7791

Part III — BUSINESS MATHEMATICS

(English Version)

Time Allowed : 3 Hours]

[Maximum Marks : 200

Instruction : Check the question paper for fairness of printing. If there is any lack of fairness, inform the Hall Supervisor immediately.

SECTION – A

N. B. : i) Answer *all* the 40 questions.

ii) Choose and write the correct answer from the four choices given.

$40 \times 1 = 40$

1. If A is a square matrix of order 3 then $|\text{Adj } A|$ is

   a) $|A|^2$ 　　　　　　　　　　　　b) $|A|$

   c) $|A|^3$ 　　　　　　　　　　　　d) $|A|^4$

2. If $A = \begin{pmatrix} 0 \cdot 8 & 0 \cdot 6 \\ -0 \cdot 6 & 0 \cdot 8 \end{pmatrix}$ then A^{-1} is

   a) $\begin{pmatrix} -0 \cdot 8 & 0 \cdot 6 \\ -0 \cdot 6 & 0 \cdot 8 \end{pmatrix}$ 　　　　　b) $\begin{pmatrix} 0 \cdot 8 & -0 \cdot 6 \\ 0 \cdot 6 & 0 \cdot 8 \end{pmatrix}$

   c) $\begin{pmatrix} 0 \cdot 8 & 0 \cdot 6 \\ 0 \cdot 6 & 0 \cdot 8 \end{pmatrix}$ 　　　　　　d) $\begin{pmatrix} 0 \cdot 2 & 0 \cdot 4 \\ -0 \cdot 4 & 0 \cdot 2 \end{pmatrix}$

[Turn over

3. The rank of a non-singular matrix of order $n \times n$ is

   a) n b) n^2

   c) 0 d) 1.

4. The equation $AX = B$ can be solved by Cramer's rule only when

   a) $|A| = 0$ b) $|A| \neq 0$

   c) $A = B$ d) $A \neq B$.

5. The number of Hawkins-Simon conditions for the viability of an input-output model is

   a) 1 b) 3

   c) 4 d) 2.

6. The length of the latus-rectum of $3x^2 + 8y = 0$ is

   a) $\dfrac{8}{3}$ b) $\dfrac{2}{3}$

   c) 8 d) $\dfrac{3}{8}$.

7. In an ellipse $e = \dfrac{3}{5}$, the length of semi-minor axis is 2. The length of major axis is
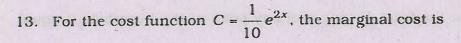
   a) 4 b) 5

   c) 8 d) 10.

8. The difference between the focal distances of any point of the hyperbola is equal to length of its

   a) transverse axis

   b) semi-transverse axis

   c) conjugate axis

   d) semi-conjugate axis.

9. Eccentricity of the rectangular hyperbola is

   a) 2

   b) $\frac{1}{2}$

   c) $\sqrt{2}$

   d) $\frac{1}{\sqrt{2}}$.

10. If 20 units of some product cost Rs. 2500 and 50 units cost Rs. 3400 to produce, the linear cost function is

   a) $y = 30x + 1900$

   b) $y = 20x + 5900$

   c) $y = 50x + 3400$

   d) $y = 10x + 900$.

11. For the function $y = 3x + 2$, the average rate of change of y when x increases from 1·5 to 1·6 is

   a) 1

   b) 0·5

   c) 0·6

   d) 3.

12. The slope of the tangent at (2, 8) on the curve $y = x^3$ is

   a) 3

   b) 12

   c) 6

   d) 8.

[Turn over

13. For the cost function $C = \dfrac{1}{10} e^{2x}$, the marginal cost is

   a) $\dfrac{1}{10}$

   b) $\dfrac{1}{5} e^{2x}$

   c) $\dfrac{1}{10} e^{2x}$

   d) $\dfrac{1}{10} e^{x}$.

