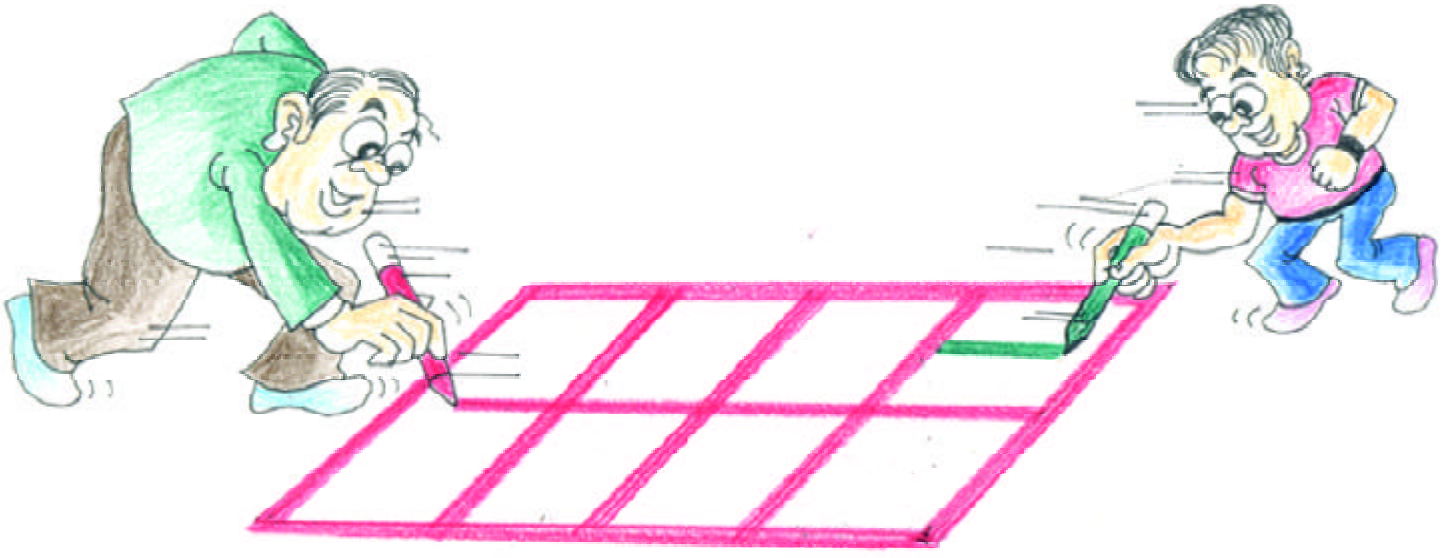
4

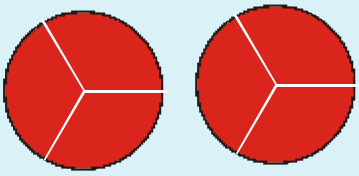
Parts of Parts



Cake math

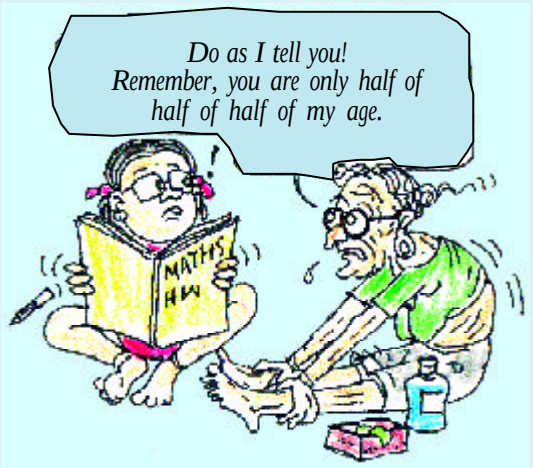
How do we divide two similar cakes equally among three people?

First we divide one of the cakes into 3 equal pieces and give them a piece each. And to with the second cake.



Each person gets two pieces. Suppose we join the two pieces.

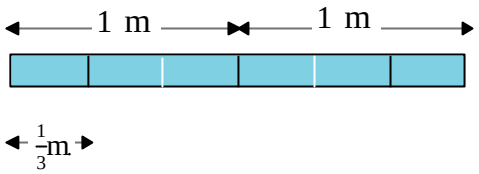
What part of a whole cake is it?



Fraction and division

3 girls divided a ribbon of length 2 meters equally between them. How much did each get?

