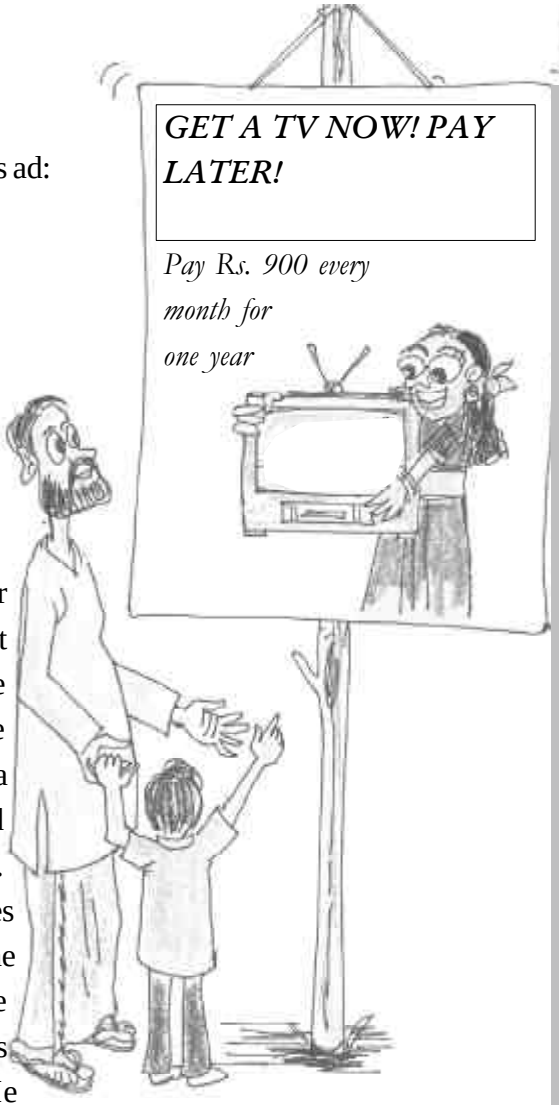
$$P\left(1+\frac{r}{100}\right)^n$$

4 Money Math

Look at this ad:



Raju's father was attracted by it and bought a TV from them.

But Babu's father decided to take out a loan from the local co-operative bank for buying a TV. They wanted details of the price. He made enquiries and found that the price of the advertised TV was 9000 rupees. He took out a loan from a bank for this amount at 12% interest and bought the TV.

How much money should Raju's father pay in one year?

Babu's father paid off the bank loan in one year. How much money did he pay the bank?

Who spent more?

Unseen traps

The colorful ads we see in the papers and TV often do not give full details and real figures. There can be a discussion in class on such hidden facts in ads.

Concession and penalty

Joseph and Thomas took out an agricultural loan of 20,000 rupees each from a bank at 10% rate of interest. The bank allows 4% concession on the interest, if the loan is paid back in two years. If the amount is not paid back within this period, they charge an additional 1% penal interest. Joseph paid all dues on the exact day of completing two years. But Thomas could do it only the next day.

How much did each pay back?

Let's first compute how much Joseph paid:

$$\text{Amount of loan} = \text{Rs. } 20{,}000$$

$$\text{Rate of interest} = 10\%$$

$$\text{Concession} = 4\%$$

$$\text{Actual rate of interest} = 6\%$$

$$\text{Interest} = 20000 \times \frac{6}{100} \times 2$$

$$= \text{Rs. } 2400$$

$$\text{Total amount paid} = 20000 + 2400$$

$$= \text{Rs. } 22400$$

What about Thomas?

$$\text{Amount of loan} = \text{Rs. } 20{,}000$$

$$\text{Rate of interest} = 10\%$$

Since he couldn't pay back in time, he doesn't get the concession on interest rate; and he has to pay penal interest as well. So, what is the rate of interest for him?

$$\text{Actual rate of interest} = 11\%$$

$$\text{Interest} = 20000 \times \frac{11}{100} \times 2 = \text{Rs. } 4400$$

$$\text{Total amount paid} = 20000 + 4400$$

$$= \text{Rs. } 24400$$

See how much more he had to pay, just for one day's delay!

Rules of the game

Thomas not only lost the 4% concession on interest rate but had to pay an additional 1% penal interest, so that he had to pay 11% interest in all. Discuss the possible reasons for Thomas's inability to pay back in time. Consider such things as the need to follow rules and the financial loss due to the ignorance of rules.

Banks and finance companies

Jayan and Binny each took out a loan of 15,000 rupees; Jayan from a private finance company which charges 5 rupees per month for 100 rupees as interest and Binny from a nationalized bank charging 12% compound interest. How much should each pay back after two years?

What is the annual rate of interest in the case of the financial company?

So, what is the total amount Jayan has to repay after two years?

In the case of the bank, interest is compounded.

Principal for the first year = Rs. 15000

Interest for the first year = ...

Principal for the second year = ...

Interest for the second year = ...

Amount to be repaid = ...

