	24 months

Calculate the E. M. I. and the total amount for the above schemes.

Solution

(i) 75% Finance

$P = ₹ 18,940$, Initial payment = 25%, Rate = 12%, Processing fee = 1%

$$\begin{aligned} \text{Processing fee} &= 1\% \text{ of } ₹ 18,940 \\ &= \frac{1}{100} \times 18940 = ₹ 189.40 \simeq ₹ 189 \end{aligned}$$

$$\begin{aligned} \text{Initial payment} &= 25\% \text{ of } ₹ 18,940 \\ &= \frac{25}{100} \times 18940 = ₹ 4,735 \end{aligned}$$

$$\text{Loan amount} = 18,940 - 4,735 = ₹ 14,205$$

$$\begin{aligned} \text{Interest} &= \frac{14205 \times 12 \times 2}{100} \\ &= ₹ 3,409.20 \simeq ₹ 3,409 \end{aligned}$$

$$\begin{aligned} \text{E. M. I.} &= \frac{\text{Loan amount} + \text{Interest}}{\text{Number of months}} \\ &= \frac{14205 + 3409}{24} = \frac{17614}{24} \\ &= ₹ 733.92 \simeq ₹ 734 \end{aligned}$$

$$\begin{aligned} \therefore \text{Total amount to be paid} &= 4,735 + 14,205 + 3,409 + 189 \\ &= ₹ 22,538 \end{aligned}$$

(ii) 100% finance

$$\begin{aligned} \text{Processing fee} &= 2\% \text{ of ₹ } 18,940 \\ &= \frac{2}{100} \times 18940 = ₹ 378.80 \simeq ₹ 379 \end{aligned}$$

$$\text{Rate of Interest} = 16\%$$

$$\begin{aligned} \text{Interest} &= 18940 \times \frac{16}{100} \times 2 \\ &= ₹ 6060.80 \simeq ₹ 6,061 \end{aligned}$$

$$\begin{aligned} \text{E. M. I.} &= \frac{\text{Loan amount} + \text{Interest}}{\text{Number of months}} \\ &= \frac{18940 + 6061}{24} = \frac{25001}{24} \\ &= ₹ 1,041.708 \simeq ₹ 1,041.71 \\ &= ₹ 1,042 \end{aligned}$$

$$\text{Total amount to be paid} = 6,061 + 18,940 + 379 = ₹ 25,380$$

(iii) 0% interest scheme

$$\begin{aligned} \text{Processing fee} &= 2\% \text{ of ₹ } 18,940 \\ &= \frac{2}{100} \times 18940 = ₹ 378.80 \simeq ₹ 379 \end{aligned}$$

$$\begin{aligned} \text{E. M. I.} &= \frac{\text{Loan amount} + \text{Interest}}{\text{Number of months}} \\ &= \frac{18940 + 0}{24} = \frac{18940}{24} \\ &= ₹ 789.166 \simeq ₹ 789 \end{aligned}$$

$$\text{Total amount to be paid} = 18,940 + 3,156 + 379 = ₹ 22,475$$

$$\text{Advance E. M. I. paid} = ₹ 789 \times 4 = ₹ 3,156$$

Hence, 0% interest scheme is the best scheme.

Example 1.35

The cost of a computer is ₹ 20,000. The company offers it in 36 months, but charges 10% interest. Find the monthly instalment the purchaser has to pay.

Solution

Cost of computer = ₹ 20,000, Interest = 10% p.a., Period = 36 months (3 years)

$$\begin{aligned}\text{Total Interest} &= 20000 \times \frac{10}{100} \times 3 \\ &= ₹ 6,000\end{aligned}$$

$$\begin{aligned}\therefore \text{Total amount to be paid} &= 20,000 + 6,000 \\ &= ₹ 26,000\end{aligned}$$

$$\begin{aligned}\text{Monthly Instalment} &= \frac{\text{Total amount}}{\text{Number of months}} \\ &= \frac{26000}{36} \\ &= ₹ 722. 22 \\ &\simeq ₹ 722\end{aligned}$$

EXERCISE 1.6

- Ponmani makes a fixed deposit of ₹ 25,000 in a bank for 2 years. If the rate of interest is 4% per annum, find the maturity value.
- Deva makes a fixed deposit of ₹ 75,000 in a bank for 3 years. If the rate of interest is 5% per annum, find the maturity value.
- Imran deposits ₹ 400 per month in a post office as R.D. for 2 years. If the rate of interest is 12%, find the amount he will receive at the end of 2 years.
- The cost of a microwave oven is ₹ 6,000. Poorani wants to buy it in 5 instalments. If the company offers it at the rate of 10% p. a. Simple Interest, find the E.M.I. and the total amount paid by her.
- The cost price of a refrigerator is ₹ 16,800. Ranjith wants to buy the refrigerator at 0% finance scheme paying 3 E.M.I. in advance. A processing fee of 3% is also collected from Ranjith. Find the E.M.I. and the total amount paid by him for a period of 24 months.
- The cost of a dining table is ₹ 8,400. Venkat wants to buy it in 10 instalments. If the company offers it for a S.I. of 5% p. a., find the E.M.I. and the total amount paid by him.

1.7 Compound Variation

In the earlier classes we have already learnt about Direct and Inverse Variation. Let us recall them.

Direct Variation

If two quantities are such that an increase or decrease in one leads to a corresponding increase or decrease in the other, we say they vary directly or the variation is Direct.

Examples for Direct Variation:

1. Distance and Time are in Direct Variation, because more the distance travelled, the time taken will be more(if speed remains the same).
2. Principal and Interest are in Direct Variation, because if the Principal is more the interest earned will also be more.
3. Purchase of Articles and the amount spent are in Direct Variation, because purchase of more articles will cost more money.

Indirect Variation or Inverse Variation:

If two quantities are such that an increase or decrease in one leads to a corresponding decrease or increase in the other, we say they vary indirectly or the variation is in inverse.

Examples for Indirect Variation:

1. Work and time are in Inverse Variation, because more the number of the workers, lesser will be the time required to complete a job.
2. Speed and time are in Inverse Variation, because higher the speed, the lower is the time taken to cover a distance.
3. Population and quantity of food are in Inverse Variation, because if the population increases the food availability decreases.

Compound Variation

Certain problems involve a chain of two or more variations, which is called as Compound Variation.

The different possibilities of variations involving two variations are shown in the following table:

Variation I	Variation II
Direct	Direct
Inverse	Inverse
Direct	Inverse
Inverse	Direct

Let us work out some problems to illustrate compound variation.

Example 1.36

If 20 men can build a wall 112 meters long in 6 days, what length of a similar wall can be built by 25 men in 3 days?

Solution:

Method 1: The problem involves set of 3 variables, namely- Number of men, Number of days and length of the wall.

Number of Men	Number of days	Length of the wall in metres
20	6	112
25	3	x

Step 1 : Consider the number of men and the length of the wall. As the number of men increases from 20 to 25, the length of the wall also increases. So it is in **Direct Variation**.

Therefore, the proportion is $20 : 25 :: 112 : x$ (1)

Step 2: Consider the number of days and the length of the wall. As the number of days decreases from 6 to 3, the length of the wall also decreases. So, it is in **Direct Variation**.

Therefore, the proportion is $6 : 3 :: 112 : x$ (2)

Combining (1) and (2), we can write

$$\left. \begin{array}{l} 20 : 25 \\ 6 : 3 \end{array} \right\} :: 112 : x$$

We know, **Product of Extremes = Product of Means.**

Extremes		Means		Extremes
20	:	25 :: 112	:	x
6	:	3		

$$\begin{aligned} \text{So, } 20 \times 6 \times x &= 25 \times 3 \times 112 \\ x &= \frac{25 \times 3 \times 112}{20 \times 6} = 70 \text{ meters.} \end{aligned}$$

Method 2

Number of Men	Number of days	Length of the wall in metres
20	6	112
25	3	x

Step 1: Consider the number of men and length of the wall. As the number of men increases from 20 to 25, the length of the wall also increases. It is in **direct variation**.

$$\text{The multiplying factor} = \frac{25}{20}$$

Step 2: Consider the number of days and the length of the wall. As the number of days decreases from 6 to 3, the length of the wall also decreases. It is in **direct variation**.

$$\text{The multiplying factor} = \frac{3}{6}$$

$$\therefore x = \frac{25}{20} \times \frac{3}{6} \times 112 = 70 \text{ meters}$$

Example 1.37

Six men working 10 hours a day can do a piece of work in 24 days. In how many days will 9 men working for 8 hours a day do the same work?

Solution

Method 1: The problem involves 3 sets of variables, namely - **Number of men, Working hours per day and Number of days**.

Number of Men	Number of hours per day	Number of days
6	10	24
9	8	x

Step 1: Consider the number of men and the number of days. As the number of men increases from 6 to 9, the number of days decreases. So it is in **Inverse Variation**.

$$\text{Therefore the proportion is } 9 : 6 :: 24 : x \quad \dots (1)$$

Step 2: Consider the number of hours worked per day and the number of days. As the number of hours working per day decreases from 10 to 8, the number of days increases. So it is **inverse variation**.

$$\text{Therefore the proportion is } 8 : 10 :: 24 : x \quad \dots (2)$$

Combining (1) and (2), we can write as

$$\left. \begin{array}{l} 9 : 6 \\ 8 : 10 \end{array} \right\} :: 24 : x$$

We know, **Product of extremes = Product of Means.**

Extremes	:	Means	:	Extremes
9	:	6 :: 24	:	x
8	:	10	:	

So,

$$9 \times 8 \times x = 6 \times 10 \times 24$$

$$x = \frac{6 \times 10 \times 24}{9 \times 8} = 20 \text{ days}$$

Note:

1. Denote the Direct variation as ↓ (Downward arrow)
2. Denote the Indirect variation as ↑ (Upward arrow)
3. Multiplying Factors can be written based on the arrows. Take the number on the head of the arrow in the numerator and the number on the tail of the arrow in the denominator.

For method two, use the instructions given in the note above .

Method 2 : (Using arrow marks)

Number of Men	Number of hours per day	Number of days
6	10	24
9	8	x

Step 1 : Consider men and days. As the number of men increases from 6 to 9, the number of days decreases. It is in **inverse variation**.

The multiplying factor = $\frac{6}{9}$

Step 2 : Consider the number of hours per day and the number of days. As the number of hours per day decreases from 10 to 8, the number of days increases. It is also in **inverse variation**.

The multiplying factor = $\frac{10}{8}$

$$\therefore x = \frac{6}{9} \times \frac{10}{8} \times 24 = 20 \text{ days.}$$

EXERCISE 1.7

1. Twelve carpenters working 10 hours a day complete a furniture work in 18 days. How long would it take for 15 carpenters working for 6 hours per day to complete the same piece of work?
2. Eighty machines can produce 4,800 identical mobiles in 6 hours. How many mobiles can one machine produce in one hour? How many mobiles would 25 machines produce in 5 hours?
3. If 14 composers can compose 70 pages of a book in 5 hours, how many composers will compose 100 pages of this book in 10 hours?
4. If 2,400 sq.m. of land can be tilled by 12 workers in 10 days, how many workers are needed to till 5,400 sq.m. of land in 18 days?
5. Working 4 hours daily, Swati can embroid 5 sarees in 18 days. How many days will it take for her to embroid 10 sarees working 6 hours daily?
6. A sum of ₹ 2,500 deposited in a bank gives an interest of ₹ 100 in 6 months. What will be the interest on ₹ 3,200 for 9 months at the same rate of interest?

1.8 Time and Work

When we have to compare the work of several persons, it is necessary to ascertain the amount of work each person can complete in one day. As time and work are of inverse variation and if more people are joined to do a work, the work will be completed within a shorter time.

In solving problems here, the following points should be remembered:

1. If a man finishes total work in ' n ' days, then in one day he does ' $\frac{1}{n}$ ' of the total work. For example, if a man finishes a work in 4 days, then in one day he does $\frac{1}{4}$ of the work.
2. If the quantity of work done by a man in one day is given, then the total number of days taken to finish the work = $1/(\text{one day's work})$. For example, if a man does $\frac{1}{10}$ of the work in 1 day, then the number of days taken to finish the work

$$= \frac{1}{\left(\frac{1}{10}\right)} = 1 \times \frac{10}{1} = 10 \text{ days.}$$

Example 1.38

A can do a piece of work in 20 days and B can do it in 30 days. How long will they take to do the work together?

Solution

Work done by A in 1 day = $\frac{1}{20}$, Work done by B in 1 day = $\frac{1}{30}$

$$\begin{aligned} \text{Work done by A and B in 1 day} &= \frac{1}{20} + \frac{1}{30} \\ &= \frac{3+2}{60} = \frac{5}{60} = \frac{1}{12} \text{ of the work} \end{aligned}$$

Total number of days required to finish the work by A and B = $\frac{1}{\frac{1}{12}} = 12$ days.

Example 1.39

A and B together can do a piece of work in 8 days, but A alone can do it 12 days. How many days would B alone take to do the same work?

Solution

Work done by A and B together in 1 day = $\frac{1}{8}$ of the work

Work done by A in 1 day = $\frac{1}{12}$ of the work

$$\text{Work done by B in 1 day} = \frac{1}{8} - \frac{1}{12} = \frac{3-2}{24} = \frac{1}{24}$$

Number of days taken by B alone to do the same work = $\frac{1}{\frac{1}{24}} = 24$ days.

Example 1.40

Two persons A and B are engaged in a work. A can do a piece of work in 12 days and B can do the same work in 20 days. They work together for 3 days and then A goes away. In how many days will B finish the work?

*Try these*

While A, B and C working individually can complete a job in 20,5,4 days respectively. If all join together and work, find in how many days they will finish the job?

Solution

$$\text{Work done by A in 1 day} = \frac{1}{12}$$

$$\text{Work done by B in 1 day} = \frac{1}{20}$$

$$\begin{aligned} \text{Work done by A and B together in 1 day} &= \frac{1}{12} + \frac{1}{20} \\ &= \frac{5+3}{60} = \frac{8}{60} = \frac{2}{15} \end{aligned}$$

$$\text{Work done by A and B together in 3 days} = \frac{2}{15} \times 3 = \frac{2}{5}$$

$$\text{Remaining Work} = 1 - \frac{2}{5} = \frac{3}{5}$$

$$\begin{aligned} \text{Number of days taken by B to finish the remaining work} &= \frac{\frac{3}{5}}{\frac{1}{20}} = \frac{3}{5} \times \frac{20}{1} \\ &= 12 \text{ days.} \end{aligned}$$

Example 1.41

A and B can do a piece of work in 12 days, B and C in 15 days, C and A in 20 days. In how many days will they finish it together and separately?

Solution

$$\text{Work done by A and B in 1 day} = \frac{1}{12}$$

$$\text{Work done by B and C in 1 day} = \frac{1}{15}$$

$$\text{Work done by C and A in 1 day} = \frac{1}{20}$$

$$\text{Work done by (A+B)+(B+C)+(C+A) in 1 day} = \frac{1}{12} + \frac{1}{15} + \frac{1}{20}$$

$$\text{Work done by (2A + 2B + 2C) in 1 day} = \frac{5+4+3}{60}$$

$$\text{Work done by 2(A + B + C) in 1 day} = \frac{12}{60}$$

$$\text{Work done by A, B and C together in 1 day} = \frac{1}{2} \times \frac{12}{60} = \frac{1}{10}$$

∴ A, B and C will finish the work in 10 days.

Work done by A in 1 day

$$\text{(i.e.) } [(A + B + C)\text{'s work} - (B + C)\text{'s work}] = \frac{1}{10} - \frac{1}{15} = \frac{3-2}{30} = \frac{1}{30}$$

∴ A will finish the work in 30 days.

Work done by B in 1 day

$$\text{(i.e.) } [(A + B + C)\text{'s work} - (C + A)\text{'s work}] = \frac{1}{10} - \frac{1}{20} = \frac{2-1}{20} = \frac{1}{20}$$

∴ B will finish the work in 20 days.