Each gets 2 out of 3 equal parts of 1 meter, right?



(See the lesson, **Joining Parts**)

We can put it another way. $\frac{2}{3}$ meter is the length of one piece, when 2 meters is divided into 3 equal parts. So $\frac{2}{3}$ meters is $\frac{1}{3}$ of 2 meters.

So what is $\frac{1}{3}$ of 3 meters?

If we divide 3 meters into 3 equal parts, what is the length of each part?

Thus, $\frac{1}{3}$ of 3 meters is 1 meter.

Now, what is $\frac{1}{3}$ of 12 meters?

If 12 meters is divided into 3 equal parts, what is the length of each part?

$\frac{1}{3}$ of 12 meters = $12 \div 3 = 4$ meters

The division operation $12 \div 3$ could be written as the fraction $\frac{12}{3}$. On the other hand, the fraction $\frac{2}{3}$ may written as the division $2 \div 3$.

That is,

$$\frac{1}{3} \text{ of } 2 = \frac{2}{3} = 2 \div 3$$

$$\frac{1}{3} \text{ of } 3 = \frac{3}{3} = 3 \div 3 = 1$$

$$\frac{1}{3} \text{ of } 12 = \frac{12}{3} = 12 \div 3 = 4$$

Now, find the answers to the following questions.

- What is the length of $\frac{1}{4}$ of an 8 centimeter long straight line?
- How many millimeters long is $\frac{1}{10}$ of a straight line, which is 2 centimeters long?
- There are 30 sweets. Lincy took $\frac{1}{2}$ of them and Johny took $\frac{1}{3}$ of them. How many sweets did they together take?
- Ramesan took $\frac{1}{2}$ of 1000 rupees and Rajan took $\frac{1}{4}$ of it. Find the remaining amount.
- A rod is 27 meters long. From it, 7 meters long pieces are cut out. How many such pieces could be got? What would be the length of the remaining piece?

Is there a remainder?

If 12 meters is divided into 3 equal parts, what is the length of each part?

What about 16 meters divided into 3 equal parts?

When 16 is divided by 3, the quotient is 5 and the remainder 1. What does this mean?

Division and common factors

What is 180 ÷ 15?

It is not difficult to find, is it? But we needn't divide directly. Since 5 is a common factor of 180 and 15, we have

$$180 \div 15 = \frac{180}{15} = \frac{36 \times 5}{3 \times 5} = \frac{36}{3} = 12$$

Find 144 ÷ 24 like this.

Thus we can remove common factors in a division.

Fraction and Remainder

What do we get on dividing 9 by 4?

Quotient 2 and remainder 1, right?

That is,

$$9 = (2 \times 4) + 1$$

Using fractions, we can write

$$\frac{9}{4} = 2 + \frac{1}{4} = 2\ \frac{1}{4}$$

Which of these to use, depends on the situation.

For example, suppose we have 9 liters of milk. The problem is to distribute it to some children, so that each gets 4 liters. How many children can we serve?

The answer is 4 children, isn't it? And 1 liter will be left.

On other hand, suppose the problem is to distribute this milk equally among 4 children. How much milk will each get?

$2\frac{1}{4}$ liters, right?

16 meters can be divided into 3 pieces of length 5 meters each. A piece of length 1 meter will remain.

Suppose we divide this 1 meter piece also into 3 equal parts?

We get 3 short pieces, each of length $\frac{1}{3}$ meter.

We can join each of these short pieces to the (5 meter) long pieces cut out first.

We now have 3 pieces, each of length $5\frac{1}{3}$ meters.

So,

$$\frac{1}{3} \text{ of } 16 \text{ meters} = 5\frac{1}{3} \text{ meters.}$$

Just as we write $15 \div 3 = 5$, we can write $16 \div 3 = 5\frac{1}{3}$.

That is,

$$\frac{16}{3} = 16 \div 3 = 5\frac{1}{3}$$

Can you write $\frac{27}{4}$ similarly?

Now try these problems.

- 17 liters of milk could fill 4 vessels of equal size. How many liters of milk does each vessel contain?
- 150 rupees is divided equally among 4 people. How many rupees and how many paise does each get?
- A rod, 24 meters long is cut into 5 equal parts. What is the length of each piece, in meters and centimeters?

Adding again and again

If 3 strings, each of length 4 meters, are laid end to end, what is the total length?

$4 + 4 + 4 = 12$ meters.

This repeated addition of 4's could be wrtten as a product.

