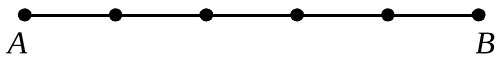
2 Ratio and Proportion

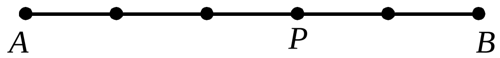
The same thing, different words

Look at this picture:



The line AB is divided into five equal parts.

Let's call the first three parts together as AP.



What are the different ways of expressing the relations between the lengths AP, BP and AB?

- AP is $\frac{3}{5}$ of AB.
- BP is $\frac{2}{5}$ of AB.
- BP is $\frac{2}{3}$ of AP.
- AP is $\frac{3}{2}$ of BP.
- The ratio of AP to BP is 3 : 2.

There is yet another way of saying this:

- If we measure lengths using $\frac{1}{5}$ of AB, then the length of AP is 3 and the length of BP is 2.

(See the lesson, *Relations of Parts* in the Class 7 textbook.)

Let's look at another problem: Ammu and Appu divided 50 rupees between them. Ammu took 30 rupees and Appu got 20. Ammu took 30 out of 50 and we know that $\frac{30}{50} = \frac{3}{5}$.

So, what part of the total did Ammu get?

And Appu?

Doing without fractions

When we measure lengths or other quantities using specific units, the results may not always turn out to be natural numbers. This gave rise to the notion of fractions. The basis for the idea of ratios is the question whether we can compare two measurements in terms of natural numbers, by choosing an appropriate unit.

For example, suppose that we measure two objects using a piece of string and find that the length of the first object is $\frac{2}{5}$ and the length of the second object is $\frac{3}{5}$. If we choose $\frac{1}{5}$ of the string as our unit of measurement, then the length of the first object would be 2 and the length of the second object would be 3. We say that the ratio of the lengths is 2 : 3.

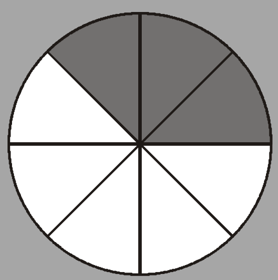
What if the length of the first object is $\frac{1}{3}$ of the string and the length of the second object is $\frac{1}{5}$ of the string?

If we take $\frac{1}{15}$ of the string as our unit, then the length of the first would be 5 and the length of the second would be 3. And we can say that the ratio of lengths is 5 : 3.

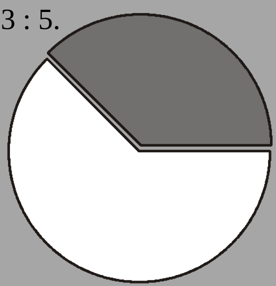
So, ratios are usually expressed in terms of natural numbers.

Ratios and fractions

A ratio can be used to compare parts of a single thing also. For example, in the picture below, the dark part is $\frac{3}{8}$ of the circle and the white part is $\frac{5}{8}$ of the circle.



These two parts together make the whole circle. The ratio of the size of the dark part to the size of the white part is 3 : 5.



Viewed thus, the ratio indicates the pair $\frac{3}{8}, \frac{5}{8}$ of fractions.

In what all ways can we say this?

- Ammu got .... of the total and Appu got ....of the total.
- Ammu got .... times what Appu got.
- Appu got .... of what Ammu got.
- The ratio of what Ammu got to what Appu got is ....

Suppose the total amount was in ten rupee notes. Then

- Ammu got .... notes and Appu got .... notes.

Now look at this problem.

> Chakkochan and Dineshan started a business together. Chakkochan invested 5000 rupees and Dineshan invested 7000 rupees.

What part of the total did Chakkochan invest? And Dineshan?

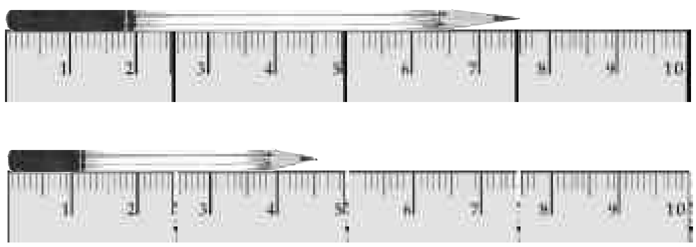
How many times the investment of Chakkochan's investment is Dineshan's investment?

What fraction of Dineshan's investment is Chakkochan's investment?

How many thousands did Chakkochan invest? And DIneshan?

What is the ratio of their investments?

Let's have one more example: Look at these pictures:



The length of the longer pencil is 7.5 centimeters and the length of the shorter one is 4.5 centimeters.

How many times the length of the shorter pencil is the length of the longer pencil?

$$\frac{7.5}{4.5} = \frac{75}{45} = \frac{5}{3}$$

So, the length of the longer pencil is $\frac{5}{3}$ times the length of the shorter pencil. It also means the length of the shorter pencil is $\frac{3}{5}$ of the longer pencil.

What if we measure the lengths using a 1.5 centimeter long piece of string?

The length of the longer pencil would be 5 strings and the length of the shorter pencil would be 3 strings.

What is the ratio of the lengths of the pencils?

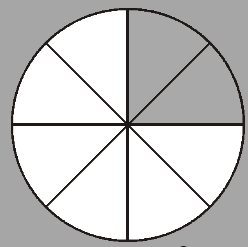
If we put the pencils end to end, then the total length would be 12 centimeters. We may say that the length of the longer pencil is $\frac{7.5}{12} = \frac{5}{8}$ of this and that the length of the shorter pencil is $\frac{4.5}{12} = \frac{3}{8}$ of this. But there is hardly any need to put the pencils together like this.