Work done by C in 1 day

$$\text{(i.e.) } [(A + B + C)\text{'s work} - (A + B)\text{'s work}] = \frac{1}{10} - \frac{1}{12} = \frac{6-5}{60} = \frac{1}{60}$$

∴ C will finish the work in 60 days.

Example 1.42

A can do a piece of work in 10 days and B can do it in 15 days. How much does each of them get if they finish the work and earn ₹ 1500?

Solution

$$\text{Work done by A in 1 day} = \frac{1}{10}$$

$$\text{Work done by B in 1 day} = \frac{1}{15}$$

$$\text{Ratio of their work} = \frac{1}{10} : \frac{1}{15} = 3 : 2$$

$$\text{Total Share} = ₹ 1500$$

$$\text{A's share} = \frac{3}{5} \times 1500 = ₹ 900$$

$$\text{B's share} = \frac{2}{5} \times 1500 = ₹ 600$$

Example 1.43

Two taps can fill a tank in 30 minutes and 40 minutes. Another tap can empty it in 24 minutes. If the tank is empty and all the three taps are kept open, in how much time the tank will be filled?

Solution

$$\text{Quantity of water filled by the first tap in one minute} = \frac{1}{30}$$

$$\text{Quantity of water filled by the second tap in one minute} = \frac{1}{40}$$

$$\text{Quantity of water emptied by the third tap in one minute} = \frac{1}{24}$$

$$\begin{aligned}
 \left. \begin{array}{l} \text{Quantity of water filled in one minute,} \\ \text{when all the 3 taps are opened} \end{array} \right\} &= \frac{1}{30} + \frac{1}{40} - \frac{1}{24} \\
 &= \frac{4 + 3 - 5}{120} = \frac{7 - 5}{120} \\
 &= \frac{2}{120} = \frac{1}{60} \\
 \text{Time taken to fill the tank} &= \frac{1}{\frac{1}{60}} = 60 \text{ minutes} \\
 &= 1 \text{ hour}
 \end{aligned}$$

EXERCISE 1.8

1. A man can complete a work in 4 days, whereas a woman can complete it in only 12 days. If they work together, in how many days, can the work be completed?
2. Two boys can finish a work in 10 days when they work together. The first boy can do it alone in 15 days. Find in how many days will the second boy do it all by himself?
3. Three men A, B and C can complete a job in 8, 12 and 16 days respectively. A and B work together for 3 days; then B leaves and C joins. In how many days, can A and C finish the work?
4. A tap A can fill a drum in 10 minutes. A second tap B can fill in 20 minutes. A third tap C can empty in 15 minutes. If initially the drum is empty, find when it will be full if all taps are opened together?
5. A can finish a job in 20 days and B can complete it in 30 days. They work together and finish the job. If ₹ 600 is paid as wages, find the share of each.
6. A, B and C can do a work in 12, 24 and 8 days respectively. They all work for one day. Then C leaves the group. In how many days will A and B complete the rest of the work?
7. A tap can fill a tank in 15 minutes. Another tap can empty it in 20 minutes. Initially the tank is empty. If both the taps start functioning, when will the tank become full?

Abbreviation: C.P. = Cost Price, S.P. = Selling Price, M.P. = Marked Price,
 P = Principal, r = Rate of interest, n = time period,
 A = Amount, C. I. = Compound Interest.



Concept Summary

Percent means per hundred. A fraction with its denominator 100 is called a percent.

In case of profit, we have

$$\text{Profit} = \text{S.P.} - \text{C.P.}; \quad \text{Profit percent} = \frac{\text{Profit}}{\text{C.P.}} \times 100$$

$$\text{S.P.} = \left(\frac{100 + \text{Profit}\%}{100} \right) \times \text{C.P.}; \quad \text{C.P.} = \left(\frac{100}{100 + \text{Profit}\%} \right) \times \text{S.P.}$$

In case of Loss, we have

$$\text{Loss} = \text{C.P.} - \text{S.P.}; \quad \text{Loss percent} = \frac{\text{Loss}}{\text{C.P.}} \times 100$$

$$\text{S.P.} = \left(\frac{100 - \text{Loss}\%}{100} \right) \times \text{C.P.}; \quad \text{C.P.} = \left(\frac{100}{100 - \text{Loss}\%} \right) \times \text{S.P.}$$

Discount is the reduction given on the Marked Price.

Selling Price is the price payable after reducing the Discount from the Marked Price.

Discount = M.P. - S.P.

$$\text{M.P.} = \frac{100}{100 - \text{Discount}\%} \times \text{S.P.}; \quad \text{S.P.} = \frac{100 - \text{Discount}\%}{100} \times \text{M.P.}$$

$$\text{C.P.} = \frac{100 - \text{Discount}\%}{100 + \text{Profit}\%} \times \text{M.P.}; \quad \text{M.P.} = \frac{100 + \text{Profit}\%}{100 - \text{Discount}\%} \times \text{C.P.}$$

$$\text{Discount Percent} = \frac{\text{Discount}}{\text{M.P.}} \times 100.$$

When the interest is

(i) compounded annually, $A = P \left(1 + \frac{r}{100} \right)^n$

(ii) compounded half - yearly, $A = P \left[1 + \frac{1}{2} \left(\frac{r}{100} \right) \right]^{2n}$

(iii) compounded quarterly, $A = P \left[1 + \frac{1}{4} \left(\frac{r}{100} \right) \right]^{4n}$

Appreciation, $A = P \left(1 + \frac{r}{100} \right)^n$; Depreciation, $A = P \left(1 - \frac{r}{100} \right)^n$

The difference between C. I. and S. I. for 2 years = $P \left(\frac{r}{100} \right)^2$

One day's work of A = $\frac{1}{\text{Number of days taken by A}}$

Work completed in 'x' days = One day's work $\times x$

Geometry

2

- 2.1 Introduction
- 2.2 Concurrency in Triangle
- 2.3 Pythagoras Theorem
- 2.4 Circles



Pythagoras
(582 - 497 BC)

The Greek Mathematician who was one of the foremost Mathematicians of all times. He was perhaps best known for the right angled triangle relation which bears his name.

2.1 Introduction

Geometry was developed by Egyptians more than 1000 years before Christ. It was abstracted by the Greeks into logical system of proofs with necessary basic postulates or axioms.

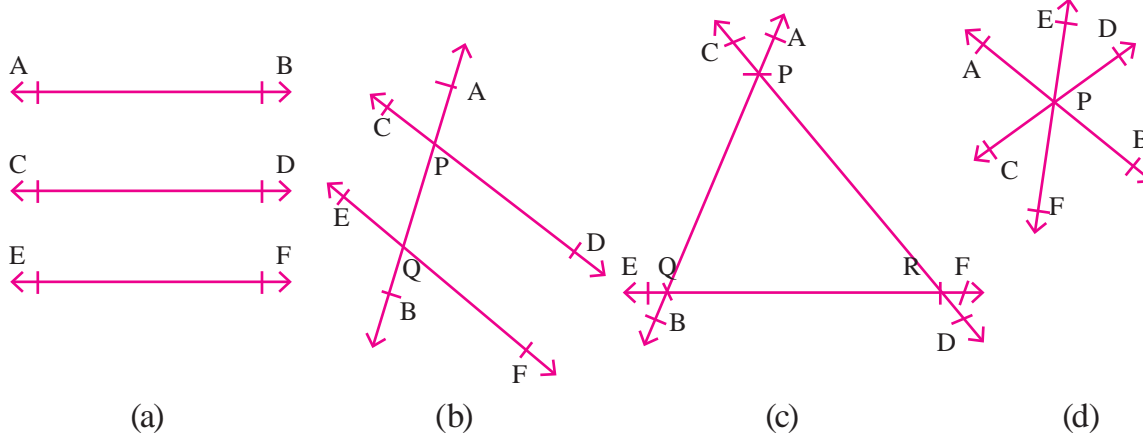
Geometry plays a vital role in our day-to-day life in many ways. Especially the triangles are used in several ways. The area of a land which is in polygon shape can be found by dividing them into triangles. The sum of the area of the triangles is the area of the land. If the triangles are in right angles then the area can be found very easily. Otherwise, the area can be found by drawing the perpendicular line from the base to the vertex.

Here we see the concurrency in triangles and some properties of them.

2.2 Concurrency in Triangles

Draw three or more lines in a plane. What are the possible ways?

The possibilities may be as follows:



In **fig (a)**, AB, CD and EF are parallel so they are **not intersecting**.

In **fig (b)**, AB and CD intersect at P, AB and EF intersect at Q. So P, Q are **two points of intersection**.

In **fig (c)**, P, Q, R are **three points of intersection**.

But in **fig (d)**, P is the only point of intersection. Here AB, CD, EF are the lines passing through the same point P. These lines are called as **concurrent lines**. The point **P** is called the **point of concurrency**.

In a triangle there are some special points of concurrence, which are Centroid of a triangle, Orthocentre of a triangle, Incentre of a triangle and Circumcentre of a triangle. Now we are going to study how to obtain these points in a triangle.

2.2.1 Centroid of a Triangle

In the adjacent figure, ABC is a triangle.

D is mid point of BC. Join AD.

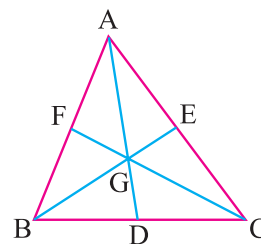
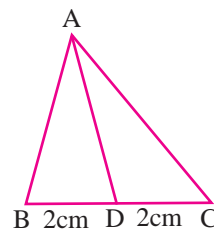
Here AD is one of the medians of $\triangle ABC$.

A **median** of a triangle is the line segment joining a vertex and the midpoint of the opposite side.

Now consider the adjacent figure, in which AD, BE, CF are the three medians of $\triangle ABC$.

They are concurrent at G. This point is called as centroid.

The three medians of a triangle are concurrent and the point of concurrency is known as **Centroid**. It is denoted by 'G'.



- Note :** (i) The Centroid divides each of the median in the ratio 2 : 1
(ii) The Centroid would be the physical centre of gravity.

2.2.2 Orthocentre of a Triangle

In the adjacent figure, ABC is a triangle .

From A, draw a perpendicular to BC ,

AD is perpendicular to BC .

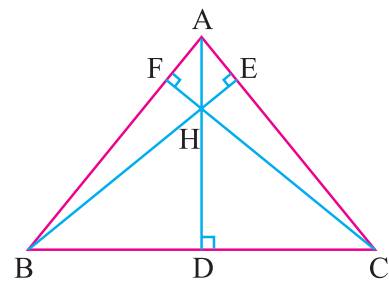
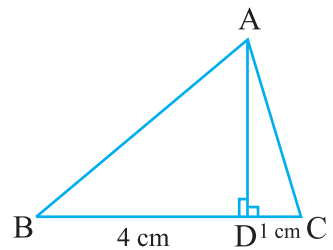
$\angle ADB = \angle ADC = 90^\circ$. Here D need not be the

mid point. Here AD is an altitude from vertex A.

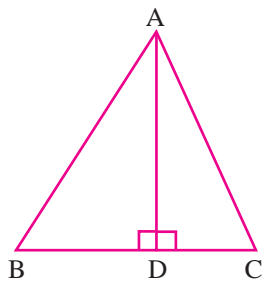
Altitude of a triangle is a perpendicular line segment drawn from a vertex to the opposite side. Now consider the figure, the triangle ABC in which AD, BE, CF are the three altitudes.

They are concurrent at H. This point is known as Orthocentre.

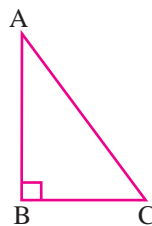
The three altitudes of a triangle are concurrent and the point of concurrency is known as **Orthocentre**.



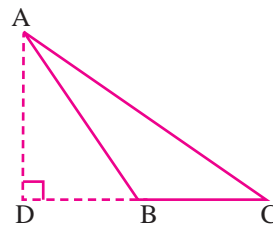
Different positions of orthocentre



(a)



(b)



(c)

Case (i) : In fig (a), ABC is an acute angled triangle .

Here orthocentre lies inside the ΔABC .

Case (ii) : In fig (b), ABC is a right angled triangle .

Here orthocentre lies on the vertex at the right angle.

Case (iii) : In fig (c), ABC is an obtuse angled triangle.

Here orthocentre lies outside the ΔABC .

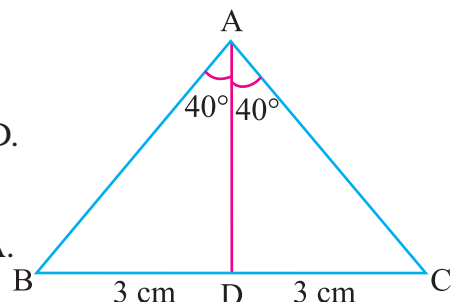
2.2.3 Incentre of a Triangle

In the adjacent figure , ABC is a triangle.

The $\angle A$ is bisected into two equal parts by AD.

Therefore $\angle BAD = \angle DAC$.

Here AD is said to be the angle bisector of $\angle A$.



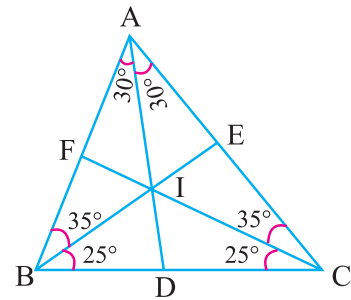
Angle bisector of a triangle is a line segment which bisects an angle of a triangle.

Now consider the figure in which AD, BE, CF are three angle bisectors of $\triangle ABC$.

They are concurrent at I.

This point is known as incentre of the triangle.

The three angle bisectors of a triangle are concurrent and the point of concurrence is called the **Incentre**.

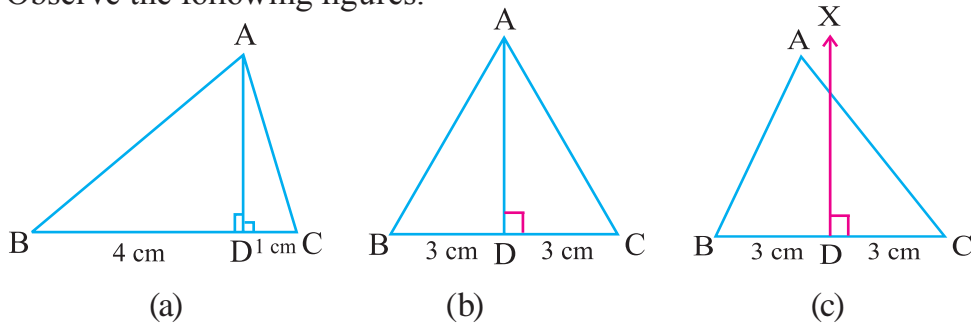


2.2.4 Circumcentre of a Triangle

We have learnt about perpendicular bisector in previous class.

What is a perpendicular bisector in a triangle?

Observe the following figures:

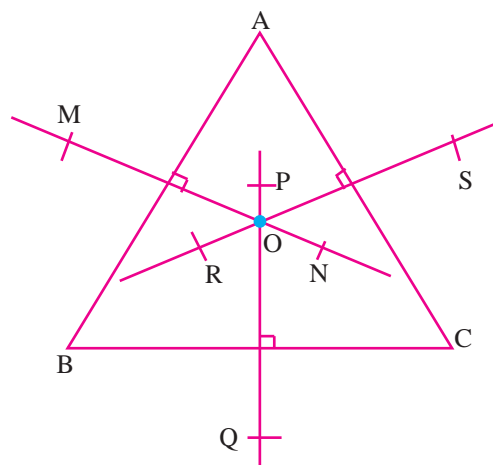


In fig (a): AD is perpendicular from A to BC but not bisecting BC.

In fig (b): AD bisects BC. Hence $BD = DC$ and AD is perpendicular to BC.

In fig (c): DX is perpendicular to BC and DX also bisecting BC. $BD = DC$ but DX need not passes through the vertex 'A'.

The **perpendicular bisector** of the side of a triangle is the line that is perpendicular to it and also bisects the side.



Now, consider the above figure.