$3 \times 4 = 4 + 4 + 4 = 12$

Likewise, if we place end to end 3 strings, each of length $\frac{1}{4}$ meter, what is the total length?

$$\frac{1}{4} + \frac{1}{4} + \frac{1}{4} = \frac{3}{4}$$

This repeated addition also, we can write as multiplication.

$$3 \times \frac{1}{4} = \frac{1}{4} + \frac{1}{4} + \frac{1}{4} = \frac{3}{4}$$

$3 \times \frac{1}{4}$ can also be wirtten $\frac{1}{4} \times 3$.

Now what does $4 \times \frac{1}{10}$ mean?

$$4 \times \frac{1}{10} = \frac{1}{10} + \frac{1}{10} + \frac{1}{10} + \frac{1}{10} = \frac{1+1+1+1}{10} = \frac{4}{10}$$

We can simplify $\frac{4}{10}$, can't we?

$$\frac{4}{10} = \frac{2\times 2}{5\times 2} = \frac{2}{5}$$

Thus

$$4 \times \frac{1}{10} = \frac{2}{5}$$

$\frac{1}{10} \times 4$ also means the same.

$$\frac{1}{10} \times 4 = 4 \times \frac{1}{10} = \frac{2}{5}$$

A new multiplication

5×3 means adding up five 3's.

Likewise, $5 \times \frac{1}{3}$ means adding up five $\frac{1}{3}$'**s.**

That is, $5 \times \frac{1}{3} = \frac{5}{3}$

But a similar meaning cannot be given to $\frac{1}{3} \times 5$.

(one third 5's doesn't mean anything, does't?)

We are giving a new meaning to multiplication here.

$\frac{1}{3} \times 5$ is defined as $5 \times \frac{1}{3}$ itself.

That is, add up five $\frac{1}{3}$'s.

But $\frac{1}{3} \times \frac{1}{5}$ doesn't have any meaning now**.** We shall assign a meaning to it later.

Sums and products

What is $3 + \frac{3}{2}$?

$\frac{3}{2} = 1\frac{1}{2}$; isn't it? So,

$$3 + \frac{3}{2} = 3 + 1 + \frac{1}{2} = 4\frac{1}{2}$$

Now, $3 \times \frac{3}{2} = \frac{3 \times 3}{2} = \frac{9}{2} = 4\frac{1}{2}$

Again

$$4 + \frac{4}{3} = 4 + 1 + \frac{1}{3} = 5\frac{1}{3}$$

$$4 \times \frac{4}{3} = \frac{4 \times 4}{3} = \frac{16}{3} = 5\frac{1}{3}$$

Can you find some more pairs of numbers whose sum and product are the same?

Are there any pair of natural numbers with these property?

Another example; what is $3 \times \frac{2}{7}$?

$$3 \times \frac{2}{7} = \frac{2}{7} + \frac{2}{7} + \frac{2}{7} = \frac{2+2+2}{7} = \frac{6}{7}$$

We can write $2 + 2 + 2$ in the numerator as 3×2. So,

$$3 \times \frac{2}{7} = \frac{3 \times 2}{7} = \frac{6}{7}$$

Let's look at one more example.

When we divide 14 by 3, the quotient is 4 and the remainder 2. That is,

$$\frac{14}{3} = 14 \div 3 = 4\frac{2}{3}$$

So,

$$7 \times \frac{2}{3} = \frac{14}{3} = 4\frac{2}{3}$$

Now look at this problem.

3 rods of length $4\frac{1}{2}$ meters each are placed end to end. What is the total length?

We must add up three $4\frac{1}{2}$s.

$4\frac{1}{2} = 4 + \frac{1}{2}$. So,

$$4\frac{1}{2} + 4\frac{1}{2} + 4\frac{1}{2} = \left(4 + \frac{1}{2}\right) + \left(4 + \frac{1}{2}\right) + \left(4 + \frac{1}{2}\right)$$

$$= (4 + 4 + 4) + \left(\frac{1}{2} + \frac{1}{2} + \frac{1}{2}\right)$$

$$= 12 + \frac{3}{2}$$

$$= 12 + 1 + \frac{1}{2}$$

$$= 13\frac{1}{2}$$

This can be written $3 \times 4\frac{1}{2} = 13\frac{1}{2}$. There is another way of doing this. $4\frac{1}{2}$ is half of 9. That is