Now see if you can say each of the following facts in various ways, using fractions and ratios:

- 6 cups of rice and 2 cups of urud were taken to make dosa.
- In a class, there are 26 girls and 24 boys.
- In a school, there are 500 students and 15 teachers.
- The length of a pencil was found to be $\frac{1}{2}$ of a string and the length of another pencil was found to be $\frac{1}{3}$ of the string.

Simple ratios

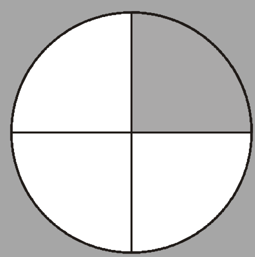
What is the ratio of the two parts of the circle shown below?



The smaller part is $\frac{2}{8}$ of the circle and the larger part is $\frac{6}{8}$ of the circle; so, we can say that the ratio is 2 : 6.

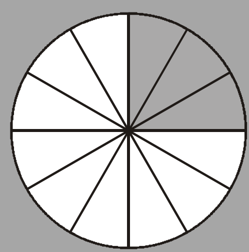
But we know that $\frac{2}{8} = \frac{1}{4}$ and $\frac{6}{8} = \frac{3}{4}$. So, we can also say that the ratio is 1 : 3.

If we divide the circle into 4 equal parts, and then take 1 part and the remaining 3 parts separately, then also we get the same pieces as before, right?



Thus the ratios 2 : 6 and 1 : 3 mean the same thing and so they are equal.

The ratio 3 : 9 is also equal to each of these:



In general, we can say that

$$a : b = ma : mb$$

Usually, ratios are given in terms of the smallest possible numbers after removing any common factors.

Using math

We have seen several explanations of the idea of ratios. They are to be used according to context. Let's look at some examples:

> To paint a house, blue and white paints are to be mixed in the ratio 3 : 2. To make 35 liters of such a mixture, how many liters of blue and how many liters of white should be taken?

What we are given is the total amount of the mixture. We know that blue and white paints are to be mixed in the ratio 3 : 2. From this we know what fraction of the mixture should be blue and what fraction should be white:

$\frac{3}{5}$ blue.

$\frac{2}{5}$ white.

We need 35 liters in all

$$\text{Amount of blue} = 35 \times \frac{3}{5} = 21 \text{ liters}$$

$$\text{Amount of white} = 35 \times \frac{2}{5} = 14 \text{ liters}$$

We can also think along different lines: since blue and white are to be mixed in the ratio 3 : 2, their actual amounts in a mixture should be 3 times and 2 times a common measure.

If this common measure is 1 liter, we get 5 liters of mixture. But we need 35 liters; and 35 is 7 times 5. So, the amount of blue should be 7 times 3 liters and the amount of white should be 7 times 2 liters.

Another problem now:

> The ratio of the number of boys to the number of girls in a school is 12 : 13. There are 360 boys in the school. How many girls are there in this school?

How should we interpret the ratio given here?

We want to find the number of girls. How many times the number of boys is the number of girls?

Rate and ratio

To make idlis, usually 2 cups of rice are mixed with 1 cup of urud. We can say that rice and urud are taken in the ratio 2 : 1. This means, for every 2 cups of rice, 1 cup of urud should be taken. Here what is important is the rate 2 to 1 of the ingredients.

In making chemical compounds also, if the ratio (rate) of the measures is changed, the property of the mixture itself maybe different.

From the ratio given, we find that the number of girls is $\frac{13}{12}$ times the number of boys; and the number of boys is given to be 360. So,

$$\text{the number of girls} = 360 \times \frac{13}{12} = 390.$$

Here also, we think in a different way: the actual number of boys is 360 and it is indicated by the number 12 in the ratio.

How many times 12 is 360?

So how many times 13 is the number of girls?

Now try your hand at these problems:

- In a rectangle, the ratio of the length to breadth is 5 : 3. The length is 2.5 meters. What is the breadth?
- Nazir invested 4000 rupees and Narayanan invested 6000 rupees to set up a partnership. In one year, they got 3000 rupees profit. If they split this in the ratio of their investments, how much would each get?
- Rama has a collection of 18 red beads and 12 green beads. Uma also has such a collection of red and green beads, 20 in all. The ratio of the number of beads in each color is the same for both. How many red beads does Uma have? And how many green beads?

Some other questions

Joy and Jayan divided some money in the ratio 3 : 5 and Jayan got 2000 rupees more than Joy. How much money did they divide? How much did each get?

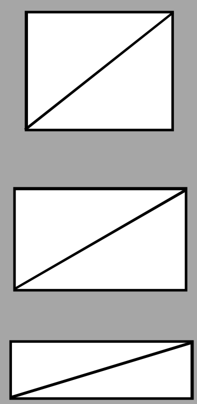
Here we don't know the amount of money divided. But from the ratio given, we find that Joy got $\frac{3}{8}$ of this amount and Jayan got $\frac{5}{8}$ of it.

Television math

The screen size of television sets are usually specified as 14 inches, 17 inches, 21 inches and so on. What does this mean?

A television screen is a rectangle, isn't it? These numbers are the lengths of their diagonals. (For some reason, television industry still uses inches instead of centimeters.)