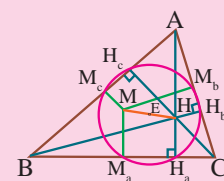
Here PQ, RS, and MN are the three perpendicular bisectors of BC, AC and AB concurrent at O. O is known as the circumcentre.

The three perpendicular bisectors of a triangle are concurrent and the point of concurrence is known as **circumcentre**.

- Note :** (i) In any triangle ABC, Circumcentre (O), Centroid (G) and Orthocentre (H) are always lie on one straight line, which is called as **Euler Line**, and $OG : GH = 1 : 2$.
- (ii) In particular for equilateral triangle, Circumcentre (O), Incentre (I), Orthocentre (H) and Centroid (G) will coincide.



Euler (1707-1783)
Switzerland



2.3 Pythagoras Theorem

2.3.1 Pythagoras Theorem

In a right angled triangle the square of the hypotenuse is equal to the sum of the squares of the other two sides.

Let us consider ΔABC with $\angle C = 90^\circ$.

$BC = a$, $CA = b$ and $AB = c$.

Then, $a^2 + b^2 = c^2$.

This was proved in number of ways by different Mathematicians.

We will see the **simple proof of Pythagoras Theorem**.

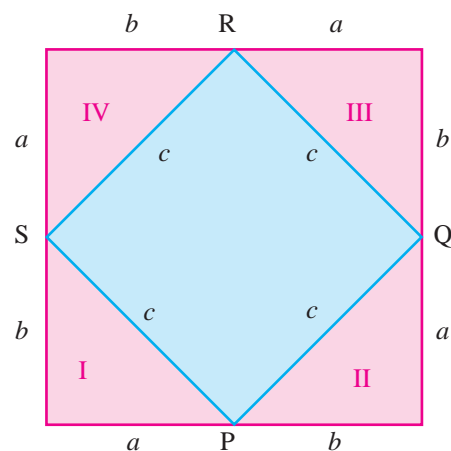
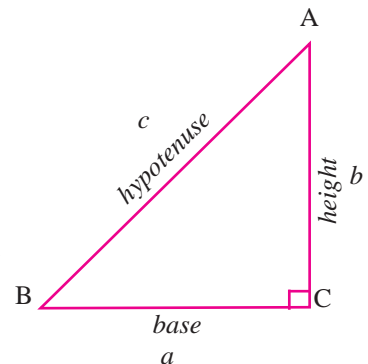
Now, we construct a square of side $(a+b)$ as shown in the figure, and using the construction we prove Pythagoras theorem. That is, we prove $a^2 + b^2 = c^2$.

We know that Area of any square is square of its side.

Area of a square of side $(a + b) = (a + b)^2$

From the figure,

Area of the square of side $(a + b) = (a + b)^2$
 = sum of the area of the triangles I, II, III and IV +
 the area of the square PQRS



i.e., $(a + b)^2 = 4$ (Area of right angled Δ) + (Area of the square PQRS)

$$(a + b)^2 = 4\left(\frac{1}{2} \times a \times b\right) + c^2$$

$$a^2 + b^2 + 2ab = 2ab + c^2$$

$$\therefore a^2 + b^2 = c^2$$

Hence we proved Pythagoras theorem.

Activity

Pythagoras Theorem

Draw a right angled triangle ABC, such that $\angle C = 90^\circ$, $AB = 5$ cm, $AC = 4$ cm and $BC = 3$ cm.

Construct squares on the three sides of this triangle.

Divide these squares into small squares of area one cm^2 each.

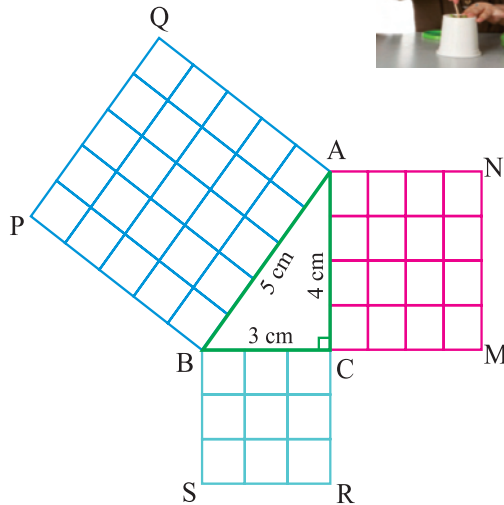
By counting the number of small squares, pythagoras theorem can be proved.

$$\text{Number of squares in ABPQ} = 25$$

$$\text{Number of squares in BCRS} = 9$$

$$\text{Number of squares in ACMN} = 16$$

$$\therefore \text{Number of squares in ABPQ} = \text{Number of squares in BCRS} + \text{Number of squares in ACMN}.$$



The numbers which are satisfying the Pythagoras theorem are called the **Pythagorean Triplets**.

Example 2.1

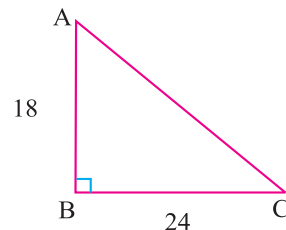
In ΔABC , $\angle B = 90^\circ$, $AB = 18$ cm and $BC = 24$ cm. Calculate the length of AC .

Solution

By Pythagoras Theorem,

$$\begin{aligned} AC^2 &= AB^2 + BC^2 \\ &= 18^2 + 24^2 \\ &= 324 + 576 \\ &= 900 \end{aligned}$$

$$\therefore AC = \sqrt{900} = 30 \text{ cm}$$



Example 2.2

A square has the perimeter 40 cm. What is the sum of the diagonals?

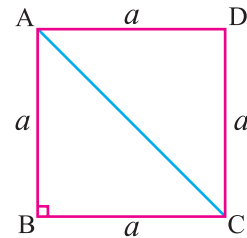
Solution

Let 'a' be the length of the side of the square. AC is a diagonal.

$$\text{Perimeter of square ABCD} = 4a \text{ units}$$

$$4a = 40\text{cm [given]}$$

$$a = \frac{40}{4} = 10\text{cm}$$



We know that in a square each angle is 90° and the diagonals are equal.

$$\begin{aligned} \text{In } \triangle ABC, AC^2 &= AB^2 + BC^2 \\ &= 10^2 + 10^2 = 100 + 100 = 200 \\ \therefore AC &= \sqrt{200} \\ &= \sqrt{2 \times 100} = 10\sqrt{2} \\ &= 10 \times 1.414 = 14.14\text{cm} \end{aligned}$$

$$\text{Diagonal AC} = \text{Diagonal BD}$$

$$\text{Hence, Sum of the diagonals} = 14.14 + 14.14 = 28.28\text{cm.}$$

Example 2.3

From the figure PT is an altitude of the triangle PQR in which PQ = 25cm, PR = 17cm and PT = 15 cm. If QR = x cm. Calculate x.

Solution From the figure, we have

$$QR = QT + TR.$$

To find : QT and TR.

In the right angled triangle PTQ ,

$$\angle PTQ = 90^\circ \text{ [PT is attitude]}$$

$$\text{By Pythagoras Theorem, } PQ^2 = PT^2 + QT^2 \quad \therefore PQ^2 - PT^2 = QT^2$$

$$\therefore QT^2 = 25^2 - 15^2 = 625 - 225 = 400$$

$$QT = \sqrt{400} = 20 \text{ cm} \quad \dots(1)$$

Similarly, in the right angled triangle PTR,

$$\text{by Pythagoras Theorem, } PR^2 = PT^2 + TR^2$$

$$\therefore TR^2 = PR^2 - PT^2$$

$$= 17^2 - 15^2$$

$$= 289 - 225 = 64$$

$$TR = \sqrt{64} = 8 \text{ cm} \quad \dots (2)$$

$$\text{Form (1) and (2) } QR = QT + TR = 20 + 8 = 28\text{cm.}$$

Example 2.4

A rectangular field is of dimension 40m by 30m. What distance is saved by walking diagonally across the field?

Solution

Given: ABCD is a rectangular field of Length = 40m, Breadth = 30m, $\angle B = 90^\circ$

In the right angled triangle ABC,

By Pythagoras Theorem,

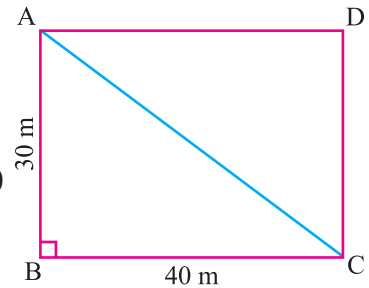
$$\begin{aligned} AC^2 &= AB^2 + BC^2 \\ &= 30^2 + 40^2 = 900 + 1600 \\ &= 2500 \end{aligned}$$

$$\therefore AC = \sqrt{2500} = 50 \text{ m}$$

Distance from A to C through B is

$$= 30 + 40 = 70 \text{ m}$$

$$\text{Distance saved} = 70 - 50 = 20 \text{ m.}$$

**EXERCISE 2.1**

- Choose the correct answer
 - The point of concurrency of the medians of a triangle is known as
(A) incentre (B) circle centre (C) orthocentre (D) centroid
 - The point of concurrency of the altitudes of a triangle is known as
(A) incentre (B) circle centre (C) orthocentre (D) centroid
 - The point of concurrency of the angle bisectors of a triangle is known as
(A) incentre (B) circle centre (C) orthocentre (D) centroid
 - The point of concurrency of the perpendicular bisectors of a triangle is known as
(A) incentre (B) circumcentre (C) orthocentre (D) centroid
- In an isosceles triangle $AB = AC$ and $\angle B = 65^\circ$. Which is the shortest side?
- PQR is a triangle right angled at P. If $PQ = 10$ cm and $PR = 24$ cm, find QR.
- Check whether the following can be the sides of a right angled triangle
 $AB = 25$ cm, $BC = 24$ cm, $AC = 7$ cm.
- $\angle Q$ and $\angle R$ of a triangle PQR are 25° and 65° . Is ΔPQR a right angled triangle? Moreover PQ is 4cm and PR is 3 cm. Find QR.
- A 15 m long ladder reached a window 12m high from the ground. On placing it against a wall at a distance x m. Find x .
- Find the altitude of an equilateral triangle of side 10 cm.
- Are the numbers 12, 5 and 13 form a Pythagorean Triplet?

9. A painter sets a ladder up to reach the bottom of a second storey window 16 feet above the ground. The base of the ladder is 12 feet from the house. While the painter mixes the paint a neighbour's dog bumps the ladder which moves the base 2 feet farther away from the house. How far up side of the house does the ladder reach?



2.4 Circles

You are familiar with the following objects. Can you say the shape of the following?

- (a) Cycle wheel
- (b) Ashoka chakra in our National Emblem
- (c) Full moon

Sure, your answer will be circle. You know that a circle is described when a point P moves in a plane such that its distance from a fixed point in the plane remains constant.

Definition of Circle

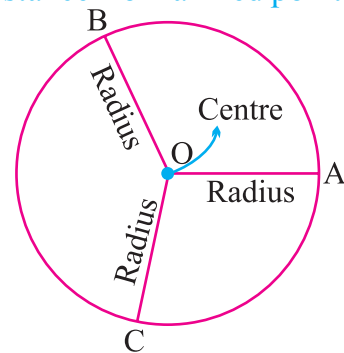
A circle is the set of all points in a plane at a constant distance from a fixed point in that plane.

The fixed point is called the centre of the circle.

The constant distance is known as the radius of the circle.

In the figure 'O' is centre and OA, OB, OC are radii of the circle.

$$\text{Here, } OA = OB = OC = r$$



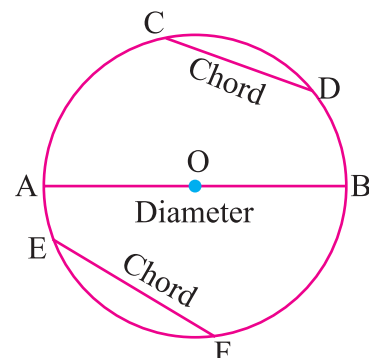
Note: All the radii of the circle are equal.

Chord

A chord is a line segment with its end points lying on a circle.

In figure, CD, AB and EF are chords.

Here AB is a special chord that passes through the centre O.



Diameter

A diameter is a chord that passes through the centre of the circle and diameter is the longest chord of a circle.

In the figure, AOB is diameter of the circle.

O is the mid point of AB and $OA = OB = \text{radius of the circle}$

Hence, **Diameter** = $2 \times \text{radius}$ (or) **Radius** = $(\text{diameter}) \div 2$

- Note :** (i) The mid-point of every diameter of the circle is the centre of the circle.
(ii) The diameters of a circle are concurrent and the point of concurrency is the centre of the circle.

Secant of a Circle

A line passing through a circle and intersecting the circle at two points is called the secant of the circle.

In the given figure, line AB is a Secant.

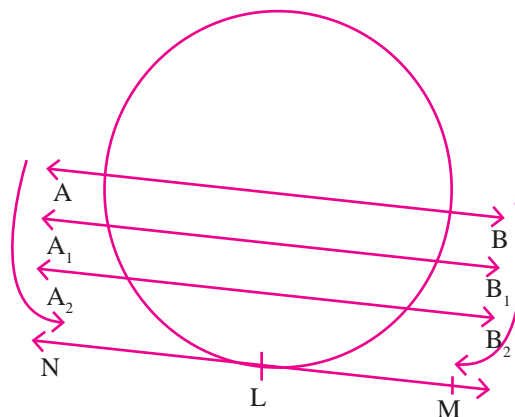
It cuts the circle at two points A and B.

Now, let us move the secant AB downwards. Then the new positions are $A_1 B_1$, $A_2 B_2$, etc.,

While secant AB moves down, the points A and B are moving closer to each other.

So distance between A and B is gradually decreases.

At one position the secant AB touches the circle at only one point L. At this position, the line LM is called as tangent and it touches the circle at only one point.

**Tangent**

Tangent is a line that touches a circle at exactly one point, and the point is known as point of contact.

Arc of a Circle

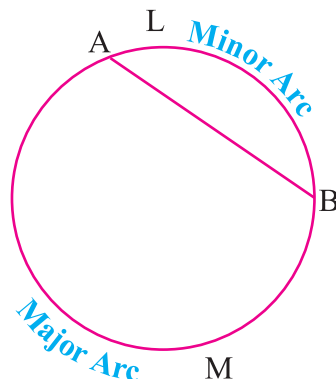
In the figure AB is a chord. The chord AB divides the circle into two parts.

The curved parts ALB and AMB are known as Arcs.

Arcs will be denoted by the symbol $\widehat{\quad}$.

The smaller arc \widehat{ALB} is the **minor arc**.

The greater arc \widehat{AMB} is the **major arc**.

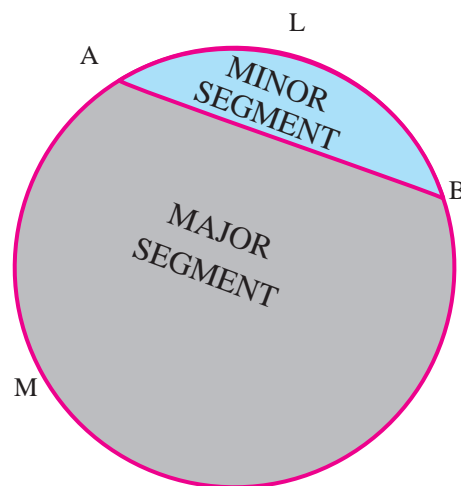


Segment of a Circle

A chord of a circle divides the circular region into two parts. Each part is called as segment of the circle.

The segment containing minor arc is called the **minor segment**.

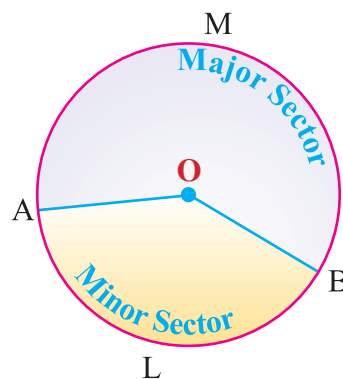
The segment containing major arc is called the **major segment**.