$$4\frac{1}{2} = \frac{9}{2}$$

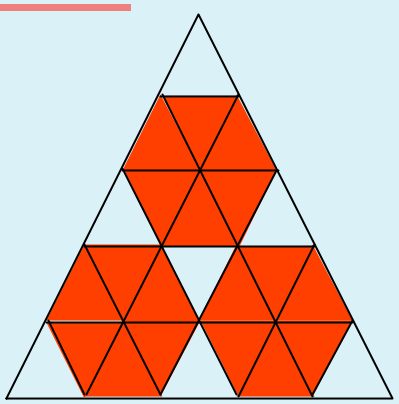
So,

$$3 \times 4\frac{1}{2} = 3 \times \frac{9}{2} = \frac{3 \times 9}{2} = \frac{27}{2} = 13\frac{1}{2}$$

Now try these problems.

- Exactly 12 pieces, each of length $\frac{1}{4}$ meter, are cut off from a string. What is the length of the string?
- The contents of a milk can were emptied into 8 bottles, each of capacity $\frac{3}{4}$ liter. What was the quantity of milk in the can?
- 5 pieces, each of length $2\frac{3}{4}$ meters, were cut off from a rod and a $\frac{1}{4}$ meter long piece was left. What is the length of the rod?
- Find the products given below:
  - $2 \times \frac{1}{3}$
  - $2 \times \frac{1}{4}$
  - $5 \times \frac{2}{3}$
  - $2 \times \frac{3}{4}$
  - $3 \times \frac{1}{3}$
  - $3 \times \frac{5}{6}$

Picture Math



In this picture, what part of the large triangle is coloured red?

The large triangle is divided into how many small triangles?

How many of them are coloured red?

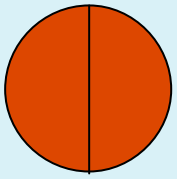
Now, we see that $\frac{18}{25}$ of the large triangle is coloured red.

We may think along another line as well. 3 shapes of the same size are coloured red. Each of these shapes contains 6 small triangles. So, the part coloured red is

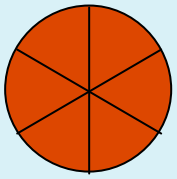
$$3 \times \frac{6}{25} = \frac{18}{25}$$

Parts of a circle

Draw a circle and divide it into two equal parts.

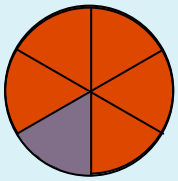


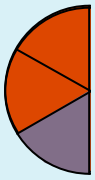
Again divide each of these parts into 3 equal parts.



The circle is now divided into how many equal parts?

Each small part is $\frac{1}{6}$ of the full circle.





Now, $\frac{1}{2}$ of $\frac{1}{3}$ means $\frac{1}{6}$ part.

Part of a fraction

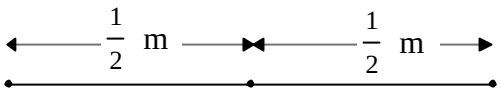
What is half of half of 1 meter? It is $\frac{1}{4}$ meter, isn't it?

It could be put another way.

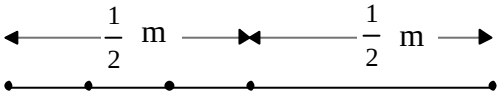
$\frac{1}{2}$ of $\frac{1}{2}$ meter is $\frac{1}{4}$ meter.

So, what is $\frac{1}{2}$ of $\frac{1}{3}$ meter?

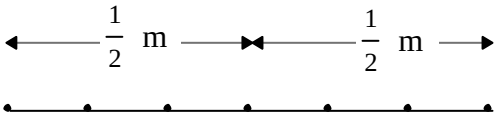
Let's draw a figure. $\frac{1}{2}$ meter is got by dividing 1 meter into two equal parts and taking one part.



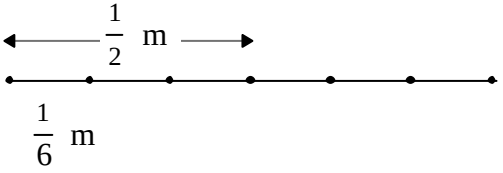
We have to find $\frac{1}{3}$ of one of these two parts.



If we divide the other $\frac{1}{2}$ meter also into 3 equal parts, then 1 meter is divided into 6 equal parts.



So, each of these small parts is $\frac{1}{6}$ meter.

