7 Mathematics of chance

Chance as a number

There are ten beads in a box - nine black and one white. If you take one out (without peeking)...

It is very likely to be black; there's an outside chance of getting white also.

In another box are five black beads and five white beads and you take one from this. It can be black or white; apart from this, we can't say anything much.

Let's put it this way: there's a high chance of drawing a black bead from the first box or in other words, there's a very low chance of getting a white. But from the second box, there's an equal chance of getting black or white. We use numbers to make this more precise.

In the first box, $\frac{9}{10}$ of the beads are black and only $\frac{1}{10}$ of the beads are white. So, we say that the *probability* of getting a black bead from this box is $\frac{9}{10}$; and the probability of getting a white bead is $\frac{1}{10}$.

What about the second box? Since $\frac{5}{10}=\frac{1}{2}$, we say that in this case, the probability of getting a black and the probability of getting a white are both equal to $\frac{1}{2}$.

Let's look at another problem: We write the numbers 1 to 25 in paper slips and put them all in a box. One slip is drawn from this. What is the probability of the number to be a multiple of 3?

Among the numbers in the box, only the eight numbers 3, 6, 9, 12, 15, 18, 21, 24 are multiples of 3. So, the probability of getting such a number is $\frac{8}{25}$.

Dicey math

Haven't you played dice-games like Snakes and Ladders? Such games were played from very old times. The picture shows a dice from India during the Indus Valley Civilization, dated about 2500 BC.

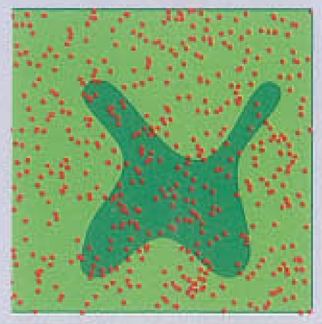
We can't predict what number will turn up on rolling a die.

The first book on the mathematics of this was written by the Gerolamo Cardano, who lived in Italy during the sixteenth century AD.

It is basically a guide to gamblers. In it, he has evaluated as numbers, the chances of getting various sums on rolling a pair of dice together.

Probability and area

We can use probability to estimate the area of complicated figures. The figure is drawn within a square and then a large number of dots are marked within the square, without any order or scheme.



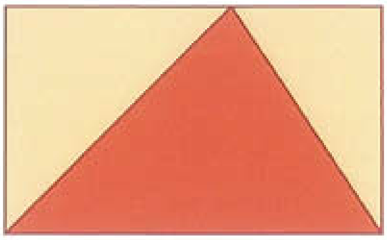
The number of dots falling within our figure, divided by the total number of dots gives an approximation of the area of the figure divided by the area of the square. And this approximation gets better, as we increase the number of dots. Both the geometric operation of marking the dots and the arithmetic operation of division can be done very fast, using computers. This is called the Monte Carlo Method.

What is the probability of getting a multiple of 4 from this box?

And the probability of getting an even number?

An odd number?

One more problem: see this picture



A figure like this is cut out and without looking, we mark a dot on it with a pencil. What's the probability that it falls within the red triangle?

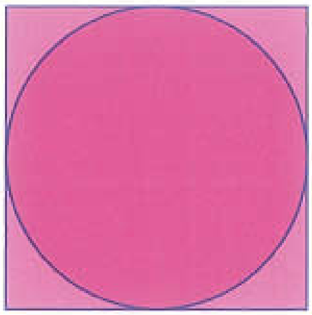
What fraction of the rectangle is the red triangle? (Remember the section, **Rectangle and triangle** of the lesson **Areas** in the Class 9 textbook)

So, the probability is $\frac{1}{2}$. In other words, the probability of the dot falling within the triangle and outside it are equal.

Now try these problems:

- A box contains 4 white balls and 6 black balls and another one, 3 white and 5 black. We can choose one box and take a ball. If we want a black ball, which box is the better choice?
- You ask someone to say a (natural) number less than 10. What is the probability that the number is a prime? What if the number asked is to be less than 100?
- A box contains paper slips with numbers written on them - 4 odd and 5 even. Two more paper slips, one with an odd number and another with an even number are put in. Does the probability of getting an odd number increase or decrease? What about the probability of getting an even number?

- A point was marked in the picture below, without looking.



What's the probability that it is within the circle? What's the probability that it is outside the circle? Calculate up to two decimal places.

Two at a time

Two slips of paper marked 1 and 2, are put in a box and three slips market 1, 2, 3 are put in another. One slip from each box is drawn. What is the probability that both show odd numbers?