Sector of a Circle

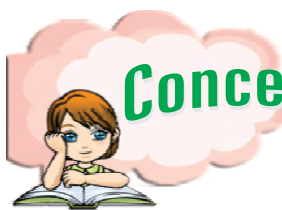
The circular region enclosed by an arc of a circle and the two radii at its end points is known as Sector of a circle.

The smaller sector OALB is called the **minor sector**. The greater sector OAMB is called the **major sector**.



EXERCISE 2.2

1. Choose the correct answer:
 - (i) The _____ of a circle is the distance from the centre to the circumference.
 (A) sector (B) segment (C) diameters (D) radius
 - (ii) The relation between radius and diameter of a circle is _____
 (A) radius = 2 × diameters (B) radius = diameter + 2
 (C) diameter = radius + 2 (D) diameter = 2 (radius)
 - (iii) The longest chord of a circle is
 (A) radius (B) secant (C) diameter (D) tangent
2. If the sum of the two diameters is 200 mm, find the radius of the circle in cm.
3. Define the circle segment and sector of a circle.
4. Define the arc of a circle.
5. Define the tangent of a circle and secant of a circle.



Concept Summary

- ↪ **Centroid** : Point of concurrency of the three Medians.
- ↪ **Orthocentre** : Point of concurrency of the three Altitudes.
- ↪ **Incentre** : Point of concurrency of the three Angle Bisectors.
- ↪ **Circumcentre** : Point of concurrency of the Perpendicular Bisectors of the three sides.
- ↪ **Circle** : A circle is the set of all points in a plane at a constant distance from a fixed point in that plane .
- ↪ **Chord** : A chord is a line segment with its end points lying on a circle.
- ↪ **Diameter** : A diameter is a chord that passes through the centre of the circle.
- ↪ A line passing through a circle and intersecting the circle at two points is called the secant of the circle.
- ↪ Tangent is a line that touches a circle at exactly one point, and the point is known as point of contact.
- ↪ **Segment of a circle** : A chord of a circle divides the circular region into two parts.
- ↪ **Sector of a circle** : The circular region enclosed by an arc of a circle and the two radii at its end points is known as Sector of a circle.

Mathematics Club Activity

PYTHAGOREAN TRIPLET

We can find many Pythagorean Triplets using the condition $m^2 + n^2, m^2 - n^2, 2mn$ here $m > n; m, n \in \mathbb{N}$.

If $m = 2$ and $n = 1$ then $m^2 + n^2 = 2^2 + 1^2 = 5, m^2 - n^2 = 2^2 - 1^2 = 3, 2mn = 2 \times 2 \times 1 = 4$

$5, 3, 4$ is a Pythagorean Triplet.

The multiple of Pythagorean Triplet is also a Pythagorean Triplet.

Eg, The multiples of $(5, 3, 4)$: $(10, 6, 8), (15, 9, 12), (20, 12, 16), \dots$ are also the Pythagorean Triplets.

Can you find more Pythagorean Triplets?

Data Handling

3

- 3.1 Introduction
- 3.2 Recalling the Formation of Frequency Table
- 3.3 Drawing Histogram and Frequency Polygon for Grouped Data
- 3.4 Construction of Simple Pie chart
- 3.5 Measures of Central Tendency

3.1 Introduction

Everyday we come across different kinds of information in the form of numbers through newspapers and other media of communication.

This information may be about food production in our country, population of the world, import and export of different countries, drop-outs of children from the schools in our state, the accidental deaths, etc.

In all these information, we use numbers. These numbers are called **data**. The data help us in making decisions. They play a vital part in almost all walks of life of every citizen. Hence, it is very important to know how to get relevant and exact information from such data.

The calculated data may not be suitable for reading, understanding and for analysing. The data should be carefully handled so that it can be presented in various forms. A common man should be able to understand and visualize and get more information on the data.



R.A. Fisher

[17th Feb., 1890 -
29th July, 1962]

Fisher was interested in the theory of errors that eventually let him to investigate statistical problems. He became a teacher in Mathematics and Physics between 1915 and 1919. He studied the design of experiments by introducing randomisation and the analysis of variance procedures now used throughout the world. He is known as “Father of Modern Statistics”.

3.2 Recalling the Formation of Frequency Table

We have learnt in seventh standard, how to form a frequency table. Let us recall it.

3.2.1 Formation of frequency table for an ungrouped data

Example 3.1

Consider the following data:

15, 17, 17, 20, 15, 18, 16, 25, 16, 15,

16, 18, 20, 28, 30, 27, 18, 18, 20, 25,

16, 16, 20, 28, 15, 18, 20, 20, 20, 25. Form a frequency table.

Solution

The frequency table is given below:

Number (x)	Tally Mark	Frequency (f)
15		4
16	 	5
17		2
18	 	5
20	 	7
25		3
27		1
28		2
30		1
	Total	30

3.2.2 Formation of frequency table for a grouped data

Example: 3.2

The marks obtained by 50 students in a Mathematics test with maximum marks of 100 are given as follows:

43, 88, 25, 93, 68, 81, 29, 41, 45, 87, 34, 50, 61, 75, 51, 96, 20, 13, 18, 35, 25, 77, 62, 98, 47, 36, 15, 40, 9, 25, 39, 60, 37, 50, 19, 86, 42, 29, 32, 61, 45, 68, 41, 87, 61, 44, 67, 30, 54, 8.

Prepare a frequency table for the above data using class interval.

Solution

$$\text{Total number of values} = 50$$

$$\begin{aligned} \text{Range} &= \text{Highest value} - \text{Lowest value} \\ &= 98 - 8 = 90 \end{aligned}$$

Let us divide the given data into 10 classes.

$$\begin{aligned} \therefore \text{Length of the Class interval} &= \frac{\text{Range}}{\text{Number of class interval}} \\ &= \frac{90}{10} = 9 \end{aligned}$$

The frequency table of the marks obtained by 50 students in a mathematics test is as follows:

Class Interval (C.I)	Tally Mark	Frequency (f)
0 - 10		2
10 - 20		4
20 - 30		6
30 - 40		7
40 - 50		9
50 - 60		4
60 - 70		8
70 - 80		2
80 - 90		5
90 - 100		3
	Total	50

Thus the given data can be grouped and tabulated as follows:

Class Interval (C.I)	0-10	10-20	20- 30	30-40	40- 50	50- 60	60- 70	70- 80	80-90	90-100
Frequency (f)	2	4	6	7	9	4	8	2	5	3

3.3 Drawing Histogram and Frequency Polygon for Grouped Data

The statistical data can be represented by means of geometrical figures or diagrams which are normally called “graphs”. The graphical representation of data makes itself interesting for reading, consuming less time and easily understandable. There are many ways of representing numerical data graphically. In this chapter, we will study the following two types of diagrams:

- (i) **Histogram**
- (ii) **Frequency Polygon**

3.3.1 Histogram

A two dimensional graphical representation of a continuous frequency distribution is called a **histogram**.

In histogram, the bars are placed continuously side by side with no gap between adjacent bars. That is, in histogram rectangles are erected on the class intervals of the distribution. The areas of rectangle are proportional to the frequencies.

3.3.1 (a) Drawing a histogram for continuous frequency distribution

Procedure:

- Step 1 :** Represent the data in the continuous (exclusive) form if it is in the discontinuous (inclusive) form.
- Step 2 :** Mark the class intervals along the X-axis on a uniform scale.
- Step 3 :** Mark the frequencies along the Y-axis on a uniform scale.
- Step 4 :** Construct rectangles with class intervals as bases and corresponding frequencies as heights.

The method of drawing a histogram is explained in the following example.

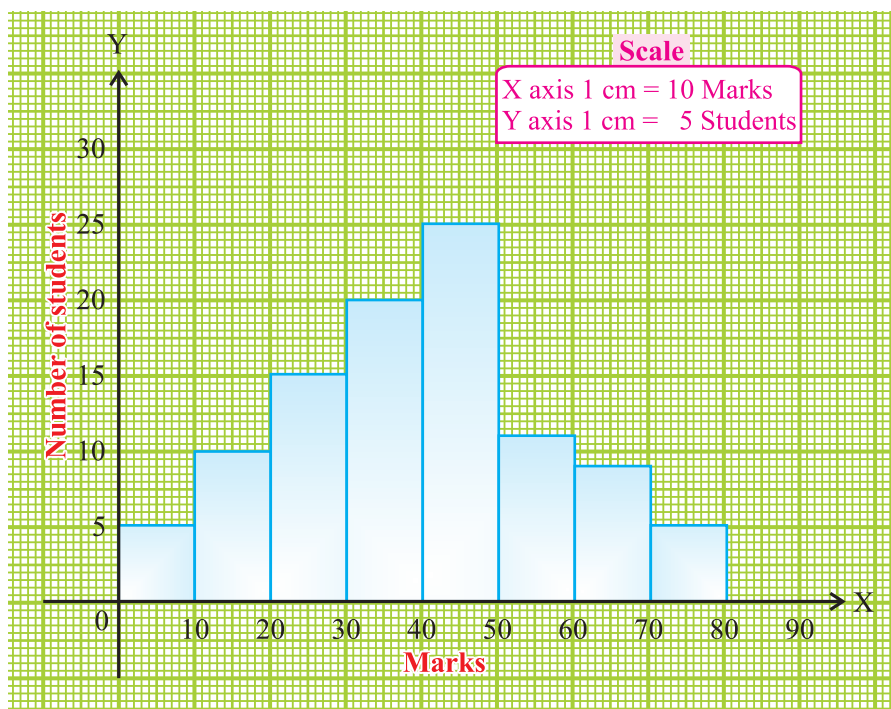
Example 3.3

Draw a histogram for the following table which represent the marks obtained by 100 students in an examination:

Marks	0-10	10-20	20-30	30-40	40-50	50-60	60-70	70-80
Number of students	5	10	15	20	25	12	8	5

Solution

The class intervals are all equal with length of 10 marks. Let us denote these class intervals along the X-axis. Denote the number of students along the Y-axis, with appropriate scale. The histogram is given below.

**Fig. 3.1**

Note: In the above diagram, the bars are drawn continuously. The rectangles are of lengths (heights) proportional to the respective frequencies. Since the class intervals are equal, the areas of the bars are proportional to the respective frequencies.

3.3.1 (b) Drawing a histogram when class intervals are not continuous**Example 3.4:**

The heights of trees in a forest are given as follows. Draw a histogram to represent the data.

Heights in metre	16-20	21-25	26-30	31-35	36-40	41-45	46-50	51-55
Number of trees	10	15	25	30	45	50	35	20

Solution

In this problem, the given class intervals are discontinuous (inclusive) form. If we draw a histogram as it is, we will get gaps between the class intervals. But in a histogram the bars should be continuously placed without any gap. Therefore we should make the class intervals continuous. For this we need an adjustment factor.

$$\text{Adjustment Factor} = \frac{1}{2} [(\text{lower limit of a class interval}) - (\text{upper limit of the preceding class interval})]$$

$$= \frac{1}{2} (21 - 20) = 0.5$$

In the above class interval, we subtract 0.5 from each lower limit and add 0.5 in each upper limit. Therefore we rewrite the given table into the following table.

Heights in metre	15.5-20.5	20.5-25.5	25.5-30.5	30.5-35.5	35.5-40.5	40.5-45.5	45.5-50.5	50.5-55.5
Number of Trees	10	15	25	30	45	50	35	20

Now the above table becomes continuous frequency distribution. The histogram is given below

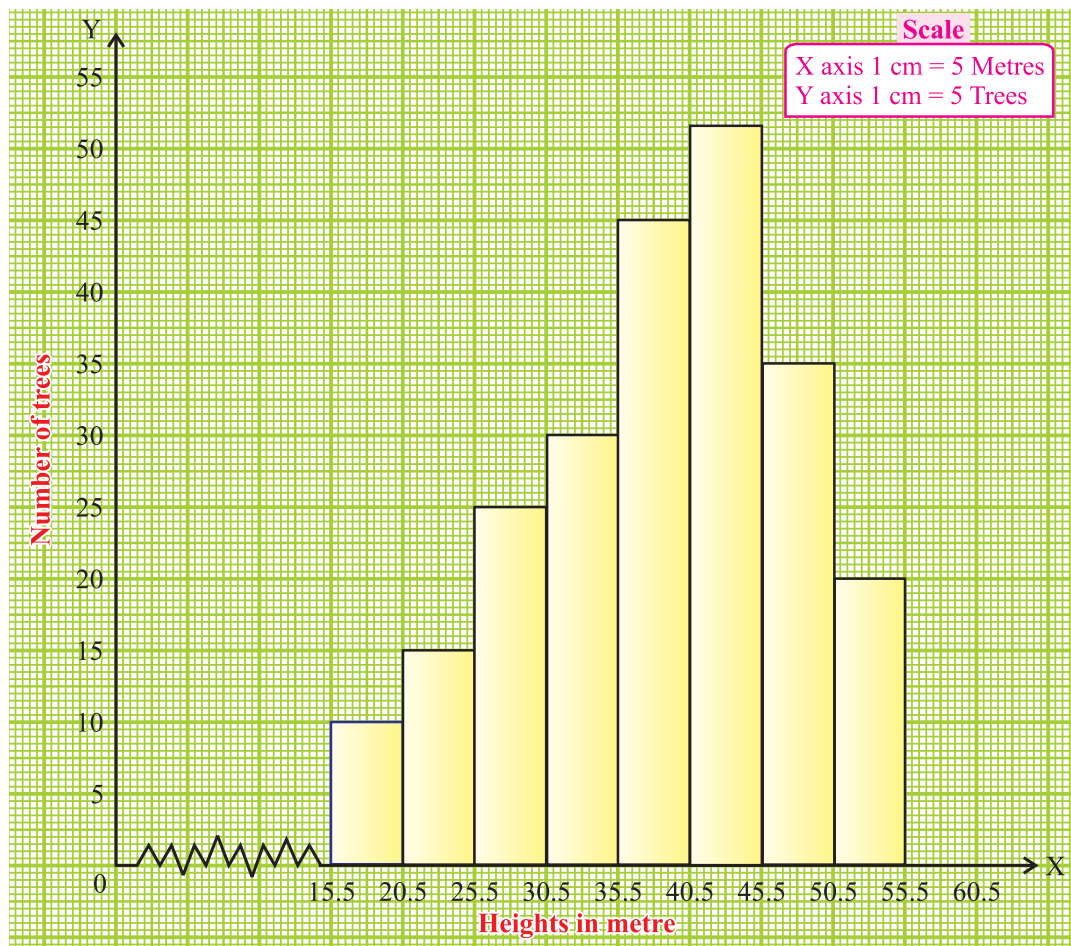


Fig. 3.2

Note: In the histogram (Fig. 3.2) along the X-axis the first value starts from 15.5, therefore a break (kink) is indicated near the origin to signify that the graph is drawn beginning at 15.5 and not at the origin.



Do you know?

The break is indicated by a Zig - Zag curve.

3.3.2 Frequency polygon

Frequency Polygon is another method of representing frequency distribution graphically.

Draw a histogram for the given continuous data. Mark the middle points of the tops of adjacent rectangles. If we join these middle points **successively** by line segment, we get a polygon. This polygon is called the **frequency polygon**. It is customary to bring the ends of the polygon down to base level by assuming a lower class of a frequency and highest class of a frequency.

Frequency Polygon can be constructed in two ways:

- (i) Using histogram
- (ii) Without using histogram.

3.3.2 (a) To draw frequency polygon using histogram

Procedure:

- Step 1 :** Obtain the frequency distribution from the given data and draw a histogram.
- Step 2 :** Join the mid points of the tops of adjacent rectangles of the histogram by means of line segments.
- Step 3 :** Obtain the mid points of two assumed class intervals of zero frequency, one adjacent to the first bar on its left and another adjacent to the last bar on its right. These class intervals are known as **imagined class interval**.
- Step 4 :** Complete the polygon by joining the mid points of first and the last class intervals to the mid point of the imagined class intervals adjacent to them.