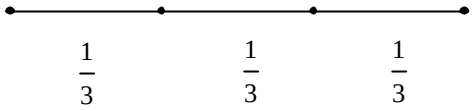


What do we see from this?

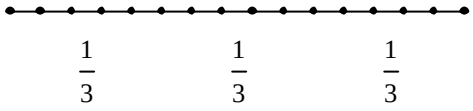
$\frac{1}{3}$ of $\frac{1}{2} = \frac{1}{6}$

Can you find $\frac{1}{5}$ of $\frac{1}{3}$ like this?

First, draw a line and divide it into 3 equal parts.

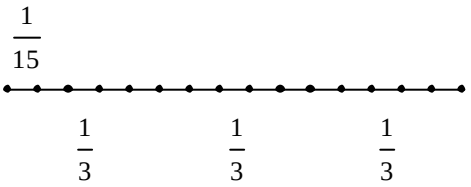


Divide each $\frac{1}{3}$ into 5 equal parts.



How many equal parts are there now?

$3 \times 5 = 15$, isn't it?



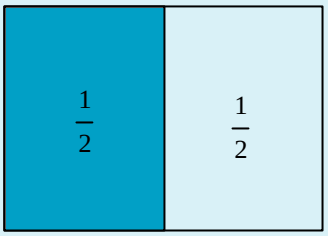
Each of these small parts is $\frac{1}{5}$ of $\frac{1}{3}$. What do we see now?

$$\frac{1}{5} \text{ of } \frac{1}{3} = \frac{1}{3\times5} = \frac{1}{15}$$

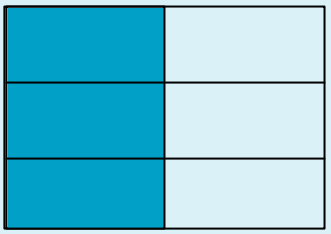
- $\frac{1}{3}$ of $\frac{1}{4}$
- $\frac{1}{3}$ of $\frac{1}{5}$
- $\frac{1}{3}$ of $\frac{1}{6}$
- $\frac{1}{2}$ of $\frac{1}{3}$
- $\frac{1}{6}$ of $\frac{1}{3}$
- $\frac{1}{4}$ of $\frac{1}{2}$

Parts of a rectangle

Draw a rectangle and divide it into two equal parts.

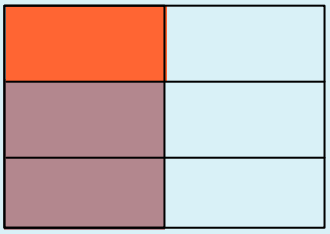


Now, divide each half into 3 equal parts.



What fraction of the whole rectangle is each of these parts?

What fraction of half the rectangle is each of these parts?



When parts join

What is $\frac{1}{5}$ of $\frac{2}{3}$ meter?

Let's draw a figure.

1 meter

Divide each $\frac{1}{3}$ part of 1 meter into 5 equal parts.

Now, 1 meter is divided into 15 equal parts. $\frac{2}{3}$ meter now consists of 10 of these parts. We want to divide $\frac{2}{3}$ into 5 equal parts. Suppose we take these 10 parts of $\frac{2}{3}$, two at a time?

Now $\frac{2}{3}$ meter is divided into how many equal parts? Each part consists of two pieces of length $\frac{1}{15}$ meter.

That is,

$$\frac{1}{5} \text{ of } \frac{2}{3} = \frac{2}{15}$$

More divisions

We know how to find $\frac{1}{5}$ of $\frac{1}{3}$

$$\frac{1}{5} \text{ of } \frac{1}{3} = \frac{1}{3\times5} = \frac{1}{15}$$

What is $\frac{1}{5}$ of $\frac{2}{3}$?

$\frac{2}{3}$ consists of two $\frac{1}{3}$'s.

So $\frac{1}{5}$ of $\frac{2}{3}$ consists of two $\frac{1}{5}$ parts of $\frac{1}{3}$ taken together.

That is,

$$\frac{1}{5} \text{ of } \frac{2}{3} = 2 \times \left(\frac{1}{5} \text{ of } \frac{1}{3}\right)$$

$$= 2 \times \frac{1}{15} = \frac{2}{15}$$

So, what is $\frac{1}{3}$ of $\frac{3}{5}$?

Now can't you do these in your head?