What is the difference in the amounts Jayan and Binny has to repay?

A computational trick

Let's see how we can compute the total amount including interest, using algebra. We denote the principal (amount invested or loan taken out) by p, the rate of compound interest by r% and compute the total amount including interest to be paid back after n years. (see the lesson, *Money for Money!* in the Class 7 textbook.)

Principal for the first year = p

Interest for the first year = $p \times \frac{r}{100}$

Principal for the second year = $p + (p \times \frac{r}{100})$

$= p\left(1+\frac{r}{100}\right)$

Interest for the second year $= p\left(1+\frac{r}{100}\right)\times\frac{r}{100}$

Principal for the third year

$$= p\,(1+\frac{r}{100}) + p\left(1+\frac{r}{100}\right)\times\frac{r}{100}$$

$$= p\left(1+\frac{r}{100}\right)\left(1+\frac{r}{100}\right)$$

$$= p\left(1+\frac{r}{100}\right)^2$$

Continuing like this, the total amount A, including interest after n years is given by

$$A = p\left(1+\frac{r}{100}\right)^n$$

What if it is simple interest, instead of compound interest?

$$A = p + \frac{prn}{100} = p\left(1+\frac{rn}{100}\right)$$

- Usha deposited 8000 rupees in a bank, which gives compound interest at 7%. How much would she get back after 3 years?

  Let's compute using our formula: here the principal is 3000 rupees, rate of interest is 7% and the number of years is 3.

  Total amount including interest

$$= 8000\times\left(1+\frac{7}{100}\right)^3$$

$$= 8000\times\left(\frac{107}{100}\right)^3$$

$$= \frac{8\times107^3}{1000}$$

Now do the computation on your own and find the actual amount.

Money math

How much would 8000 rupees become after 3 years, if we add compound interest at 7%? According to math, it is $8000\times\left(\frac{107}{100}\right)^3$, which on computation gives 9800.344.

In financial transactions, amounts less than 0.5 rupee (that is, 50 paise) are ignored and amounts greater than or equal to 0.5 rupee is rounded to 1 rupee. So, in our problem, the total amount would be 9800 rupees.

Let's find out

- Mahadevan took out a loan of 5,000 rupees from a bank, which charges compound interest. The rate of interest was 10% for the first year, but it was increased to 12% during the second year. How much money should Mahadevan pay at the end of two years to settle the debt?
- Compute the compound interest for 25,000 rupees at 9%, for 2 years.
- Verghese borrowed 16,000 rupees from a bank at 8% compound interest. After 2 years, the interest rate was lowered to 6%. What is the total amount that Verghese has to repay at the end of 3 years?

Changing times

Ravi took out an agricultural loan of 12,500 rupees from the co-operative bank. They charge 8% interest. How much should Ravi pay back after 6 months?

Amount of loan = Rs. 12,500

Rate of interest = 8%

Period = 6 months

$= \frac{6}{12}$ year

Interest $= 12{,}500 \times \frac{8}{100} \times \frac{6}{12}$

= .....

Amount to be repaid = ...

Raji borrowed 12,500 rupees for 3 months from the same bank. How much should she pay back at the end of this period?

Amount of loan = Rs. 12,500

Rate of interest = 8%

Period = 3 months

$= \frac{3}{12}$ year $= \frac{1}{4}$ year

$$\text{Interest} = 12{,}500 \times \frac{8}{100} \times \frac{1}{4}$$

Amount to be repaid = ...

For deposits and loans, sometimes interest is calculated every 6 months (half-yearly) and added to the principal; and sometimes interest is calculated every 3 months (quarterly) and added to the principal.

Ramu deposited 15,000 rupees in a bank, which gives 12% interest compounded half-yearly. How much would he get back after one year?

Principal for the first half-year = Rs. .....

$$\text{Interest for the first half-year} = 15{,}000 \times \frac{12}{100} \times \frac{1}{2}$$

= .....

Principal for the second half-year = ... + ...

= Rs. .....

$$\text{Interest for the second half-year} = 15{,}900 \times \frac{12}{100} \times \frac{1}{2}$$

= .....

Amount got back after one year = ... + ...

= .....

Suppose that Ramu had deposited in a bank which gives interest at the same rate, but compounded quarterly. What is the amount he would have got back after one year?

Principal for the first quarter = Rs. .....

$$\text{Interest for the first quarter} = 15{,}000 \times \frac{12}{100} \times \frac{1}{4}$$

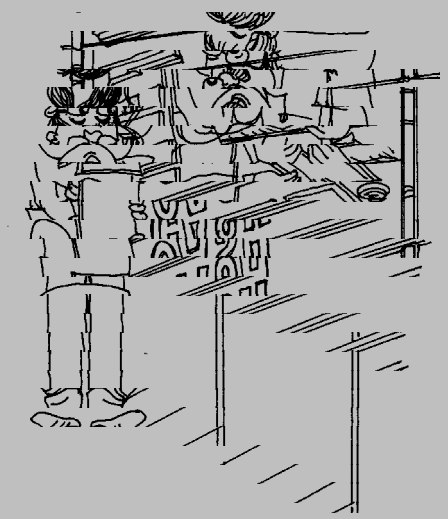
= Rs. .....