Example 3.5

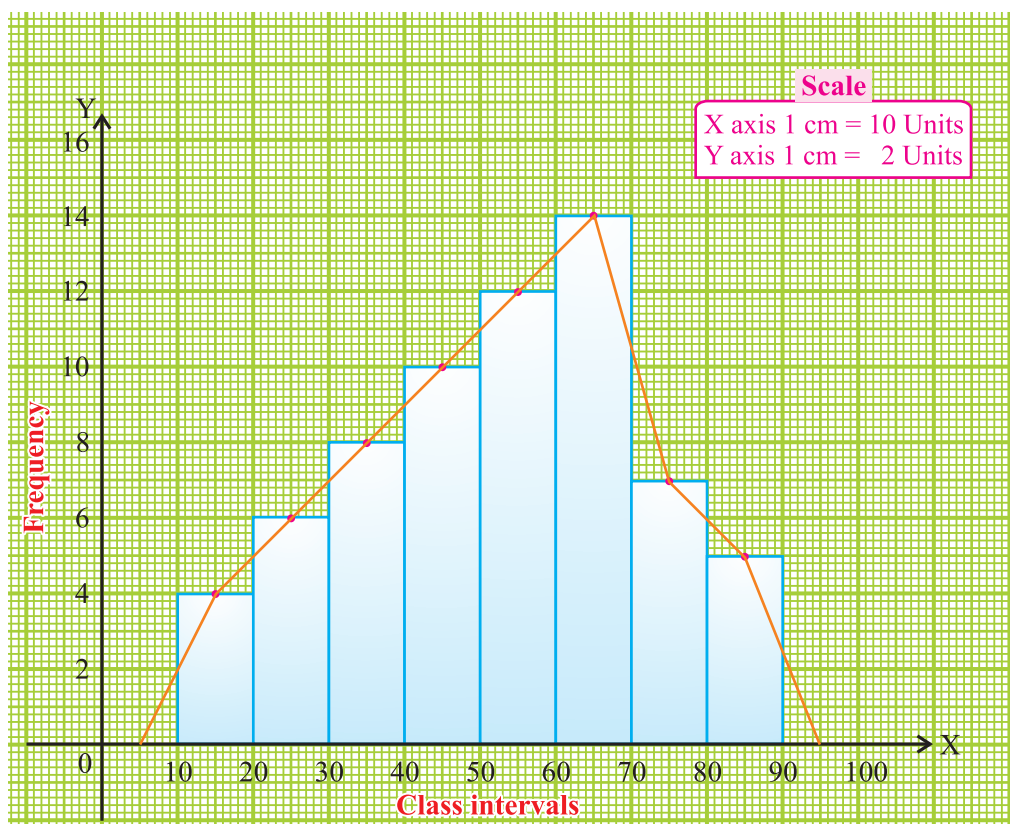
Draw a frequency polygon imposed on the histogram for the following distribution

Class interval	10-20	20-30	30-40	40-50	50-60	60-70	70-80	80-90
Frequency	4	6	8	10	12	14	7	5

Solution

Take the class-intervals along the X-axis and frequencies along the Y-axis with appropriate scale as shown in the Fig 3.3.

Draw a histogram for the given data. Now mark the mid points of the upper sides of the consecutive rectangles. We also mark the midpoints of the assumed class intervals 0-10 and 90-100. The mid points are joined with the help of a ruler. The ends of the polygon are joined with the mid points of 0-10 and 90-100. Now, we get the frequency polygon. Refer Fig 3.3.

**Fig. 3.3****Example 3.6**

Draw a frequency polygon of the following data using histogram

Class interval	0 - 10	10 - 20	20 - 30	30 - 40	40 - 50	50 - 60
Frequency	5	10	25	16	12	8

Solution

Mark the class intervals along the X-axis and the frequencies along the Y-axis with appropriate scale shown in Fig 3.4.

Draw a histogram for the given data. Now, mark the mid points of the upper sides of the consecutive rectangles. Also we take the imagined class interval $(-10) - 0$ and $60 - 70$. The mid points are joined with the help of a ruler. The ends of the polygon are joined with the mid points of the imagined class intervals $(-10)-0$ and $60 - 70$. Now we get the frequency polygon. (Refer Fig 3.4).

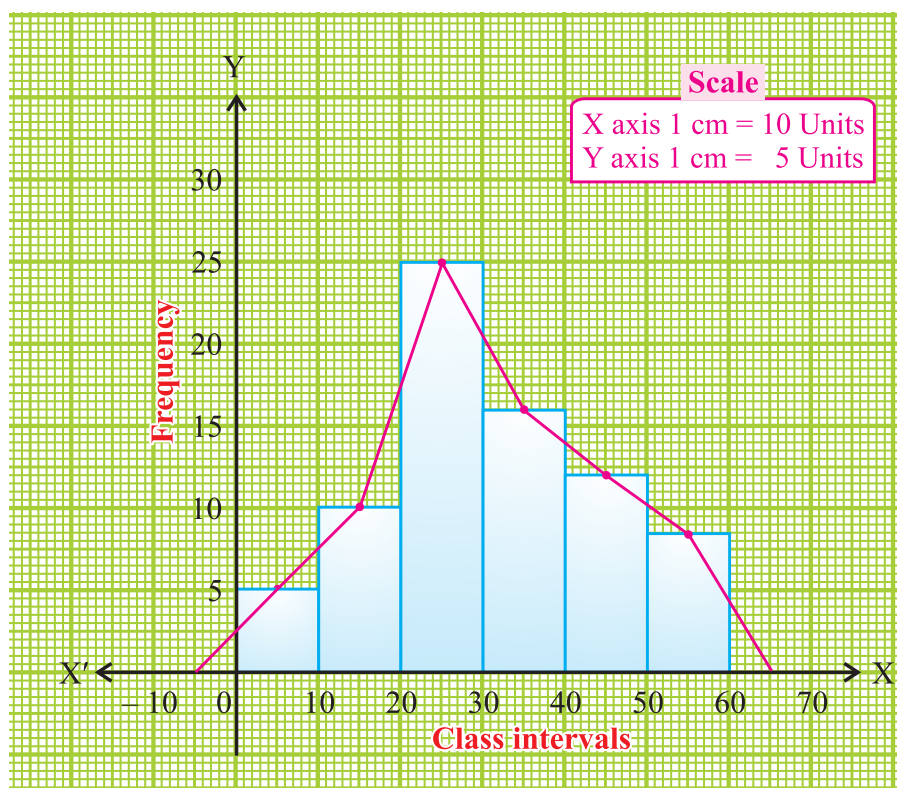


Fig. 3.4

Note: Sometimes imagined class intervals do not exist. **For example**, in case of marks obtained by the students in a test, we cannot go below zero and beyond maximum marks on the two sides. In such cases, the extreme line segments are only partly drawn and are brought down vertically so that they meet at the mid points of the vertical left and right sides of first and last rectangles respectively.

Using this note, we will draw a frequency polygon for the following example:

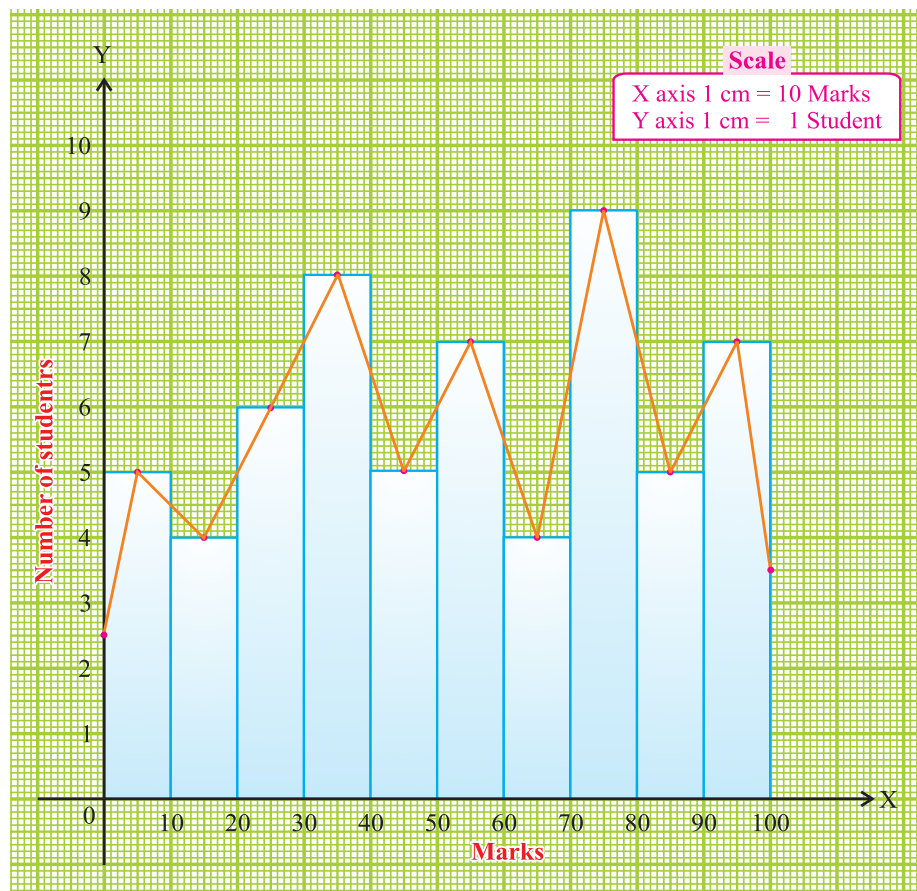
Example 3.7

Draw a frequency polygon for the following data using histogram

Marks	0-10	10-20	20-30	30-40	40-50	50-60	60-70	70-80	80-90	90-100
Number of students	5	4	6	8	5	7	4	9	5	7

Solution

Mark the class intervals along the X-axis and the number of students along the Y-axis. Draw a histogram for the given data. Now mark the mid points of the upper sides of the consecutive rectangles. The mid points are joined with the help of a ruler. Note that, the first and last edges of the frequency polygon meet at the mid point of the vertical edges of the first and last rectangles.

**Fig. 3.5****3.3.2 (b) To draw a frequency polygon without using histogram****Procedure:**

- Step 1 :** Obtain the frequency distribution and compute the mid points of each class interval.
- Step 2 :** Represent the mid points along the X-axis and the frequencies along the Y-axis.
- Step 3 :** Plot the points corresponding to the frequency at each mid point.
- Step 4 :** Join these points, by straight lines in order.
- Step 5 :** To complete the polygon join the point at each end immediately to the lower or higher class marks (as the case may be at zero frequency) on the X-axis.

Example 3.8

Draw a frequency polygon for the following data without using histogram.

Class interval	10-20	20-30	30-40	40-50	50-60	60-70	70-80	80-90
Frequency	4	6	8	10	12	14	7	5

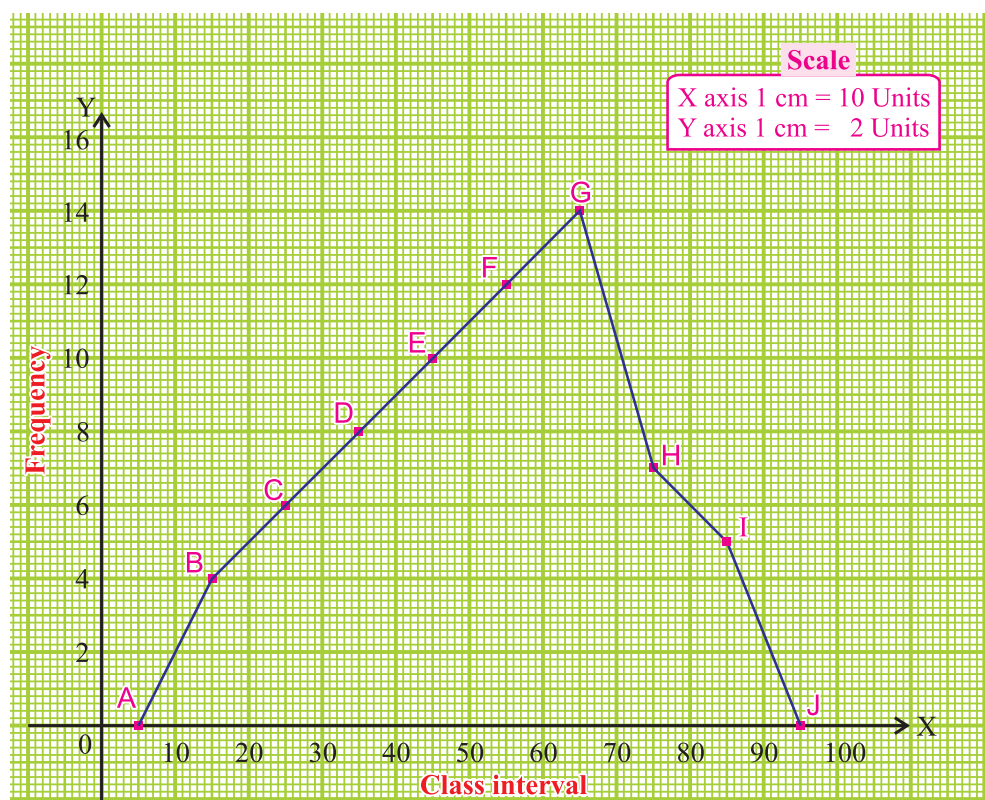
Solution:

Mark the class intervals along the X-axis and the frequency along the Y-axis. We take the **imagined classes** 0-10 at the beginning and 90-100 at the end, each with frequency zero. We have tabulated the data as shown.

Using the adjacent table, plot the points A (5, 0), B (15, 4), C (25, 6), D (35, 8), E (45, 10), F (55, 12), G (65, 14), H (75, 7), I (85, 5) and J (95, 0).

Class interval	Midpoints	Frequency
0-10	5	0
10-20	15	4
20-30	25	6
30-40	35	8
40-50	45	10
50-60	55	12
60-70	65	14
70-80	75	7
80-90	85	5
90-100	95	0

We draw the line segments AB, BC, CD, DE, EF, FG, GH, HI, IJ to obtain the required frequency polygon **ABCDEFGHJI**, which is shown in Fig 3.6.

**Fig. 3.6**

EXERCISES 3.1

1. Draw a histogram to represent the following data

Class intervals	0-10	10-20	20-30	30-40	40-50	50-60
Frequency	8	12	6	14	10	5

2. Draw a histogram with the help of the following table

Yield per acre (Quintal)	11-15	16-20	21-25	26-30	31-35	36 - 40
Number of rice field	3	5	18	15	6	4

3. Draw a histogram to represent the following data of the spectators in a cricket match

Age in years	10-19	20-29	30-39	40-49	50-59	60-69
Number of spectators	4	6	12	10	8	2

4. In a study of diabetic patients in a village, the following observations were noted

Age (in years)	10-20	20-30	30-40	40-50	50-60	60-70
Number of patients	3	6	13	20	10	5

Represent the above data by a frequency polygon using histogram.

5. Construct a histogram and frequency polygon for the following data

Class interval	0-10	10-20	20-30	30-40	40-50	50-60
Frequency	7	10	23	11	8	5

6. The following table shows the performance of 150 candidates in an Intelligence test. Draw a histogram and frequency polygon for this distribution

Intelligent Quotient	55-70	70-85	85-100	100-115	115-130	130-145
Number of candidates	20	40	30	35	10	15

7. Construct a frequency polygon from the following data using histogram.

Marks	0-10	10-20	20-30	30-40	40-50	50-60	60-70	70-80	80-90	90-100
No of students	9	3	4	6	2	3	4	5	7	8

8. Draw a frequency polygon for the following data without using histogram

Age (in years)	0-10	10-20	20-30	30-40	40-50	50-60	60-70
Number of persons	6	11	25	35	18	12	6

9. Construct a frequency polygon for the following data without using histogram

Class interval	30-34	35-39	40-44	45-49	50-54	55-59
Frequency	12	16	20	8	10	4

10. The following are the marks obtained by 40 students in an English examination (out of 50 marks). Draw a histogram and frequency polygon for the data
29, 45, 23, 40, 31, 11, 48, 1, 30, 24, 25, 29, 25, 32, 31, 22, 9, 49, 19, 13, 32, 39, 25, 3, 27, 41, 12, 13, 2, 44, 7, 43, 15, 35, 40, 3, 12, 48, 49, 18.