But does this number alone determine the size of the screen? Rectangles with equal diagonals can have different lengths and breadths.



Whatever be the size of the screen, the ratio of its length to its breadth is usually 4 : 3 for most television sets. Nowadays, there are also (the so called wide-screen) television sets, in which this ratio is 16 : 9. The picture below shows the difference between two type of screens for the same length of the diagonal:

4 : 3

16 : 9

This ratio is called the aspect ratio of the rectangle.

Can you compute the length and breadth of a 4 : 3 television screen of 17 inches? Remember Pythagoras.

The additional 2000 rupees that Jayan got, is the difference of these fractions of the total.

What is the difference of $\frac{5}{8}$ and $\frac{3}{8}$?

$$\frac{5}{8} - \frac{3}{8} = \frac{2}{8} = \frac{1}{4}$$

So, $\frac{1}{4}$ of the total amount is 2000 rupees. Now can't you find the total and the amount each got?

There is another way of looking at the problem: Joy and Jayan got 3 times and 5 times a specific amount. So, the extra money Jayan got is 2 times this amount; and that is 2000 rupees. This means the specific amount is $\frac{1}{2} \times 2000 = 1000$ rupees. Now we can easily see that Joy got $3 \times 1000 = 3000$ rupees and Jayan got $5 \times 1000 = 5000$ rupees.

Look at another problem:

> The ratio of the number of boys to the number of girls in a class is 2 : 3. If $\frac{1}{4}$ of the boys leave the class, what would be this ratio?

Here, the number of girls is $1\frac{1}{2}$ times the number of boys. When $\frac{1}{4}$ of the boys leave, $\frac{3}{4}$ of them remain. Now $1\frac{1}{2}$ is 2 times $\frac{3}{4}$. So, the ratio becomes 1 : 2.

Can you think of other ways of doing this?

One more question:

> In ΔABC, we have $AB : BC = 1 : 2$ and $BC : AC = 3 : 5$. What is $AB : AC$?

Here we must find out how many times (or what fraction of) the length of the side AC is the length of the side AB.

To and fro

The length of a rectangle is 33 centimeters and its breadth is 1 centimeter. Another rectangle has length 11 centimeters and breadth 6 centimeters. What is the ratio of the perimeters of these rectangles? What is the ratio of their areas?

Can you find other pairs of rectangles like this?

What fraction of the length of BC is the length of AB?

Since $AB : BC = 1 : 2$, we have

$$AB = \frac{1}{2}\,BC$$

Now what fraction of the length of AC is the length of BC?

$BC : AC = 3 : 5$. From this

$$BC = \frac{3}{5}\,AC$$

So, what is the relation between the lengths of AB and AC?

$$AB = \frac{1}{2} \times \frac{3}{5}\ AC = \frac{3}{10}\,AC$$

From this we find that

$$AB : AC = 3 : 10$$

See if you can find out other ways of doing this problem also.

Now do these problems on your own.

- The ratio of illiterate to literate people in a locality is 1 : 19. The population of that locality is 64000. How many among these are literate? And how many are illiterate?
- In a cow farm, the ratio of the number of cows which give milk to the number of cows which don't is 8 : 3. The number of cows which don't give milk is 144. How many cows do give milk? And how many cows are there in all?
- The ratio of the number of boys to the number of girls in a school is 14 : 15. There are 27 more girls than boys. How many girls are there in this school? How many boys?
- The length and breadth of a rectangle are in the ratio 8 : 5. The length is 10.5 centimeters more than the breadth. What are the length and breadth of the rectangle?
- The ratio of the number of men and women who attended a meeting is 3 : 5. After some time, half the men and one-third the women left. What is the ratio of men to women now?

Decimals

The length of a rectangle is 2.5meters and its breadth is 1.5meters. What is the ratio of the length to the breadth?

The length is 5 times 0.5 meter and the breadth is 3 times 0.5 meter. So, length and breadth are in the ratio 5 : 3.

There is another way of looking at this.

We can say that the length of this rectangle is $\frac{2.5}{1.5}$ times that of the breadth. Also,

$$\frac{2.5}{1.5} = \frac{25}{15} = \frac{5}{3}$$

So, the ratio of length to breadth is 5 : 3.

Now can you find the ratio of the length of a rectangle of length 3.5 meters and breadth 2.25meters?

Ratio and area

There are two rectangles of equal perimeter; the ratio of length to breadth is 2 : 1 for one rectangle and 3 : 2 for the other. Which of them have the greater area?

Since the perimeters are equal, the sum of the length and breadth of the two rectangles are equal. Denoting this length by s, the length and breadth of the first rectangle are $\frac{1}{3}$ s, and $\frac{2}{3}$ s . So, its area is $\frac{2}{9}$ s^2..

What about the second rectangle? Its area is

$$\frac{2}{5}s\times\frac{3}{5}s=\frac{6}{25}s^2.$$

Which of the fractions $\frac{2}{9}$ and $\frac{6}{25}$ is the larger one?

Since $2 \times 25 < 6 \times 9$, we have

$$\left(\frac{2}{9} < \frac{6}{25}\right)$$

So, the second square has the larger area.