14. The slope of the curve $x = y^2 - 6y$ at the point where it crosses the y-axis is

   a) 5

   b) -5

   c) $\dfrac{1}{6}$

   d) $\dfrac{1}{16}$

15. The stationary value of x for $f(x) = 3(x - 1)(x - 2)$ is

   a) 3

   b) $\dfrac{3}{2}$

   c) $\dfrac{2}{3}$

   d) $-\dfrac{3}{2}$.

16. If $f(x, y) = 2x + ye^{-x}$ then $f_y(1, 0)$ is equal to

   a) e

   b) $\dfrac{1}{e}$

   c) e^2

   d) $\dfrac{1}{e^2}$.

17. If the marginal revenue is Rs. 25 and the elasticity of demand with respect to price is 2 then the average revenue is

   a) Rs. 50

   b) Rs. 25

   c) Rs. 27

   d) Rs. 12·50.

18. The production function for a firm is $P = 3L^2 - 5kL + 2k^2$. The marginal productivity of capital (K) when $L = 2$ and $k = 3$ is

    a)   5   b)   3

    c)   6   d)   2.

19. $\int_{-2}^{2} x^4 \, dx$ is

    a) $\dfrac{32}{5}$ b) $\dfrac{64}{5}$

    c) $\dfrac{16}{5}$ d) $\dfrac{8}{5}$.

20. The area bounded by the curve $y = e^x$, the x-axis and the lines $x = 0$ and $x = 2$ is

    a) $e^2 - 1$ b) $e^2 + 1$

    c) e^2 d) $e^2 - 2$.

21. If the marginal cost function $MC = 2 - 4x$, then cost function is

    a) $2x - 2x^2 + k$ b) $2 - 4x^2$

    c) $\dfrac{2}{x} - 4$ d) $2x - 4x^2$.

22. The order and degree of the differential equation $\left[1 + \left(\dfrac{dy}{dx} \right)^2 \right]^{\frac{2}{3}} = \dfrac{d^2 y}{dx^2}$ are

    a)   3 and 2   b)   2 and 3

    c)   3 and 3   d)   2 and 2.

[Turn over

23. The solution of $\dfrac{dp}{dt} = ke^{-t}$ (k is constant) is

     a) $c - \dfrac{k}{e^t} = p$ b) $p = ke^t + c$

     c) $t = \log \dfrac{c - p}{k}$ d) $t = \log_c p$

24. The integrating factor of $x\dfrac{dy}{dx} - y = e^x$ is

     a) $\log x$ b) $e^{-\frac{1}{x}}$

     c) $\dfrac{1}{x}$ d) $-\dfrac{1}{x}$

25. The complementary function of the differential equation $\left(D^2 - D\right)y = e^x$ is

     a) $A + Be^x$ b) $(Ax + B)e^x$

     c) $A + Be^{-x}$ d) $(A + Bx)e^{-x}$.

26. When $h = 1$, $\Delta\left(x^2\right) =$

     a) $2x$ b) $2x - 1$

     c) $2x + 1$ d) 1.

27. $\nabla f(x + 3h) =$

     a) $f(x + 2h)$ b) $f(x + 3h) - f(x + 2h)$

     c) $f(x + 3h)$ d) $f(x + 2h) - f(x - 3h)$.

28. If the probability density function of a random variable X is defined as

$f(x) = cx(2-x), 0 < x < 2$ then the value of c is

a) $\dfrac{4}{3}$ b) $\dfrac{6}{4}$

c) $\dfrac{3}{4}$ d) $\dfrac{3}{5}$.

29. If a random variable X has the following probability distribution :

X	-1	-2	1	2
$P(x)$	$\dfrac{1}{3}$	$\dfrac{1}{6}$	$\dfrac{1}{6}$	$\dfrac{1}{3}$

then the expected value of X is

a) $\dfrac{3}{2}$ b) $\dfrac{1}{6}$

c) $\dfrac{1}{2}$ d) $\dfrac{1}{3}$.