3.4 Construction of Simple Pie Chart

Have you ever come across the data represented in a circular form as shown in Figure 3.7 and Figure 3.8 ?

The time spent by a school student during a day (24 hours).

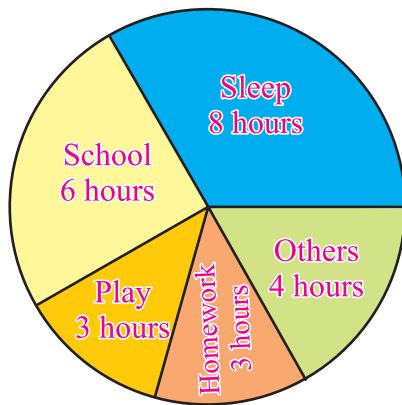


Fig. 3.7

Viewers watching different types of channels on TV.

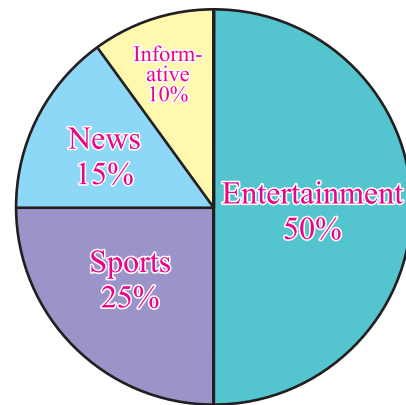


Fig. 3.8

The figures similar to the above are called circle graphs. A **circle graph** shows the relationship between a whole and its parts. Here, the whole circle is divided into sectors. The size of each sector is proportional to the activity or information it represents. Since, the sectors resemble the slices of a pie, it is called a **pie chart**.



Do you know?



Pie is an American food item

For example, in the pie chart (Fig 3.7).

$$\begin{aligned} \left. \begin{array}{l} \text{The proportion of the sector} \\ \text{for hours spent in sleeping} \end{array} \right\} &= \frac{\text{number of sleeping hours}}{\text{whole day}} \\ &= \frac{8 \text{ hours}}{24 \text{ hours}} = \frac{1}{3} \end{aligned}$$

So, this sector is drawn $\frac{1}{3}$ rd part of the circle.

$$\begin{aligned} \left. \begin{array}{l} \text{The proportion of the sector} \\ \text{for hours spent in school} \end{array} \right\} &= \frac{\text{number of school hours}}{\text{Whole day}} \\ &= \frac{6 \text{ hours}}{24 \text{ hours}} = \frac{1}{4} \end{aligned}$$

So, this sector is drawn $\frac{1}{4}$ th of the circle.

$$\begin{aligned} \left. \begin{array}{l} \text{The proportion of the sector} \\ \text{for hours spent in play} \end{array} \right\} &= \frac{\text{number of play hours}}{\text{Whole day}} \\ &= \frac{3 \text{ hours}}{24 \text{ hours}} = \frac{1}{8} \end{aligned}$$

So, this sector is drawn $\frac{1}{8}$ th of the circle.

$$\begin{aligned} \left. \begin{array}{l} \text{The proportion of the sector} \\ \text{for hours spent in homework} \end{array} \right\} &= \frac{\text{number of home work hours}}{\text{whole day}} \\ &= \frac{3 \text{ hours}}{24 \text{ hours}} = \frac{1}{8} \end{aligned}$$

So, this sector is drawn $\frac{1}{8}$ th of the circle.

$$\begin{aligned} \left. \begin{array}{l} \text{The proportion of the sector} \\ \text{for hours spent in others} \end{array} \right\} &= \frac{\text{number of others hours}}{\text{whole day}} \\ &= \frac{4 \text{ hours}}{24 \text{ hours}} = \frac{1}{6} \end{aligned}$$

So, this sector is drawn $\frac{1}{6}$ th of the circle.

Adding the above fractions for all activities,

$$\begin{aligned} \text{We get the total} &= \frac{1}{3} + \frac{1}{4} + \frac{1}{8} + \frac{1}{8} + \frac{1}{6} \\ &= \frac{8 + 6 + 3 + 3 + 4}{24} = \frac{24}{24} = 1. \end{aligned}$$

The sum of all fractions is equal to one. Here the time spent by a school student during a day is represented using a circle and the whole area of the circle is taken as one. The different activities of the school student are represented in various sectors by calculating their proportion. This proportional part can also be calculated using the measure of angle. Since, the sum of the measures of all angles at the central point is 360° , we can represent each sector by using the measure of angle.

In the following example, we are going to illustrate how a pie chart can be constructed by using the measure of angle.

Example 3.9

The number of hours spent by a school student on various activities on a working day, is given below. Construct a pie chart using the angle measurement.

Activity	Sleep	School	Play	Homework	Others
Number of hours	8	6	3	3	4

Solution

Number of hours spent in different activities in a day of 24 hours are converted into component parts of 360° . Since the duration of sleep is 8 hours, it should be represented by $\frac{8}{24} \times 360^\circ = 120^\circ$.

Therefore, the sector of the circle representing sleep hours should have a central angle of 120° .

Similarly, the sector representing other activities such as school, play, homework, and others are calculated in the same manner in terms of degree, which is given in the following table :

Activity	Duration in hours	Central angle
Sleep	8	$\frac{8}{24} \times 360^\circ = 120^\circ$
School	6	$\frac{6}{24} \times 360^\circ = 90^\circ$
Play	3	$\frac{3}{24} \times 360^\circ = 45^\circ$
Homework	3	$\frac{3}{24} \times 360^\circ = 45^\circ$
Others	4	$\frac{4}{24} \times 360^\circ = 60^\circ$
Total	24	360°

Drawing pie chart

Now we draw a circle of any convenient radius. In that circle, start with any radius and measure 120° at the centre of the circle and mark the second arm of this angle. This sector represents the hours spent in sleeping.

From this arm now mark off a second sector, by measuring an angle of 90° in the same sense as before. This sector represents the school hours. Proceeding like this, we construct the sectors for play and home work. The remaining sector will represent the last class (i.e. others).

The sectors may be shaded or coloured differently to distinguish one from the other. The completed pie chart is represented in the figure as shown above.

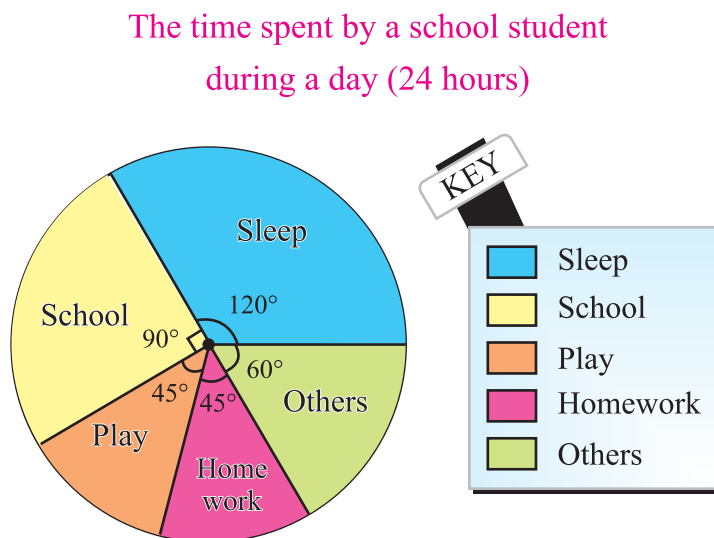


Fig. 3.9

Note: In a pie chart, the various observations or components are represented by the sectors of a circle and the whole circle represents the sum of the value of all the components. Clearly, the total angle of 360° at the centre of the circle is divided according to the values of the components.

$$\text{The central angle of a component} = \left[\frac{\text{Value of the component}}{\text{Total value}} \times 360^\circ \right]$$

Sometimes, the value of the components are expressed in percentages. In such cases, the central angle of a component = $\left[\frac{\text{Percentage value of the component}}{100} \times 360^\circ \right]$.

Steps for construction of pie chart of a given data:

1. Calculate the central angle for each component, using the above formula.
2. Draw a circle of convenient radius.
3. Within this circle, draw a horizontal radius.
4. Draw radius making central angle of first component with horizontal radius; this sector represents the first component. From this radius, draw next radius with central angle of second component; this sector represents second component and so on, until we exhaust all components.
5. Shade each sector differently and mark the component it represents.
6. Give a key.
7. Give the heading.

Thus, we obtain the required pie chart for the given data.

Example 3.10

The following table shows the monthly budget of a family

Particulars	Food	House Rent	Clothing	Education	Savings	Miscellaneous
Expenses (in ₹)	4800	2400	1600	800	1000	1400

Draw a pie chart using the angle measurement.

Solution

The central angle for various components may be calculated as follows:

Particulars	Expenses (₹)	Central angle
Food	4800	$\frac{4800}{12000} \times 360^\circ = 144^\circ$
House rent	2400	$\frac{2400}{12000} \times 360^\circ = 72^\circ$
Clothing	1600	$\frac{1600}{12000} \times 360^\circ = 48^\circ$
Education	800	$\frac{800}{12000} \times 360^\circ = 24^\circ$
Savings	1000	$\frac{1000}{12000} \times 360^\circ = 30^\circ$
Miscellaneous	1400	$\frac{1400}{12000} \times 360^\circ = 42^\circ$
Total	12000	360°

Clearly, we obtain the required pie chart as shown below.

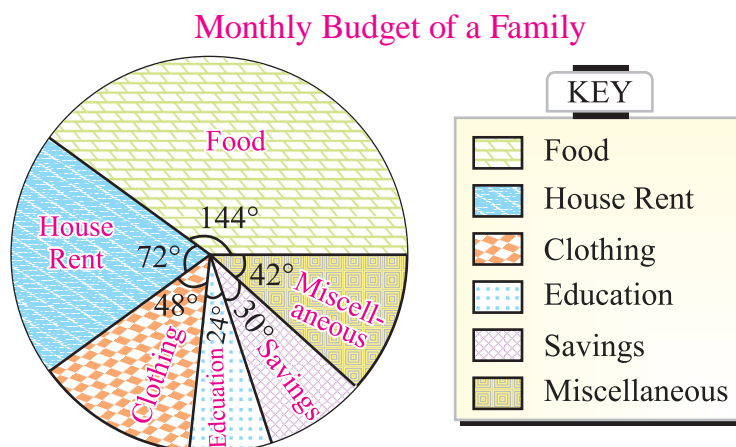


Fig. 3.10

Example 3.11

The S.S.L.C Public Examination result of a school is as follows:

Result	Passed in first class	Passed in second class	Passed in third class	Failed
Percentage of students	25%	35%	30%	10%

Draw a pie chart to represent the above information.

Solution

$$\text{Central angle for a component} = \frac{\text{Percentage value of the component}}{100} \times 360^\circ$$

We may calculate the central angles for various components as follows:

Result	Percentage of students	Central angle
Passed in first class	25%	$\frac{25}{100} \times 360^\circ = 90^\circ$
Passed in second class	35%	$\frac{35}{100} \times 360^\circ = 126^\circ$
Passed in third class	30%	$\frac{30}{100} \times 360^\circ = 108^\circ$
Failed	10%	$\frac{10}{100} \times 360^\circ = 36^\circ$
Total	100%	360°

Clearly, we obtain the required pie chart as shown below:

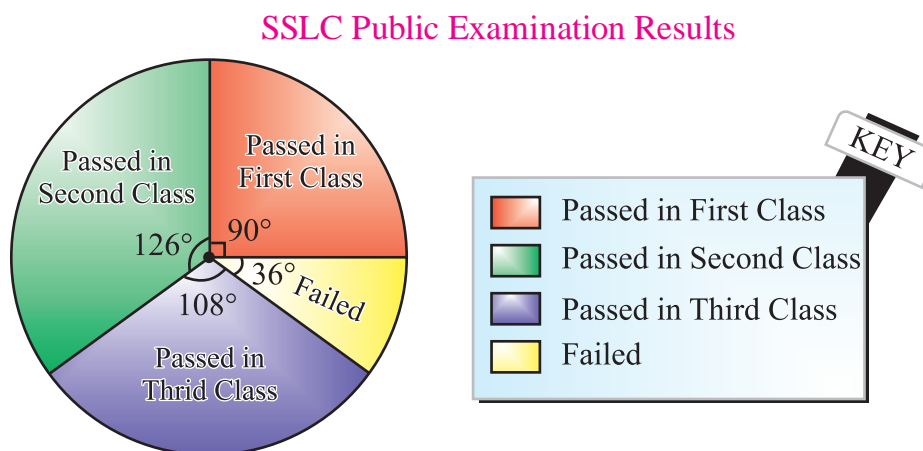


Fig. 3.11

EXERCISE 3.2

1. Yugendran's progress report card shows his marks as follows:

Subject	Tamil	English	Mathematics	Science	Social science
Marks	72	60	84	70	74

Draw a pie chart exhibiting his mark in various subjects.

2. There are 36 students in class VIII. They are members of different clubs:

Clubs	Mathematics	N.C.C	J.R.C	Scout
Number of students	12	6	10	8

Represent the data by means of a pie chart.

3. The number of students in a hostel speaking different languages is given below:

Languages	Tamil	Telugu	Malayalam	Kannada	English	Others
Number of students	36	12	9	6	5	4

Represent the data in a pie chart.

4. In a school, the number of students interested in taking part in various hobbies from class VIII is given below. Draw a pie chart.

Hobby	Music	Pottery	Dance	Drama	Social service
Number of students	20	25	27	28	20

5. A metal alloy contains the following metals. Represent the data by a pie chart.

Metal	Gold	Lead	Silver	Copper	Zinc
Weights (gm)	60	100	80	150	60

6. On a particular day, the sales (in ₹) of different items of a baker's shop are given below. Draw a pie chart for this data.

Item	Ordinary Bread	Fruit Bread	Cakes	Biscuits	Others
Cost (₹)	320	80	160	120	40

7. The money spent on a book by a publisher is given below:

Item	Paper	Printing	Binding	Publicity	Royalty
Money spent (₹)	25	12	6	9	8

Represent the above data by a pie chart.

8. Expenditure of a farmer for cultivation is given as follows:

Item	Ploughing	Fertilizer	Seeds	Pesticides	Irrigation
Amount (₹)	2000	1600	1500	1000	1100

Represent the data in a pie chart.

9. There are 900 creatures in a zoo as per list below:

Creatures	Wild animal	Birds	Other land animals	Water animals	Reptiles
Number of Creatures	400	120	135	170	75

Represent the above data by a pie chart.

10. In a factory, five varieties of vehicles were manufactured in a year, whose break up is given below. Represent this data with the help of a pie chart.

Vehicle	Scooter	Motorbike	Car	Jeep	Van
Number	3000	4000	1500	1000	500

11. A food contains the following nutrients. Draw a pie chart representing the data.

Nutrients	Protein	Fat	Carbohydrates	Vitamins	Minerals
Percentage	30%	10%	40%	15%	5%

12. The favorite flavours of ice cream for students of a school is given in percentages as follows

Flavours	Chocolate	Vanilla	Strawberry	Other flavours
Percentage of Students preferring the flavours	40%	30%	20%	10%

Represent this data by a pie chart.

13. The data on modes of transport used by students to come to school are given below:

Mode of transport	Bus	Cycle	Walking	Scooter	Car
Percentage of students	40%	30%	15%	10%	5%

Represent the data with the help of a pie chart.

14. Mr. Rajan Babu spends 20% of his income on house rent, 30% on food and 10% for his children's education. He saves 25%, while the remaining is used for other expenses. Make a pie chart exhibiting the above information.

15. The percentage of various categories of workers in a state are given in following table

Category of workers	Cultivators	Agricultural labours	Industrial Workers	Commercial Workers	Others
Percentage	40%	25%	12.5%	10%	12.5%

Represent the information given above in the form of a pie chart.