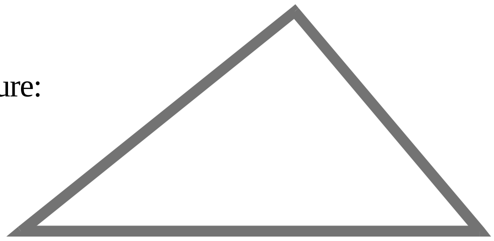
Now what can you say about a rectangle of the same perimeter, but with the ratio of sides 1 : 3? Which has the largest area?

Compare the difference between the length and breadth for all these rectangles. Then check some more rectangles of equal perimeter.

- In a school, the ratio of the number of children in lower primary classes to the number of children in the upper primary classes is 2 : 3; and the ratio of the number of children in the upper primary to the number of children in the secondary classes is 4 : 5. What is the ratio of the number of children in lower primary to the number of children in the secondary?
- Fatima, Ganga and Heera bought two packets of sweets, 140 in all. The ratio of the number of sweets Fatima took to the number of sweets Ganga took is 3 : 4. The ratio of the number of sweets Ganga and Heera took is 6 : 7. How many sweets did each take?

Three measures

Look at this picture:



A triangle made with *eerkkil* bits. Its sides are 8 centimeters. 6 centimeters and 4 centimeters.

So, what length of *eerkkil* was used to make this triangle?

$8 + 4 + 6 = 18$ centimeters, right?

What fraction of this is the bottom side?

$$\frac{8}{18} = \frac{4}{9}$$

What about the other two sides?

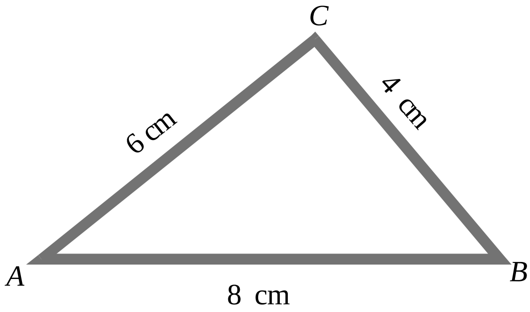
$$\frac{4}{18} = \frac{2}{9}$$

$$\frac{6}{18} = \frac{3}{9}$$

So, the sides are $\frac{4}{9}$, $\frac{2}{9}$ and $\frac{3}{9}$ of the sum of their lengths.

We can put this differently: the sides of this triangle are in the ratio 4 : 2 : 3.

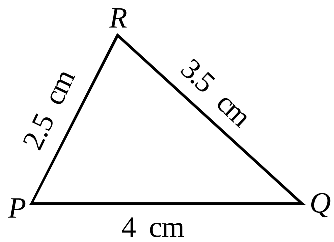
To make this a little more precise, we name the triangle.



In this triangle, the lengths of the sides *AB*, *BC*, *CA* are in the ratio 4 : 2 : 3.

We can consider this in a different manner: if we measure the lengths using a 2 centimeter long string, then the lengths of *AB*, *BC*, *CA* would be 4, 2, 3.

Now look at this triangle:



What is the ratio of the sides?

What is the sum of the sides (that is, the perimeter)?

$$PQ + QR + RP = 4 + 3.5 + 2.5 = 10$$

What fraction of this is each side?

$$\frac{4}{10} = \frac{2}{5}$$

$$\frac{3.5}{10} = \frac{35}{100} = \frac{7}{20}$$

$$\frac{2.5}{10} = \frac{25}{100} = \frac{1}{4}$$

The denominators of these fractions are not equal; so, how do we find the ratio?

Changing ratio

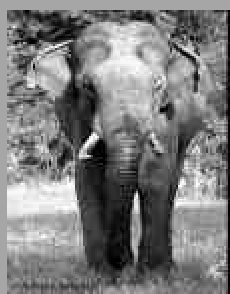
Look at this photo:

Its width is 1.5 centimeters and height is 2.25 centimeters. This means the ratio of width to height is 2 : 3.

What happens if we double the width and height?

Width is now 3 centimeters and height is 4.5 centimeters. The width to height ratio remains the same.

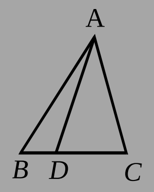
Now let's make the width of the first photo 3 centimeters, but instead of doubling the height as before, let's add 1.5 centimeters to height also to make it 3.75 centimeters, so that the ratio is 4 : 5.



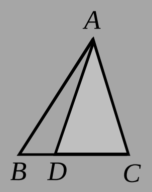
What happens to the picture?

Area ratio

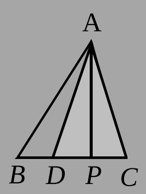
Look at this picture:



What is the ratio of the areas of the triangles *ABD* and *ACD?*



Draw a perpendicular from *A* to *BC*.



If we take the length of this perpendicular as *h*,

$$\text{area of } \Delta ABD = \frac{1}{2} h \times BD$$

$$\text{area of } \Delta ACD = \frac{1}{2} h \times CD$$

So

$$\frac{\text{area of } \Delta ABD}{\text{area of } \Delta ACD} = \frac{BD}{CD}$$

That is, the ratio of these areas is equal to the ratio of the lengths *BD* and *CD*.

So, how do we divide a triangle into two triangles of equal area?

What if we want the area of one triangle to be double the area of the other?

How about making all denominators equal to 20?

$$\frac{2}{5} = \frac{8}{20}$$

$$\frac{1}{4} = \frac{5}{20}$$

So, the lengths of PQ, QR, RP are $\frac{8}{20}$, $\frac{7}{20}$, $\frac{5}{20}$ of the perimeter. Thus

$$PQ : QR : RP = 8 : 7 : 5$$

We can do this in a different way. If we measure the lengths using a 0.5 centimeter long string, the lengths of PQ, QR, RP would be 8, 7, 5.