Principal for the second quarter = ... + ...

= Rs. 15,450

Different interests

Collect the details of various schemes for deposits and loans from different banks and compare the rates of interests.



Interest for the second quarter = $15{,}450 \times \frac{12}{100} \times \frac{1}{4}$

= Rs. 463.50

Principal for the third quarter = ...

Interest for the third quarter = ...

Principal for the fourth quarter = ...

Interest for the fourth quarter = ...

Total amount got back after one year = ...

Now can't you do the following problems on your own?

- Rahim, Johny and Gopu deposited 10,000 rupees each, under different schemes, in a bank which gives 8% interest. Rahim gets simple interest, Johny gets interest compounded half-yearly and Gopu gets interest compounded quarterly. Compute the total amount each of them gets back after one year and compare these amounts.

- A person deposited 20,000 rupees in a bank which gives 6% interest compounded half-yearly. How much would he get back after 2 years?

- Ravi deposited 50,000 rupees in a bank, which gives 6% interest, compounded quarterly. How much would he get back after 9 months?

- Lathika deposited 25,000 rupees in a bank which gives 8% interest compounded half-yearly. Najma deposited the same amount at the same rate of interest, but compounded quarterly. What is the difference in the amounts they would get back after one year?

Interest computation

To compute compound interest under different schemes, we must know the principal amount, the rate of interest, the period of deposit or loan and also the number of times interest is compounded during this period. For example, if the period is 2 years, the number of years is 2, the number of half-years is 4 and the number of quarter years is 8.

Past interest

Bhaskara was a mathematician who lived in India during the twelfth century AD. One of his works is titled *Siddhantashiromani*. The first chapter of this work, named *Lilavati* gained much popularity and can be considered a mathematical text on its own. One problem from it, about the computation of interests, is as follows:

Oh! Mathematician! If the interest for 100 is 5 per month, say what the interest for 16, for one year is. Then from principal and interest, say what the period is and from period and interest, say what the principal is.

- A man deposits 50,000 rupees in a financial company, which gives 9% interest compounded every three months. How much would he get back after one year?

Same trick

A company which manufactures computers increases its production by 10% every year. In 2008, the company produced 75,000 computers. How many computers would it produce in 2010?

Here, the number of computers produced every year is 10% more than the number produced the year before.

So, starting with 75,000, we have to find the number of computers produced every year after that for two years. Try to find out.

It is the same computation as that of calculating the total amount on compounding interest, isn't it?

Look at another problem:

The price of a car is 3 lakh rupees and it depreciates by 4% every year. What would be the price after 2 years?

Here the price every year is 4% less than the previous year's price.

$$\text{First year's price} = \text{Rs } 3{,}00{,}000$$

$$\text{First year's depreciation} = 3{,}00{,}000 \times \frac{4}{100}$$

$$= \text{Rs. } 12{,}000$$

$$\text{Second year's price} = \ldots\ldots\ldots$$

$$\text{Second year's depreciation} = \ldots\ldots\ldots \times \frac{4}{100}$$

$$= \text{Rs. } \ldots\ldots\ldots .$$

Price at the end of two years = ... – 11520

= Rs. .....

Here, since the depreciation is at the same rate every year, we can write

Price at the end of two years = $3,00,000 \times \left(1-\frac{4}{100}\right)^2$

Now do these problems on your own

- The population of Kerala increases by 2% every year. The current population is 3 crores. What would be the population after 3 years?
- A TV manufacturer reduces the price of a particular model by 5% every year. The current price of this model is 8000 rupees. What would be the price after 3 years?
- Abu, Babu and Sabu deposited 10000 rupees each, under various schemes which give 10% interest. Abu gets interest compounded annually, Babu gets interest compounded every half-year and Sabu gets interest compounded every quarter. Compute the total amount each would get after 2 years.
- A financial company claims that it charges only 20% interest on loans. But if a person takes out a loan of 100 rupees, he would get only 80 rupees, after subtracting the annual interest of 20 rupees at the outset. And he has to pay back 100 rupees after one year. What is their actual rate of interest?
- Dileep and Manoj deposited the same amount under different schemes giving 10% interest. Dileep gets only simple interest, while Manoj gets interest compounded annually. After 2 years, Manoj had 50 rupees more than Dileep. How much money did each deposit?

Simple and compound

The simple interest for a certain amount of money for two years is 50 rupees. For the same amount at the same rate of interest, if interest is compounded annually, the interest would be 55 rupees. What is the rate of interest? And what is the amount?

- A person deposits 6400 rupees in a bank, which gives 12% interest. How much interest would he get after one month? What would be the interest for one month at 6%? What if it is 3%?
- What is the interest for 16,000 rupees at $12\frac{1}{2}$% for one year? What would be the interest if the rate of interest is only $6\frac{1}{4}$%?