3.5 Measures of Central Tendency

Even after tabulating the collected mass of data, we get only a hazy general picture of the distribution. To obtain a more clear picture, it would be ideal if we can describe the whole mass of data by a single number or representative number. To get more information about the tendency of the data to deviate about a particular value, there are certain measures which characterise the entire data. These measures are called the **Measures of Central Tendency**. Such measures are

- (i) Arithmetic Mean, (ii) Median and (iii) Mode

3.5.1 Arithmetic Mean (A.M)

The arithmetic mean is the ratio of the sum of all the observations to the total number of observations.

3.5.1. (a) Arithmetic mean for ungrouped data

If there are n observations $x_1, x_2, x_3, \dots, x_n$ for the variable x then their arithmetic mean is denoted by \bar{x} and it is given by $\bar{x} = \frac{x_1 + x_2 + x_3 + \dots + x_n}{n}$.

In Mathematics, the symbol in Greek letter Σ , is called **Sigma**. This notation is used to represent the summation. With this symbol, the sum of $x_1, x_2, x_3, \dots, x_n$ is denoted as $\sum_{i=1}^n x_i$ or simply as Σx_i . Then we have $\bar{x} = \frac{\Sigma x_i}{n}$.

Note: Arithmetic mean is also known as **Average** or **Mean**.

More about Notation : Σ

$$\sum_{k=1}^3 k = 1 + 2 + 3 = 6$$

$$\sum_{n=3}^6 n = 3 + 4 + 5 + 6 = 18$$

$$\sum_{n=2}^4 2n = 2 \times 2 + 2 \times 3 + 2 \times 4 = 18$$

$$\begin{aligned} \sum_{k=1}^3 5 &= \sum_{k=1}^3 5 \times k^0 \\ &= 5 \times 1^0 + 5 \times 2^0 + 5 \times 3^0 \\ &= 5 + 5 + 5 = 15 \end{aligned}$$

$$\sum_{k=2}^4 (k-1) = (2-1) + (3-1) + (4-1) = 6$$

Example 3.12

The marks obtained by 10 students in a test are 15, 75, 33, 67, 76, 54, 39, 12, 78, 11. Find the arithmetic mean.

Solution

Here, the number of observations, $n = 10$

$$\begin{aligned} \text{A. M} = \bar{x} &= \frac{15 + 75 + 33 + 67 + 76 + 54 + 39 + 12 + 78 + 11}{10} \\ \bar{x} &= \frac{460}{10} = 46. \end{aligned}$$

Example 3.13

If the average of the values 9, 6, 7, 8, 5 and x is 8. Find the value of x .

Solution

Here, the given values are 9, 6, 7, 8, 5 and x , also $n = 6$.

$$\text{By formula, A.M.} = \bar{x} = \frac{9 + 6 + 7 + 8 + 5 + x}{6} = \frac{35 + x}{6}$$

$$\text{By data, } \bar{x} = 8$$

$$\text{So, } \frac{35 + x}{6} = 8$$

$$\text{i.e. } 35 + x = 48$$

$$x = 48 - 35 = 13.$$

Example 3.14

The average height of 10 students in a class was calculated as 166 cm. On verification it was found that one reading was wrongly recorded as 160 cm instead of 150 cm. Find the correct mean height.

Solution

$$\text{Here, } \bar{x} = 166 \text{ cm and } n = 10$$

$$\text{We have } \bar{x} = \frac{\Sigma x}{n} = \frac{\Sigma x}{10}$$

$$\text{i.e. } 166 = \frac{\Sigma x}{10} \text{ or } \Sigma x = 1660$$

$$\text{The incorrect } \Sigma x = 1660$$

$$\begin{aligned} \text{The correct } \Sigma x &= \text{incorrect } \Sigma x - \text{the wrong value} + \text{correct value} \\ &= 1660 - 160 + 150 = 1650 \end{aligned}$$

$$\text{Hence, the correct A.M.} = \frac{1650}{10} = 165 \text{ cm.}$$

3.5.1 (b) Arithmetic mean for grouped data

Arithmetic mean for grouped data can be obtained in two methods which are

(i) Direct Method and (ii) Assumed Mean Method

(i) To calculate Arithmetic Mean (Direct Method)

Suppose we have the following frequency distribution

Variable	x_1	x_2	x_3	\dots	x_n
Frequency	f_1	f_2	f_3	\dots	f_n

Then this table is to be interpreted in the following manner:

The value : x_1 occurs f_1 times

x_2 occurs f_2 times

x_3 occurs f_3 times

.....

.....

x_n occurs f_n times.

Here $x_1, x_2, x_3, \dots, x_n$ are the distinct values of the variable x .

In this case, the total number of observations is usually denoted by N .

$$\text{(i.e.,)} \quad f_1 + f_2 + f_3 + \dots + f_n = N \quad \text{(or)} \quad \sum_{i=1}^n f_i = N$$

Then the total values observed

$$\begin{aligned} &= (x_1 + x_1 + x_1 + \dots : f_1 \text{ times}) + (x_2 + x_2 + x_2 + \dots : f_2 \text{ times}) + \dots + (x_n + x_n + x_n + \dots : f_n \text{ times}) \\ &= f_1 \times x_1 + f_2 \times x_2 + \dots + f_n \times x_n = \sum f_i x_i \end{aligned}$$

$$\text{Hence, } \bar{x} = \frac{\text{Total values observed}}{\text{Total number of observations}} = \frac{\sum f_i x_i}{\sum f_i}$$

$$\text{Usually, it is written as } \bar{x} = \frac{\sum fx}{\sum f} = \frac{\sum fx}{N}, \quad \text{where } N = \sum f.$$

Example 3.15

Calculate the Arithmetic mean of the following data by direct method

x	5	10	15	20	25	30
f	4	5	7	4	3	2

Solution

x	f	fx
5	4	20
10	5	50
15	7	105
20	4	80
25	3	75
30	2	60
Total	$N = 25$	$\sum fx = 390$

$$\begin{aligned} \text{Arithmetic Mean, } \bar{x} &= \frac{\sum fx}{N} \\ &= \frac{390}{25} = 15.6. \end{aligned}$$

(ii) To calculate Arithmetic Mean (Assumed Mean Method)

In the above example multiplication looks very simple, since the numbers are small. When the numbers are huge, their multiplications are always tedious or uninteresting and leads to errors.

To overcome this difficulty another simpler method is devised. In this method we assume one of the values as mean (A). This assumed value A is known as assumed mean. Then we calculate the deviation $d_1, d_2, d_3, \dots, d_n$ of each of the variables $x_1, x_2, x_3, \dots, x_n$ from the assumed mean A.

$$\text{i.e. } d_1 = x_1 - A, d_2 = x_2 - A, d_3 = x_3 - A, \dots, d_n = x_n - A$$

Now, multiply $d_1, d_2, d_3, \dots, d_n$ respectively by $f_1, f_2, f_3, \dots, f_n$ and add all these values to get Σfd . Now,

$$\text{Arithmetic mean } \bar{x} = A + \frac{\Sigma fd}{\Sigma f}$$

$$\bar{x} = A + \frac{\Sigma fd}{N} \quad (\text{Here } A \text{ is assumed mean and } N = \Sigma f)$$

Now, we can calculate the A.M. for the above problem (example 8.15) by assumed mean method.

Take the assumed mean $A = 15$

x	f	$d = x - A$	fd
5	4	-10	-40
10	5	-5	-25
15	7	0	0
20	4	5	20
25	3	10	30
30	2	15	30
Total	N = 25		$\Sigma fd = 15$

$$\begin{aligned} \text{Arithmetic Mean} = \bar{x} &= A + \frac{\Sigma fd}{N} \\ &= 15 + \frac{15}{25} = 15 + \frac{3}{5} = \frac{75 + 3}{5} = \frac{78}{5} \\ &= 15.6. \end{aligned}$$

3.5.2 Weighted Arithmetic Mean (W.A.M.)

Sometimes the variables are associated with various weights and in those cases the A.M. can be calculated, such an arithmetic mean is known as **Weighted Arithmetic Mean (W.A.M.)**.

For example, let us assume that the variable x_1 is associated with the weight w_1 , x_2 is associated with the weight w_2 etc. and finally, x_n is associated with the weight w_n then

$$\text{W. A. M.} = \frac{w_1 x_1 + w_2 x_2 + w_3 x_3 + \dots + w_n x_n}{w_1 + w_2 + w_3 + \dots + w_n} = \frac{\Sigma wx}{\Sigma w}$$

Example 3.16

Find the weighted A. M of the price for the following data:

Food stuff	Quantity (in kg) w_i	Price per kg (in ₹) x_i
Rice	25	30
Sugar	12	30
Oil	8	70

Solution

Here the x -values are the price of the given food stuff and the weights associated are the quantities (in Kg)

$$\begin{aligned} \text{Then, the W.A.M} &= \frac{w_1x_1 + w_2x_2 + w_3x_3 + \cdots + w_nx_n}{w_1 + w_2 + w_3 + \cdots + w_n} \\ &= \frac{25 \times 30 + 12 \times 30 + 8 \times 70}{25 + 12 + 8} = \frac{1670}{45} \\ &= ₹ 37.11 . \end{aligned}$$

3.5.3 Median

Another measure of central tendency is the **Median**.

3.5.3 (a) To find Median for ungrouped data

The median is calculated as follows:

- (i) Suppose there are an **odd number** of observations, write them in ascending or descending order. Then the middle term is the Median.

For example: Consider the five observations 33, 35, 39, 40, 43. The middle most value of these observation is 39. It is the Median of these observation.

- (ii) Suppose there are an **even number** of observations, write them in ascending or descending order. Then the average of the two middle terms is the Median.

For example, the median of 33, 35, 39, 40, 43, 48 is $\frac{39 + 40}{2} = 39.5$.

Note: The Median is that value of the variable which is such that there are as many observations above and below it.

Example 3.17

Find the median of 17, 15, 9, 13, 21, 7, 32.

Solution

Arrange the values in the ascending order as 7, 9, 13, 15, 17, 21, 32,

Here, $n = 7$ (odd number)

Therefore, Median = Middle value

$$= \left(\frac{n+1}{2}\right)^{\text{th}} \text{ value} = \left(\frac{7+1}{2}\right)^{\text{th}} \text{ value} = 4^{\text{th}} \text{ value.}$$

Hence, the median is 15.

Example 3.18

A cricket player has taken the runs 13, 28, 61, 70, 4, 11, 33, 0, 71, 92. Find the median.

Solution

Arrange the runs in ascending order as 0, 4, 11, 13, 28, 33, 61, 70, 71, 92.

Here $n = 10$ (even number).

There are two middle values 28 and 33.

$$\begin{aligned} \therefore \text{Median} &= \text{Average of the two middle values} \\ &= \frac{28 + 33}{2} = \frac{61}{2} = 30.5 . \end{aligned}$$

3.5.3 (b) To find Median for grouped data**Cumulative frequency**

Cumulative frequency of a class is nothing but the total frequency upto that class.

Example 3.19

Find the median for marks of 50 students

Marks	20	27	34	43	58	65	89
Number of students	2	4	6	11	12	8	7

Solution

Marks (x)	Number of students (f)	Cumulative frequency
20	2	2
27	4	(2 + 4 =) 6
34	6	(6 + 6 =) 12
43	11	(11 + 12 =) 23
58	12	(23 + 12 =) 35
65	8	(35 + 8 =) 43
89	7	(43 + 7 =) 50

Here, the total frequency, $N = \Sigma f = 50$

$$\therefore \frac{N}{2} = \frac{50}{2} = 25.$$

The median is $\left(\frac{N}{2}\right)^{\text{th}}$ value = 25th value.

Now, **25th value occurs in the cumulative frequency 35, whose corresponding marks is 58.**

Hence, the median = 58.

3.5.4 Mode

Mode is also a measure of central tendency.

The Mode can be calculated as follows:

3.5.4 (a) To find Mode for ungrouped data (Discrete data)

If a set of individual observations are given, then the Mode is the value which occurs most often.

Example 3.20

Find the mode of 2, 4, 5, 2, 1, 2, 3, 4, 4, 6, 2.

Solution

In the above example the number 2 occurs maximum number of times. ie, 4 times. Hence mode = 2.

Example 3.21

Find the mode of 22, 25, 21, 22, 29, 25, 34, 37, 30, 22, 29, 25.

Solution

Here 22 occurs 3 times and 25 also occurs 3 times

\therefore Both 22 and 25 are the modes for this data. We observe that there are two modes for the given data.

Example 3.22

Find the mode of 15, 25, 35, 45, 55, 65,

Solution

Each value occurs exactly one time in the series. Hence there is no mode for this data.

3.5.4 (b) To find Mode for grouped data (Frequency distribution)

If the data are arranged in the form of a frequency table, the class corresponding to the maximum frequency is called the modal class. The value of the variate of the modal class is the **mode**.

Example: 3.23

Find the mode for the following frequency table

Wages (₹)	250	300	350	400	450	500
Number of workers	10	15	16	12	11	13

Solution

Wages (₹)	Number of workers
250	10
300	15
350	16
400	12
450	11
500	13

We observe from the above table that the maximum frequency is 16. **The value of the variate (wage) corresponding to the maximum frequency 16 is 350.** This is the mode of the given data.

**Do you know?**

Unimodal	Bimodal	Trimodal	Multimodal
If there is only one mode in a given series, then it is called Unimodal .	If there are two modes in a given series, then it is called Bimodal .	If there are three modes in a given series, then it is called Trimodal .	If there are more than three modes in the series it is called Multimodal .
Example : 10, 15, 20, 25, 15, 18, 12, 15. Here, Mode is 15.	Example: 20, 25, 30, 30, 15, 10, 25. Here 25, 30 are Bimodal.	Example: 60, 40, 85, 30, 85, 45, 80, 80, 55, 50, 60. Here 60, 80, 85 are Trimodal.	Example: 1, 2, 3, 8, 5, 4, 5, 4, 2, 3, 1, 3, 5, 2, 7, 4, 1. Here 1, 2, 3, 4, 5 are Multimodal.

EXERCISE 3.3**I. Problems on Arithmetic Mean**

- Find the mean of 2, 4, 6, 8, 10, 12, 14, 16.
- If the average of the values 18, 41, x , 36, 31, 24, 37, 35, 27, 36, is 31. Find the value of x .
- If in a class of 20 students, 5 students have scored 76 marks, 7 students have scored 77 marks and 8 students have scored 78 marks, then compute the mean of the class.
- The average height of 20 students in a class was calculated as 160 cm. On verification it was found that one reading was wrongly recorded as 132 cm instead of 152 cm. Find the correct mean height.

5. Calculate the Arithmetic mean of the following data:

x	15	25	35	45	55	65	75	85
f	12	20	15	14	16	11	7	8

6. The following data give the number of boys of a particular age in a class of 40 students. Calculate the mean age of the students

Age (in years)	13	14	15	16	17	18
Number of students	3	8	9	11	6	3

7. Obtain the A.M of the following data:

Marks	65	70	75	80	85	90	95	100
Number of students	6	11	3	5	4	7	10	4

8. The following table shows the weights of 12 workers in a factory

Weight (in Kg)	60	64	68	70	72
Number of workers	3	4	2	2	1

Find the mean weight of the workers.

9. For a month, a family requires the commodities listed in the table below. The weights to each commodity is given. Find the Weighted Arithmetic Mean.

Commodity	Weights (in kg)	Price per kg (in ₹)
Rice	25	30
Wheat	5	20
Pulses	4	60
Vegetables	8	25
Oil	3	65

10. Find the Weighted Arithmetic Mean for the following data:

Item	Number of Item	Cost of Item
Powder	2	₹ 45
Soap	4	₹ 12
Pen	5	₹ 15
Instruments box	4	₹ 25.50

II. Problems on Median

1. Find the median of the following set of values:

(i) 83, 66, 86, 30, 81.

(ii) 45, 49, 46, 44, 38, 37, 55, 51.

(iii) 70, 71, 70, 68, 67, 69, 70.

(iv) 51, 55, 46, 47, 53, 55, 51, 46.

2. Find the median for the following data:

x	1	2	3	4	5	6	7	8
f	9	11	5	6	8	1	3	7

3. The height (in cm) of 50 students in a particular class are given below:

Height (in cm)	156	155	154	153	152	151	150
Number of students	8	4	6	10	12	3	7

Find the median.