Let's look at another example: Ali put up 40000 rupees, Jose 20000 rupees and John 50000 rupees to start an agency. What is the ratio of their investments?

The total investment is 40000 + 20000 + 50000 = 110000, right? What fraction of did Ali invest?

$$\frac{40000}{110000} = \frac{4}{11}$$

Jose?

$$\frac{20000}{110000} = \frac{2}{11}$$

And John?

$$\frac{50000}{110000} = \frac{5}{11}$$

So, Ali, Jose and John invested in the ratio 4 : 2 : 5.

This can be seen also like this: reckoning by ten thousands, Ali invested 4 ten thousands, Jose 2 ten thousands and John 5 ten thousands.

Now see if you can do these problems on your own:

- In a contest, the first gets 1000 rupees as prize, the second 600 rupees and the third 400 rupees. What is the ratio of the prize money?

- The perimeter of a triangle is 10 meters and the lengths of two of its sides are $2\frac{1}{2}$ meters and $3\frac{1}{2}$ meters. What is the ratio of its sides?
- To make *unniyappam*, 1 kilogram rice, 250 grams banana and 750 grams jaggery were mixed. What is the ratio of the ingredients?

Let's look at a few more examples.

The perimeter of a triangle is 60 centimeters and its sides are in the ratio 3 : 4 : 5. What are the lengths of the sides?

Since the ratio of the sides is 3 : 4 : 5, their lengths are $\frac{3}{12}$, $\frac{4}{12}$, $\frac{5}{12}$ of the perimeter. So, the actual lengths (in centimeters)

$$60 \times \frac{3}{12} = 15$$

$$60 \times \frac{4}{12} = 20$$

$$60 \times \frac{5}{12} = 25$$

Can we do it in a different way?

The sides are 3, 4, 5 times a specific length, aren't they?

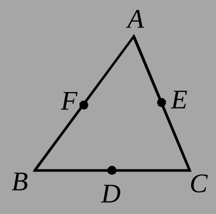
What about the perimeter then?

Now another problem:

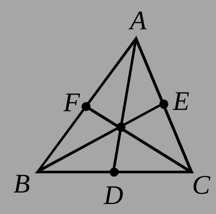
Vijayan, Gopan and Mukundan took up some work on contract. Vijayan worked for 3 days, Gopan for 5 days and Mukundan 6 days. When they divided the money they got for the work, Gopan got 500 rupees more than Vijayan. How much money did they get in all? How much did each get?

Center of a triangle

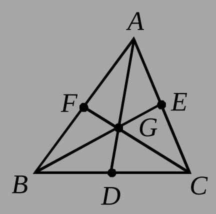
Draw a triangle and mark the midpoints of its sides.



Now join each of these midpoints to the opposite vertex of the triangle.



These lines are called the *medians* of the triangle. They all pass through a single point within the triangle, don't they?



This point is called the *centroid* of the triangle.

This point divides each of the medians in the ratio 2 : 1; that is, in our picture,

$$\frac{AG}{GD} = \frac{BG}{GE} = \frac{CG}{GF} = 2$$

This point has another significance.Cut out a picture like this in cardboard. You can balance the triangle on the tip of a pencil at this point

This means, the centroid of a triangle is its center of gravity.

The number of days they worked is in the ratio 3 : 5 : 6; and this is the ratio in which they divide the money. So, Vijayan got 3 times a specific amount, Gopan got 5 times this amount and Mukundan got 6 times this amount.

This means the difference in what Gopan and Vijayan got is 2 times this specific amount; and we know that it is 500 rupees.

So, this specific amount is $\frac{1}{2} \times 500$ = Rs. 250.

Now we can easily compute how much each got.

Vijayan's share	$= 3 \times 250$	=	Rs. 750
Gopan's share	$= 5 \times 250$	=	Rs.1250
Mukundan's share	$= 6 \times 250$	=	Rs. 1500

Now can't you do these problems on your own?

- The three angles of a triangle are in the ratio 1 : 3 : 5. How much is each angle?
- To make gunpowder, carbon, sulphur and potassium nitrate are to be mixed in the ratio 3 : 2 : 1. To make 1.2 kilograms of gunpowder, how much of each is needed?
- The sides of a triangle are in the ratio 2 : 3 : 4 and the longest side is 20 centimeters longer than the shortest side. Compute the length of each side.
- Can the sides of a triangle be in the ratio 1 : 2 : 3?
- In ΔABC, we have $AB : BC = 2 : 3$ and $BC : CA = 4 : 5$. What is $AB : BC : CA$?

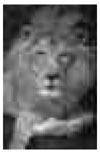
Teacher-student ratio

The teacher-student ratio is an important indicator of the state of education in a country. According to the figures for the year 2005, this ratio is 1 : 27 for the schools in Keralam. This means, there is one teacher for every 27 students, on average. If we consider the whole of India, this average is about 1 : 40.

- Molly, Nafeeza and Omana started a tailoring shop. The money they invested for this is in the ratio 5 : 7 : 8. They divide the profit also in this ratio. One year, Omana got 1800 rupees more than Molly. How much did each get that year?

Changing and unchanging

Look at these photos:



The same photo in different sizes, right?

The first photo is 1 centimeter wide and 1.5 centimeters high.