30. If X is a Poisson variate with $P(X=1) = P(X=2)$ the mean of the Poisson variate is equal to

a) 1 b) 2

c) -2 d) 3.

31. If $X \sim N(8, 64)$, then the standard normal variate Z will be

a) $\dfrac{X-64}{8}$ b) $\dfrac{X-8}{64}$

c) $\dfrac{X-8}{8}$ d) $\dfrac{X-8}{\sqrt{8}}$.

[Turn over

32. The standard error of the sample mean is

    a) Type I error

    b) Type II error

    c) Standard deviation of the sampling distribution of the mean

    d) Variance of the sampling distribution of the mean.

33. The Z value that is used to establish a 95% confidence interval for the estimation of a population parameter is

    a) 1·28

    b) 1·65

    c) 1·96

    d) 2·58.

34. Probability of rejecting the null hypothesis when it is true, is

    a) Type I error

    b) Type II error

    c) Sampling error

    d) Standard error.

35. The number of ways in which one can select 2 customers out of 10 customers is

    a) 90

    b) 60

    c) 45

    d) 50.

36. The components of a time series attached to long term variation is termed as

    a)   cyclic variation   b)   secular trend

    c)   irregular variation   d)   all of these.

37. Most commonly used index number is

    a)   diffusion index number   b)   price index number

    c)   value index number   d)   none of these.

38. Chance variation in the manufactured product is

    a)   controllable   b)   not controllable

    c)   both (a) & (b)   d)   none of these.

39. If X and Y are two variates, there can be at most

    a)   one regression line   b)   two regression lines

    c)   three regression lines   d)   none of these.

40. Scatter diagram of the variate values (X, Y) gives the idea about

    a)   functional relationship   b)   regression model

    c)   distribution of errors   d)   none of these.

[Turn over

SECTION – B

N. B. : Answer any *ten* questions. $10 \times 6 = 60$

41. Verify $(AB)^{-1} = B^{-1}.A^{-1}$ when $A = \begin{pmatrix} 3 & 1 \\ 2 & -1 \end{pmatrix}$ and $B = \begin{pmatrix} -6 & 0 \\ 0 & 9 \end{pmatrix}$.

42. Find the rank of the matrix $A = \begin{pmatrix} 4 & 5 & 2 & 2 \\ 3 & 2 & 1 & 6 \\ 4 & 4 & 8 & 0 \end{pmatrix}$.

43. Find the equation of the hyperbola whose eccentricity is $\sqrt{3}$, focus is $(1, 2)$ and the corresponding directrix is $2x + y = 1$.

44. If $y = \dfrac{1 - 2x}{2 + 3x}$, find $\dfrac{E_y}{E_x}$. Obtain the value of η when $x = 0$ and $x = 2$.

45. At what points on the curve $3y = x^3$ are the tangents inclined at $45°$ to the x-axis ?

46. Find EOQ for the data given below. Also verify that the carrying cost is equal to ordering cost at EOQ.

    Monthly requirement 9000

    Ordering cost per unit Rs. 200

    Carrying cost per unit Rs. 3·60.

47. The elasticity of demand with respect to price P for a commodity is $\dfrac{x - 5}{x}$, $x > 5$.

    When the demand is x, find the demand function if the price is 2 when the demand is 7. Also find the revenue function.

48. Solve : $\dfrac{dy}{dx} + y \cos x = \dfrac{1}{2} \sin 2x$.

49. Solve : $\left(D^2 - 14D + 49\right)y = 3 + e^{7x}$.

50. From the following data find $f(3)$:

x :	1	2	3	4	5
$f(x)$	2	5	—	14	32

51. Using Lagrange's formula find $y(11)$ from the following data :

x	6	7	10	12
y	13	14	15	17

52. Ten coins are thrown simultaneously. Find the probability of getting at least 7 heads.

53. A random sample of 50 branches of State Bank of India out of 200 branches in a district showed a mean annual profit of Rs. 75 lakhs and a standard deviation of 10 lakhs. Find the 95% confidence limits for the estimate of mean profit of 200 branches.