4. The hearts of 60 patients were examined through X-ray and the observations obtained are given below:

Diameter of heart (in mm)	130	131	132	133	134	135
Number of patients	7	9	15	12	6	11

Find the median.

5. The salary of 43 employees are given in the following table. Find the median.

Salary (in ₹)	4000	5500	6000	8250	10,000	17,000	25,000
Number of employees	7	5	4	3	13	8	3

III. Problems on Mode

1. Find the mode of the following data:

i) 74, 81, 62, 58, 77, 74.

iii) 43, 36, 27, 25, 36, 66, 20, 25.

ii) 55, 51, 62, 71, 50, 32.

vi) 24, 20, 27, 32, 20, 28, 20.

2. Find the mode for the following frequency table:

x	5	10	15	20	25	30
f	14	25	37	16	8	5

3. Find the mode for the following table:

Temperature in °c	29	32.4	34.6	36.9	38.7	40
Number of days	7	2	6	4	8	3

4. The demand of different shirt sizes, as obtained by a survey, is given below.

Size	38	39	40	41	42	43	44
Number of persons (wearing it)	27	40	51	16	14	8	6

Calculate the Mode.

IV. Problems on Mean, Median and Mode

1. Find the mean, median and mode for the following frequency table:

x	10	20	25	30	37	55
f	5	12	14	15	10	4

2. The age of the employees of a company are given below.

Age (in years)	19	21	23	25	27	29	31
Number of persons	13	15	20	18	16	17	13

Find the mean, median and mode.

3. The following table shows the weights of 20 students.

Weight (in kg)	47	50	53	56	60
Number of students	4	3	7	2	4

Calculate the mean, median and mode.



- ↪ Histogram and frequency polygon are the two types of graphical representations of a frequency distribution.
- ↪ In the graphical representation of histogram and frequency polygon the class-intervals are taken along the X-axis and the corresponding frequencies are taken along the Y-axis.
- ↪ In a histogram the rectangles are placed continuously side by side with no gap between the adjacent rectangles.
- ↪ The polygon obtained by joining the mid points of the top of the adjacent rectangles of a histogram and also the midpoints of the preceding and succeeding class intervals is known as a frequency polygon.
- ↪ The central angle of a component = $\left[\frac{\text{Value of the component}}{\text{Total value}} \times 360^\circ \right]$
- ↪ Arithmetic mean is the ratio of the sum of all the observations to the total number of observations.
- ↪ Formula for finding A.M.
 - (i) $\bar{x} = \frac{\sum x}{n}$ (ii) $\bar{x} = \frac{\sum fx}{\sum f}$
 - (iii) $\bar{x} = A + \frac{\sum fd}{\sum f}$ when A is the assumed mean and $d = x - A$.
- ↪ The weighted Arithmetic mean (W.A.M.) = $\frac{\sum w_i x_i}{\sum w_i}$.
- ↪ The median is that value of the variable which is such that there are as many observations above and below it.
- ↪ Mode is that value which occurs most frequently in a distribution.

4

Practical Geometry



Brahmagupta

[598 - 670 A.D.]

The great 7th century Indian mathematician of Rajasthan Brahmagupta wrote several books on Mathematics and astronomy. He became the head of astronomical observatory at Ujjain. He wrote the famous book “Brahmasphuta-siddhanta”.

He found that $\pi \simeq \sqrt{10}$ and the sum of the squares of the “n” natural numbers =
$$\frac{n(n+1)(2n+1)}{6}$$

4.1 Introduction



4.2 Concentric Circles

4.1 Introduction

Ancient Egyptians demonstrated practical knowledge of geometry through surveying and construction of projects. Ancient Greeks practised experimental geometry in their culture. They have performed variety of constructions using ruler and compass.

Geometry is one of the earliest branches of Mathematics. Geometry can be broadly classified into Theoretical Geometry and Practical Geometry. Theoretical Geometry deals with the principles of geometry by explaining the construction of figures using rough sketches. Practical Geometry deals with constructing of exact figures using geometrical instruments.

We have already learnt in the previous classes, the definition, properties and formulae for the area of some plane geometrical figures. In this chapter let us learn to draw concentric circles,

4.2 Concentric Circles

In this section, we are going to learn about Concentric Circles. Already we are familiar with Circles.

4.2.1. Motivation

When a small stone was dropped in still water, you might have seen circular ripples were formed. Which is the centre of these circles? Is it not the place where the stone was dropped? **Yes.**

The circles with different measures of radii and with the same centre are called **concentric circles**. The centre is known as common centre.

The Concentric Circles

Circles drawn in a plane with a common centre and different radii are called **concentric circles**. See Fig. 4.1 and 4.2.

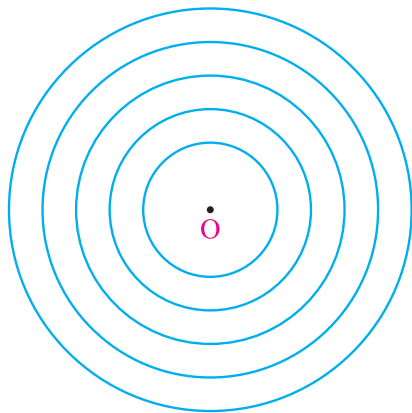


Fig. 4.1

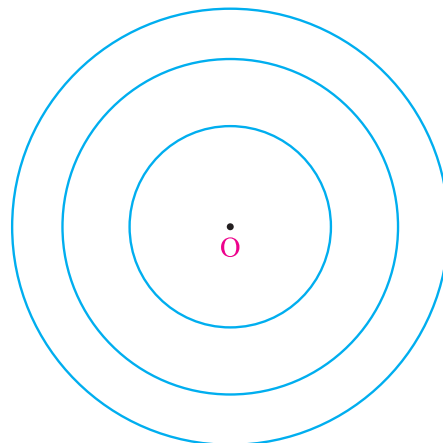


Fig. 4.2

Look at the following two figures:

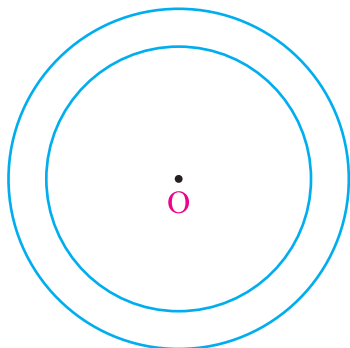


Fig. 4.3

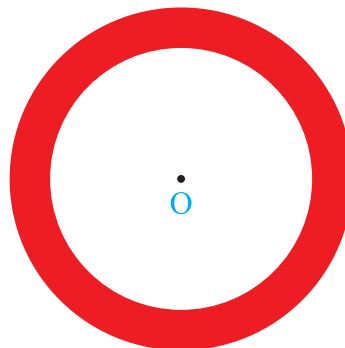


Fig. 4.4

Fig. 4.3 represents two concentric circles.

In Fig. 4.4 the area between the two concentric circles are shaded with colour. The coloured area is known as **circular ring**.

Description : Circular Ring

In Fig. 4.5, C_1 and C_2 are two circles having the same centre O with different radii r_1 and r_2 respectively.

Circles C_1 and C_2 are called concentric circles.

The area bounded between the two circles is known as circular ring.

Width of the circular ring = $OB - OA = r_2 - r_1$

$$(r_2 > r_1)$$

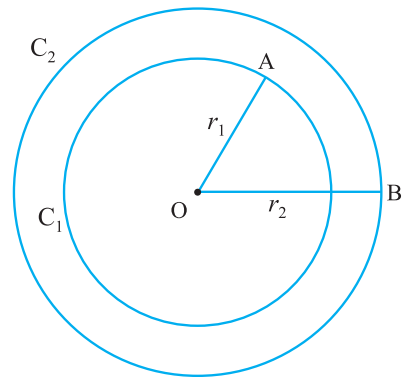


Fig. 4.5

4.2.2. Construction of concentric circles when the radii are given.**Example 4.1**

Draw concentric circles with radii 3 cm and 5 cm and shade the circular ring. Find its width.

Solution

Given: The radii are 3 cm and 5 cm.

Rough Diagram

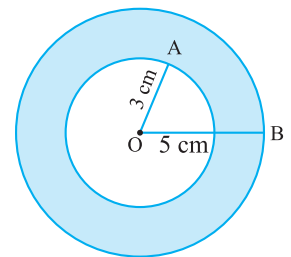


Fig. 4.6

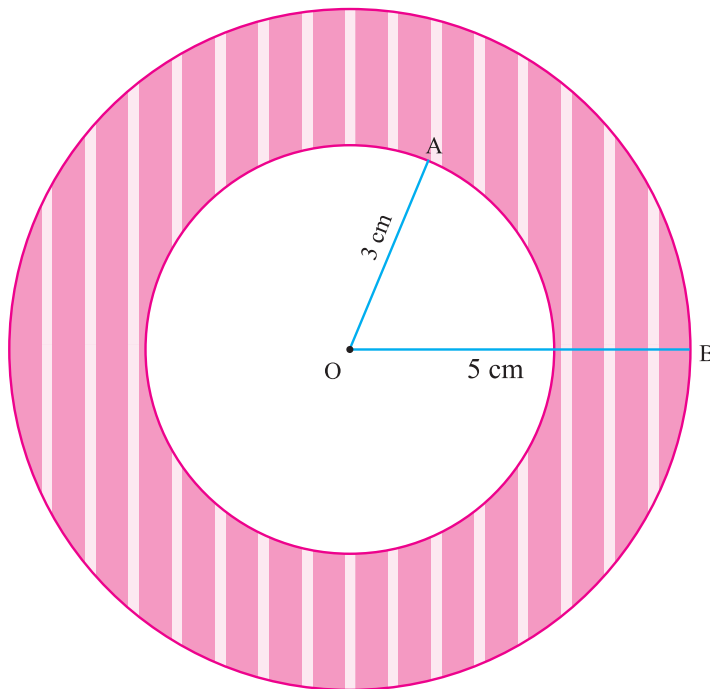
To construct concentric circles

Fig. 4.7

Steps for construction

Step 1 : Draw a rough diagram and mark the given measurements.

Step 2 : Take any point O and mark it as the centre.

Step 3 : With O as centre and draw a circle of radius $OA = 3$ cm

Step 4 : With O as centre and draw a circle of radius $OB = 5$ cm.

Thus the concentric circles C_1 and C_2 are drawn.

$$\begin{aligned}\text{Width of the circular ring} &= OB - OA \\ &= 5 - 3 \\ &= 2 \text{ cm.}\end{aligned}$$

EXERCISE 4.1

1. Draw concentric circles for the following measurements of radii. Find out the width of each circular ring.

- (i) 4 cm and 6 cm.
- (ii) 3.5 cm and 5.5 cm.
- (iii) 4.2 cm and 6.8 cm.
- (iv) 5 cm and 6.5 cm.
- (v) 6.2 cm and 8.1 cm.
- (vi) 5.3 cm and 7 cm.



- ✦ Circles drawn in a plane with a common centre and different radii are called concentric circles.
- ✦ The area bounded between two concentric circles is known as circular ring.
- ✦ Width of the circular ring $= r_2 - r_1$; $(r_2 > r_1)$

ANSWERS

Chapter 1

Exercise 1.1

1. i) D ii) C iii) B iv) B v) A
2. i) 200 litres ii) 20,000 km iii) ₹ 1,550
iv) 50 minutes v) ₹ 50
3. ₹ 40,000 4. 5000, 3750
5. i) 90% ii) 94% iii) 98% iv) 88% v) 95% vi) 93%
6. 5 7. ₹ 9,000 8. ₹ 1,020 9. 180, 1320 10. 6 kgs.
11. i) 26,100 ii) 5,220 12. 25%, ₹ 8,600
13. She scored better in maths by 20% 14. ₹ 6,250 15. 20%

Exercise 1.2

1. i) ₹ 7490 ii) ₹ 500 iii) ₹ 9,000 iv) ₹ 2,246 v) ₹ 6,57,500
2. i) Profit ₹ 64, Profit % = 20% ii) Profit ₹ 200, Profit % = 8%
iii) Loss ₹ 19, Loss % = 5% iv) S.P. = ₹ 38, Loss % = 5%
v) S.P. = ₹ 5,500, Profit % = 10%.
3. i) ₹ 787.50 ii) ₹ 1,260 iii) ₹ 2,835
4. ₹ 1,200 5. $33\frac{1}{3}\%$ 6. 25%
7. ₹ 22,80,000 8. ₹ 34,40,000
9. $11\frac{1}{9}\%$ 10. Overall gain ₹ 113.68

Exercise 1.3

1. i) A ii) D iii) B iv) B v) C 2. ₹ 360
3. ₹ 8,000 4. ₹ 49,220 5. ₹ 18,433.40 6. ₹ 4,950
7. ₹ 13,000 8. 33% 9. ₹ 9,832.50 10. 20% 11. ₹ 1,310.40 per shirt
12. i) Amount of discount = ₹ 460; S.P. = ₹ 1,840
ii) Amount of discount = ₹ 35; Rate of discount = 25%
iii) M.P. = ₹ 20,000; Amount of discount = ₹ 4,000
iv) Rate of discount = 5%; Amount of discount = ₹ 725
v) Amount of discount = ₹ 403; S.P. = ₹ 2,821

Exercise 1.4

1. i) $A = ₹ 1,157.63$, Interest = ₹ 157.63
 ii) $A = ₹ 4,840$, Interest = ₹ 840
 iii) $A = ₹ 22,869$, Interest = ₹ 4,869
2. ₹ 2,125 3. i) ₹ 88,200 ii) ₹ 4,410 4. $A = ₹ 27,783$, C.I. = ₹ 3,783
5. ₹ 9,826 6. C.I. = ₹ 1,951 7. ₹ 20,000 8. ₹ 36,659.70
9. i) ₹ 92,400 ii) ₹ 92,610, Difference = ₹ 210 10. ₹ 6
11. ₹ 25 12. ₹ 2,000 13. Suja pays more interest of ₹ 924.10
14. $P = ₹ 1,25,000$ 15. 2 years 16. 10%

Exercise 1.5

1. 2,205 2. ₹ 2,55,150 3. ₹ 46,000 4.5,31,616.25
5. 5,415 6. ₹ 20,000 7. 10,000

Exercise 1.6

1. ₹ 27,000 2. ₹ 86,250 3. ₹ 10,800
4. ₹ 1,250 5. E.M.I. = ₹ 700, Total Amount = ₹ 19,404
6. E.M.I. = ₹ 875, Total Amount = ₹ 8,750

Exercise 1.7

1. 24 days 2. 10; 1250 3. 10 composers 4. 15 Workers
5. 24 days 6. ₹ 192

Exercise 1.8

1. 3 days 2. 30 days 3. 2 days 4. 12 minutes
5. $A = ₹ 360$, $B = ₹ 240$ 6. 6 days 7. 1 hour

Chapter 2

Exercise 2.1

1.i)D ii) C iii) A iv) B

2.The shortest side is BC. 3. $QR = 26$ cm 4.It forms a right angled triangle5. $QR = 5$ cm 6. $x = 9$ m 7.Altitude $x = 5\sqrt{3}$ cm8.Yes 9. $2\sqrt{51}$ ft

Exercise 2.2

1. i) D ii) D iii) C 2. radius = 5 cm.

Chapter 3

Exercise 3.3

I. Problems on Arithmetic Mean

1. 9 2. $x = 25$ 3. 77.15 4. 161 cm 5. 45
 6. 15.45 7. 82.1 8. 65.33 9. ₹ 33 10. ₹ 21

II. Problems on Median

1. i) 81 ii) 45.5 iii) 70 iv) 51
 2. 3 3. 153 4. 132 5. ₹ 10,000

III. Problems on Mode

1. i) 74 ii) No mode iii) 25 and 36 iv) 20
 2. 15 3. 38.7°C 4. 40

IV. 1. Mean 28; Median 25; Mode 30

2. Mean 25; Median 25; Mode 23

3. Mean 53.05; Median 53; Mode 53