What about the second photo?

2 centimeters wide and 3 centimeters high.

That is, both width and height are doubled.

Here, both width and height are changed.

What remains unchanged?

In the first photo, height is $\frac{3}{2}$ times the width.

What about the second?

Can't we say this without using fractions?

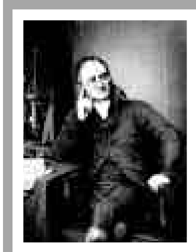
In both photos, the width and height are in the ratio 2 : 3.

What about this photo?

Ratio in Chemistry

In chemistry, things are classified as elements and compounds. In the eighteenth century, Joseph Proust found out that the masses of the elements in a compound are in a definite ratio. For example, he found through experiments that in any quantity of copper carbonate, the mass of copper is 5.3 times and the mass of oxygen is 4 times that of carbon.

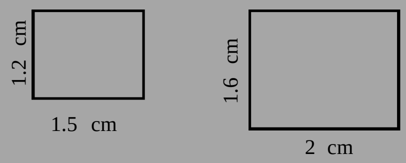
The idea of atom may have been motivated by the thought that, if we consider very small particles of elements, such comparisons for compounds can be done using natural numbers. This idea was first presented by John Dalton in the nineteenth century.



According to Dalton's theory, compounds are formed by the combination of very small particles of elements called atoms; and in any particular compound, the number of atoms of the various elements are in a definite ratio.

Ratio check

In the two rectangles below, are the ratios of the sides equal?



In the first rectangle, the longer side is $\frac{1.5}{1.2}$ times the shorter side. What about the second rectangle? The longer side is $\frac{2}{1.6}$ times the shorter side.

If the ratios are to be equal, these fractions must be the same.

Are they?

We can think about this in a different way. The longer side of the second rectangle is $\frac{2}{1.5}$ times the longer side of the first rectangle. What about the shorter sides? The shorter side of the second rectangle is $\frac{1.6}{1.2}$ times the shorter side of the second rectangle. If the ratios are to be equal, these fractions must also be the same.

Simplify the fractions and check.

What does the first fraction show?

In both rectangles, the longer side is $\frac{5}{4}$ times the shorter side.

What about the second fraction?

The sides of the second rectangle are $\frac{4}{3}$ times the sides of the second rectangle.

Look at another piece of math:

Appu's mother usually takes 2 cups of rice and 1 cup of urud for making idlis. One day, when they had guests, she took 3 cups of rice and $1\frac{1}{2}$ cups of urud.

Here the amount of rice is $1\frac{1}{2}$ times the usual. What about the amount of urud?

What is the ratio of rice to urud in both occasions?

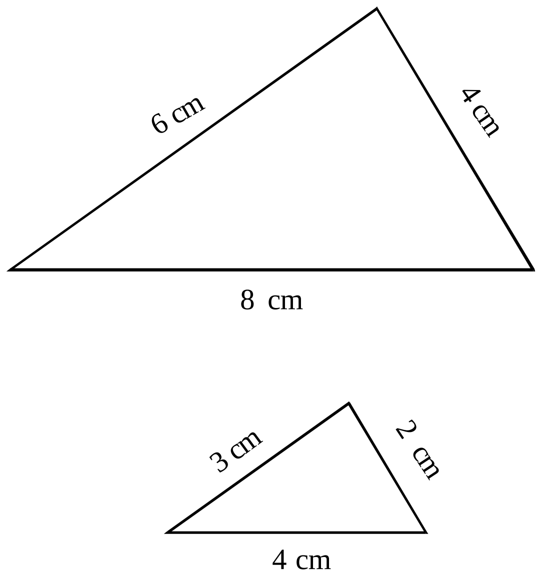
There is a name in mathematics for such cases where the individual quantities change, but the ratio remains the same: *proportion*.

Thus in the case of the two photographs at the beginning, the width and height are proportional.

What about the case of idlis?

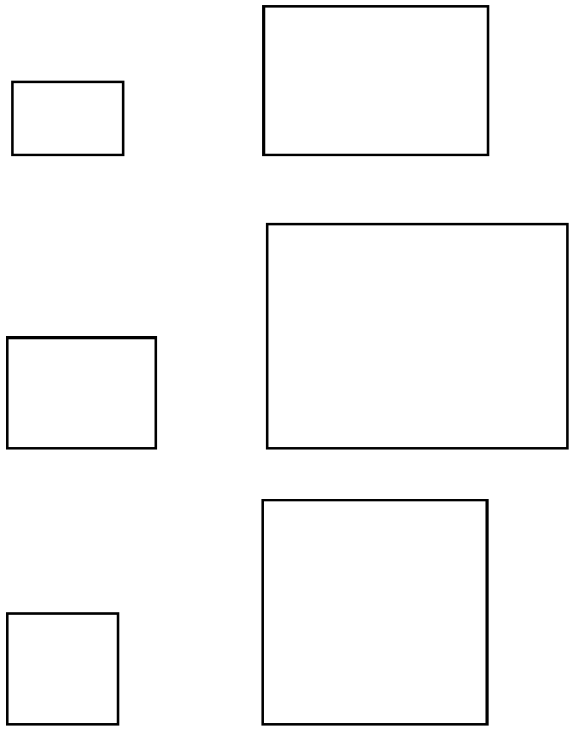
The amounts of rice and urud in the two occasions are proportional.