Drawing one slip from each box, we get a pair of numbers. What are the possibilities? It can be 1 from the first box, 2 from the second; or 1 from both boxes; there are several, right? Let's write down all possible pairs:

(1, 1)   (1, 2)   (1, 3)

(2, 1)   (2, 2)   (2, 3)

Six pairs altogether. Our interest is in those pairs in which both numbers are odd. How many such pairs are there among these?

Only two, isn't it?

So, what is the probability of this happening? $\frac{2}{6}=\frac{1}{3}$

What is the probability of getting one odd number and one even number?

A problem

The famous scientist Galileo writes about a problem asked by a gambler friend. He had computed that when three dice are rolled together, 9 or 10 can occur as the sum in 6 different ways:

	9	10
1.	1 + 2 + 6	1 + 3 + 6
2.	1 + 3 + 5	1 + 4 + 5
3.	1 + 4 + 4	2 + 2 + 6
4.	2 + 2 + 5	2 + 3 + 5
5.	2 + 3 + 4	2 + 4 + 4
6.	3 + 3 + 3	3 +3+4

But then in actual experience, he found 10 occurring more often than 9 as the sum. He wanted an explanation of this.

In the list above (1,2,6) for example, stands for 1 coming up in some die, 2 in some other die and 6 in yet an other die. Galileo argued instead of this, he must denote by the triple (1,2,6), the occurrence of 1 in the first die, 2 in the second die, 6 in the third die; by the triple (1,6,2), the occurrence of 1 in the first die, 6 in the second die and 2 in the third die and so on. This gives six different triples, (1,2,6),(1,6,2),(2,1,6), (2,6,1),(6,1,2),(6,2,1) all denoting the occurrence of the same numbers 1, 2, 6 in the three dice. Expanding other triples also like this, Galileo shows that the sum 9 can occnr in 25 different ways, while 10 can ocur in 27 different ways. (Try it)

Theory and reality

When we toss a coin, we may get head or tail. And mathematically, it is logical to take the probability of each as $\frac{1}{2}$.

But this does not mean, if we toss a coin twice we would get head once and tail once. Nor are we sure to get five heads and five tails exactly, if we toss it ten times. It only means that if we toss a coin a large number of times, the number of heads and the number of tails would be more or less the same. For example, in 1000 tosses, we may get 510 heads and 490 tails.

Likewise, if we roll a die 1200 times, each number may not turn up exactly 200 times (this is more likely not to occur). One number may come up 220 times, another number 180 times and so on.

How about increasing the number of slips? Suppose one box contains numbers from 1 to 5 and the other contains numbers from 1 to 10. What can you say about the probabilities we have seen earlier, in this new set up?

How many number pairs are possible in this case? It is a bit tedious to write out all the possibilities, as in the first example (and there is no charm in it). Can we compute this number?

Let's think about it like this: how many pairs are possible with the first number (that is, the number from the first box) 1? How many with this 2?

In short, there are 5 possibilities for the first number; and in each of these, there are 10 possibilities for the second. (You may find it helpful to imagine all these pairs written out in rows and columns, as in the first example: a row of 10 pairs with 1 as the first number; below it, another row of ten pairs with 2 as the first number and so on, giving 5 rows, each containing 10 pairs.)

So, 50 pairs in all. How many of these pairs have both numbers odd?

For such pairs, the first number should one of the three numbers 1, 3, 5. And the second number?

Thus we can see that there are $3 \times 5 = 15$ such pairs. (Do you understand this? Picture these in rows and columns, if you want.)

So, the probability of getting two odd numbers in this case is $\frac{15}{50} = \frac{3}{10}$

Can you find like this, the probability of getting both even and also the probability of getting one odd and one even?

Let's look at another problem: It's about a game played by two children. Both raise some of their fingers. If the total number of fingers raised by both is odd, the first player wins; if it is even, the second player wins. Who has more chance of winning?

In this game, the number of fingers each raises can be any number from 1 to 10. So, if we take the possible pairs of the number of fingers each can raise, how many different pairs do we get?

Out of these 100 possible pairs (how did we get this hundred?), in how many do we get an odd sum?

For the sum to be odd, one number must be odd and the other even.

How many pairs are there with the first odd and the second even? $5 \times 5 = 25$ (how is that?) And how many, the other way round?