54. Using three year moving averages determine the trend values for the following data :

Year	1983	1984	1985	1986	1987	1988	1989	1990	1991	1992
Production in tonnes	21	22	23	25	24	22	25	26	27	26

55. Find the coefficient of correlation for the data given below :

X	10	12	18	24	23	27
Y	13	18	12	25	30	10

SECTION – C

N. B. : Answer any *ten* questions. $10 \times 10 = 100$

56. Solve by determinant method the equations :

$2x + 2y - z - 1 = 0, \quad x + y - z = 0, \quad 3x + 2y - 3z = 1.$

57. In an economy there are two industries P and Q and the following table gives the supply and demand positions in crores of rupees.

Producer	User		Final demand	Total output
	P	Q		
P	10	25	15	50
Q	20	30	10	60

Determine the outputs when the final demand changes to 35 for P and 42 for Q.

58. Find the centre, vertices, eccentricity, foci, latus rectum and directrices of the ellipse $9x^2 + 16y^2 + 36x - 32y - 92 = 0$.

59. If $A.R$ and $M.R$ denote the average and marginal revenue at any output level, show that elasticity of demand is equal to $\dfrac{A.R}{A.R - M.R}$. Verify this for the linear demand law $P = a + bx$ where P is price and x is the quantity.

60. Investigate the maxima and minima of the function $2x^3 - 15x^2 + 24x - 15$.

61. The demand function for a commodity Y is $q_1 = 12 - p_1^2 + p_1 p_2$. Find the partial elasticities when $p_1 = 10$ and $p_2 = 4$.

62. Evaluate : $\displaystyle\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \dfrac{dx}{1 + \sqrt{\tan x}}$

63. Find the producer's surplus and the consumer's surplus under market equilibrium if the demand function $P_d = 20 - 3x - x^2$ and the supply function is $P_s = x - 1$.

64. The rate of increase in the cost c of ordering and holding as the size q of the order increases is given by the differential equation $\dfrac{dc}{dq} = \dfrac{c^2 + q^2}{2cq}$. Find the relationship between c and q, if $c = 4$ and $q = 2$.

65. Fit a straight line to the following data :

x	4	8	12	16	20	24
y	7	9	13	17	21	25

66. Find the mean and variance for the following probability distribution :

$$f(x) = \begin{cases} 2e^{-2x} & ; \quad x \geq 0 \\ 0 & ; \quad x < 0 \end{cases}$$

67. In a sample of 1000 candidates the mean of certain test is 45 and standard deviation is 15.

Assuming the normality of the distribution find the following :

i) How many candidates score between 40 and 60 ?

ii) How many candidates score above 50 ?

iii) How many candidates score below 30 ?

Given :

Z	1	0·33	0·34
Area	0·3413	0·1293	0·1331

68. A sample of 400 students is found to have a mean height of 171·38 cm. Can it reasonably be regarded as a sample from a large population with mean height of 171·17 cm and standard deviation of 3·3 cm (Test at 5% level) ?

69. Solve the following, using graphical method :

Minimize $\qquad Z = 3x_1 + 2x_2$

subject to the constraints $\quad 5x_1 + x_2 \geq 10,$

$$2x_1 + 2x_2 \geq 12,$$

$$x_1 + 4x_2 \geq 12,$$

$$x_1, \ x_2 \geq 0$$

70. Calculate Fisher's Index number from the following data and show that it satisfies time reversal test and factor reversal test :

Commodity	Base Year 1997		Current Year 1998	
	Price	Quantity	Price	Quantity
A	10	10	12	8
B	8	12	8	13
C	12	12	15	8
D	20	15	25	10
E	5	8	8	8
F	2	10	4	6

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