Three quantities can also be sometimes proportional. Look at these pictures:

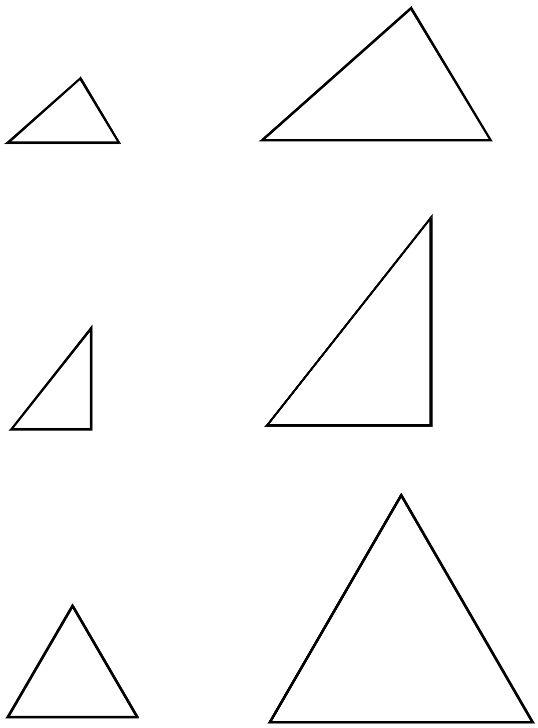


What is the ratio of the lengths of the sides in each triangle?

For each pair of rectangles shown below, first try to guess whether the sides are proportional or not. Then check, by actually measuring the sides.



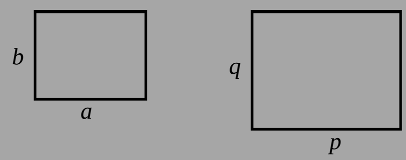
Now do this for the pairs of triangles below.



Using algebra

How do we check whether the sides of two rectangles are proportional or not?

Let the lengths of sides of one rectangle be a, b and let the lengths of another rectangle be p, q.



One way is to find for each rectangle, how many times (or what fraction of) one side is the other side. That means, we must check whether the fractions $\frac{a}{b}$, $\frac{p}{q}$ are the same.

Or we can check how many times (or what fraction of) the sides of one rectangle are the sides of the other rectangle. That is, we must check whether the fractions $\frac{a}{p}$, $\frac{b}{q}$ are the same.

For $\frac{a}{b} = \frac{p}{q}$ or for $\frac{a}{p} = \frac{b}{q}$, we must have $aq = bp$. On the other hand, if these products are equal, then so are the fractions. Thus to check for proportionality, we need only check the equality of these products.

The sides of one rectangle are 2.5 centimeters and 1.5 centimeters; the sides of another rectangle are 2 centimeters and 1.2 centimeters. Are the sides proportional?

Let's find out

- Mathew invested 30000 rupees and Stephen, 50000 rupees to start a business. In one month they made a profit of 2400 rupees. Mathew took 900 rupees and Stephen took 1500 rupees as their shares of the profit. What is the ratio of their investments? What is the ratio of their shares of the profit? Are the shares proportional to the investments?
- Ramu worked 8 hours and got 400 rupees as wages. Benny worked 6 hours and got 300 rupees. Are the wages proportional to the hours of work?
- 10 litres of blue paint and 15 litres of white paint are mixed in one can; 12 litres of blue and 17 litres of white are mixed in another. Will the shades of blue in the two cans be the same? Why?

Proportional growth

If two quantities are proportional and one of them increases, so does the other. But simply because two quantities are related in such a way that one increases when the other increases, the quantities need not be proportional.

For example, if the length of the sides of a square increase, the area also increases. But the area of a square of side 1 centimeter is 1 square centimeter, and the area of a square of side 2 centimeters is 4 square centimeters. The side to area ratio is 1 : 1 in the first square and 2 : 4 = 1 : 2 in the second square. Since these two ratios are not equal, the change in the area of a square is not proportional to the change in side.

Are the side and perimeter of a square proportional?

Constancy in change

What is the perimeter of a square of side 3 centimeters?

What about the perimeter of a square of side 5 centimeters?

Whatever be the side, the perimeter of a square is four times the length of the side, isn't it? This can be stated in another manner: in a square, the ratio of the side to perimeter is 1 : 4.

As the length of the sides of a square changes, so does the perimeter; but their ratio does not change. So, the earlier statement can also be put this way: in a square, the perimeter is proportional to the side.

This is true for equilateral triangles also, isn't it?

Let's look at another situation. The price of a pencil is $1\frac{1}{2}$ rupees. What is the cost of 10 such pencils?

What about the price of 20 pencils? Is the price proportional to the number of pencils?

Let's look at the algebraic form of these:

- If the length of the side of a square is x and its perimeter is y, then

$$y = 4x$$

- If the length of the side of an equilateral triangle is x and its perimeter is y, then

$$y = 3x$$

- If the number of pencils is x and the total price is y, then

$$y = \frac{3}{2}x$$

In general, if two quantities x and y are proportional, then the relation between all their different values is

$$y = kx$$

where the value of k does not change with the values of x and y. This number k is called the *constant of proportionality*.

In our first example, the constant of proportionality is 4 and it is the number of sides; in the second example also, the constant of proportionality 3 is the number of sides. In the third example, the constant of proportionality is $\frac{3}{2} = 1\frac{1}{2}$ and it is the price of a single pencil.