Thus there are $25 + 25 = 50$ pairs with the sum odd. So, the possibility of the odd-player winning is $\frac{50}{100} = \frac{1}{2}$

We can now say without any actual listing that the probability of the even - player winning is the same (how?)

One more problem: there are 50 mangoes in a basket, in which 20 are not ripe; in another basket, there are 40 mangoes with 15 of them not ripe. One mango is taken from each basket. What is the probability of getting at least one ripe mango?

In how many different ways can we take two mangoes, one from each basket? (You can think of the mangoes in the first basket to be numbered 1 to 50 and the mangoes in the other numbered 1 to 40, and all possible mango pairs arranged in rows and columns, if that helps).

We can classify these 2000 mango pairs as follows:

(i) both unripe

(ii) both ripe

(iii) one ripe and the other unripe

How many pairs are possible with both unripe?

$20 \times 15 = 300$, right?

What about the pairs with both ripe? In the first basket, $50 - 20 = 30$ are ripe, and in the second, $40 - 15 = 25$ are ripe; so $30 \times 25 = 750$ pairs are possible with both ripe.

Now with the first mango (that is, from the first basket) ripe and the second unripe, $30 \times 15 = 450$ pairs are possible. What about the other way round? With the first mango unripe and the second ripe, $20 \times 25 = 500$ pairs are possible. So how many pairs in all in the third group? $450 + 500 = 950$.

Probability and frequency

We said that when a coin is tossed a large number of times, the number of heads and tails are almost equal, and so we can take the probability of each coming up as $\frac{1}{2}$. But due to some reason, such as a manufacturing defect in the coin, it may happen that the probability of head coming up is higher. How do we recognize this?

We suspect such a case, if in a large number of tosses, one side comes up very much more than the other. Then we toss the coin more and more times and tabulate the number of times each side comes up. For example, see this table:

Tosses	Heads	Tails
10	6	4
100	58	42
1000	576	424
10000	5865	4135

This shows that, instead of taking the probability of each face as 0.5, it is better to take the probability of head as 0.6 and the probability of tail as 0.4.

There are mathematical methods for making such assignments of probabilities more accurate, which we will see in the further study of the branch of mathematics called *Probability Theory*.

The pairs with at least one mango ripe are those in the second or third groups; and there are $750 + 950 = 1700$ pairs in these groups put together. So, the probability of getting at least one ripe mango is

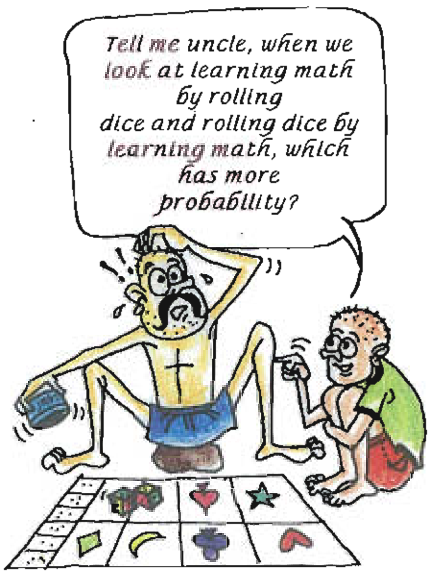
$$\frac{1700}{2000} = \frac{17}{20}$$

We can write this as 0.85 also.

Instead of finding the number of pairs in each of the three groups, we could have computed this probability from just the number of pairs in the first group alone. How?

Now some problems for you:

- There are two boxes, each containing slips numbered 1 to 5. One slip is drawn from each box and their numbers added. What are the possible sums? Compute the probability of each sum.
- In the finger-raising game, which number has the maximum probability of occurrence as the sum? What is this probability?
- Suppose you ask someone to say a two-digit number.
    - What is the probability of this number having both digits the same?
    - What is the probability of the first digit being larger than the second?
    - What is the probability of the first digit being smaller than the first?



Measuring uncertainty

Have you noticed that calendars show the time of sunrise and sunset for each day? It is possible to compute these, since the earth and sun move according to definite mathematical laws.

Because of this, we can also predict the months of rain and shine. But we may not be able to predict a sudden shower during summer. It is the largeness of the number of factors affecting rainfall and the complexity of their inter-relations that makes such predictions difficult.

But in such instances also, we can analyse the context mathematically and compute probabilities. This is why weather predictions are often given as possibilities. And unexpected changes in the circumstances sometimes make these predictions wrong.

If we look at such situations rationally, we can see that such probability predictions are more reliable than predictions sounding exact, but made without any scientific basis.