Now work these out on your own:

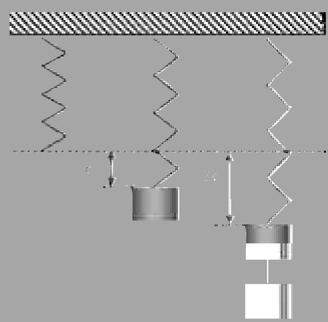
- Rajan deposited 10000 rupees in a bank, which gives 6% simple interest. Write an equation connecting the number of years of deposit and the total interest. Is the total interest proportional to the number of years? What if interest is compounded?

Ratios for weighing

Have you seen a spring balance?

When a weight is hung on its hook, the spring is stretched down, and a pointer attached to it shows the weight on a scale marked on its cover.

What is the principle behind its working? The seventeenth century English scientist Robert Hooke discovered that the stretch produced by weights suspended on a spring is proportional to the weight.



Using this idea, it is easy to mark the positions for various weights on the spring balance. First, the position of the pointer with no weights on the hook is marked. Then the position when a definite weight is suspended is marked. For example, suppose a 1 kilogram weight produces 2 centimeters stretch. Then, the positions for 1,2,3,4 kilograms can be marked 2,4,6,8 centimeters from the first position. If the distance between consecutive marks are subdivided into ten equal parts, we can also mark positions for such weights as 1.1 kilograms or 2.3 kilograms.

Universal proportion

A basic principle of physics is that all objects in the Universe attract one another. This principle says that this force of attraction is directly proportional to the product of the masses and inversely proportional to the square of the distance.

This was formulated by the famous English scientist, Isaac Newton in the seventeenth century.

- Mary gets an increment of 200 rupees on her salary every year. Write an equation connecting the number of years she works and the total increase in salary over the years. Is the total increase in salary proportional to the number of years of work?

- When an object falls from a height, its speed, v meters per second after t seconds of fall, is given by the equation

$$v = 9.8t$$

  Is the speed proportional to the time?

- When an object falls from a height, the distance s (meters), it travels in time t (seconds) is given by the equation

$$s = 4.9t^2$$

  Is the distance travelled proportional to time?

Inverse change

How many rectangles of area 30 square centimeters can we draw?

We can take the lengths of sides as 10 centimeters and 3 centimeters; or 6 centimeters and 5 centimeters. We can also have fractional lengths such as 12.5 centimeters and 2.4 centimeters.

So, we can draw as many such rectangles as we wish. Here, when we increase the length of one side, the length of the other side is decreased. So, the lengths of the sides of various such rectangles are definitely not proportional. What exactly is the relation between the lengths of sides?

If we take the lengths of the sides of such a rectangle as x and y, we have

$$xy = 30$$

We can write this equation as

$$y = \frac{30}{x} = 30 \times \frac{1}{x}$$

So, y is proportional to $\frac{1}{x}$

If two quantities x and y change according to the rule

$$y = \frac{k}{x}$$

where k does not change with x and y, then they are said to be in *inverse proportion*.

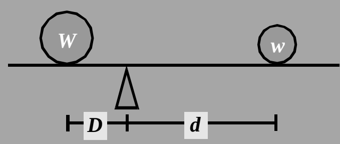
In this context, the proportional quantities we discussed earlier are said to be in *direct proportion*.

Now some questions:

- Are the lengths of sides of various rectangles with the same perimeter in inverse proportion?
- Write an equation connecting the average speed and the time of travel of a vehicle travelling 200 kilometers. Is the time inversely proportional to the average speed?
- 1000 rupees is to be equally divided among some persons. Is the amount each gets directly or inversely proportional to the number of persons?
- Compare the prices of the same brand of tooth paste of different weights such as 50 grams, 100 grams, 150 grams and so on. Is the price directly proportional to the weight?

Opposite forces

In the picture, the object on the left is heavier. Still the objects are in balance. How is it possible?



For balancing like this, the weights and their distances from the point of balance should be inversely proportional. That is, if the weights are W, w and their distances from the point of balance are D, d, then

$$W \times D = w \times d$$

This was discovered the famous Greek scientist Archimedes, who lived in the second century BC. This is known as the lever principle.

He found that as an application of this principle, heavy objects can be raised up using a small force, using a long rod. (Have you seen rocks lifted using iron rods?)

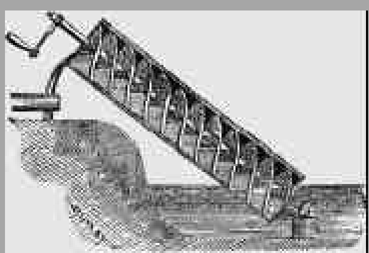
"With a place to stand and a lever long enough, I can move the earth" is an oft-quoted remark of Archimedes.

Archimedes

Archimedes is the most famous scientist of ancient times and in fact one of the greatest scientists of all times. The other Greek scientists of his time were interested only in theoretical questions; but Archimedes was also interested in practical application of his theories.

Ancient historians have recorded that when Syracuse, the hometown of Archimedes was attacked by Romans, he used machines made according to the principle of levers, to topple their ships!

The Archimedes screw is a machine he invented to raise water from a lower level to irrigate fields at an upper level.



Such screws are still used in large machines for waste-water treatment and also in small machines which maintain blood circulation during heart surgery.

- Get the world record times of men's sprints in 100 meters, 200 meters, 400 meters and 800 meters. Are the times directly proportional to the distances?
- Are the sides of a triangle and the altitudes to them inversely proportional?