- $\frac{1}{5}$ of $\frac{2}{3}$
- $\frac{1}{2}$ of $\frac{3}{5}$
- $\frac{1}{2}$ of $\frac{4}{5}$
- $\frac{1}{8}$ of $\frac{3}{4}$

The last part

What is $\frac{2}{3}$ of $\frac{4}{5}$?

First we find $\frac{1}{3}$ of $\frac{4}{5}$.

$$\frac{1}{3} \text{ of } \frac{4}{5} = 4 \times \frac{1}{15} = \frac{4}{15}$$

$\frac{2}{3}$ consists of two $\frac{1}{3}$'s.

So,

$$\frac{2}{3} \text{ of } \frac{4}{5} = \left(\frac{1}{3} \text{of} \frac{4}{5} \right) \times 2 = \frac{4}{15} \times 2 = \frac{8}{15}$$

Now, how can we find $\frac{5}{8}$ of $\frac{3}{4}$?

First we find $\frac{1}{8}$ of $\frac{1}{4}$.

$$\frac{1}{8} \text{ of } \frac{1}{4} = \frac{1}{8 \times 4} = \frac{1}{32}$$

What next?

$$\frac{1}{8} \text{ of } \frac{3}{4} = 3 \times \frac{1}{32} = \frac{3}{32}$$

Then?

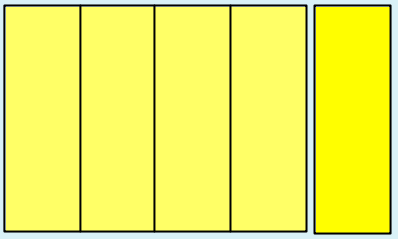
$$\frac{5}{8} \text{ of } \frac{3}{4} = \frac{3}{32} \times 5 = \frac{15}{32}$$

Now try these problems.

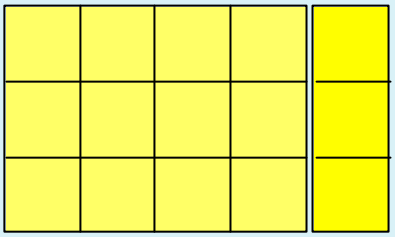
- Draw a line 15 centimeters long. Mark $\frac{4}{5}$ of it. What is its length? Now mark $\frac{2}{3}$ of this shorter line. What is its length?
- What is $\frac{4}{5}$ of $\frac{2}{3}$?
- What is $\frac{3}{4}$ of $\frac{5}{8}$?

Yet another rectangle

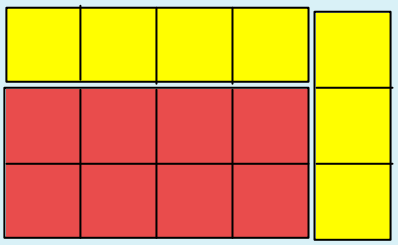
Draw a rectangle of length 5 centimeters and breadth 3 centimeters. Divide it into 5 equal parts and cut off one of the parts as shown below.



Now we have $\frac{4}{5}$ of the rectangle. In order to find $\frac{2}{3}$ of this, we divide the rectangle into 3 equal parts horizontally.



If we now cut off the top row, we are left with $\frac{2}{3}$ of this.



Now the red part is $\frac{2}{3}$ of $\frac{4}{5}$ of the rectangle. Count the small rectangles and find out what part of the original rectangle, the red part is.

A New Multiplication

$2 \times \frac{1}{3}$ means adding up four $\frac{1}{3}$'s.

That is,

$$4 \times \frac{1}{3} = \frac{1}{3} + \frac{1}{3} + \frac{1}{3} + \frac{1}{3} = \frac{4}{3}$$

But we cannot give a similar meaning to $\frac{1}{4} \times \frac{1}{3}$.

We have seen that $\frac{4}{3}$ is $\frac{1}{3}$ of 4. Then $4 \times \frac{1}{3}$ means $\frac{1}{3}$ of 4.

Similarly, $\frac{1}{4} \times \frac{1}{3}$ is given the meaning $\frac{1}{3}$ of $\frac{1}{4}$.

$$\frac{1}{3} \text{ of } \frac{1}{4} = \frac{1}{4 \times 3} = \frac{1}{12}$$

So,

$$\frac{1}{4} \times \frac{1}{3} = \frac{1}{4 \times 3} = \frac{1}{12}$$

- What is $\frac{2}{5}$ of $\frac{3}{7}$?
- What is $\frac{2}{7}$ of $\frac{3}{5}$?
- What is $\frac{3}{5}$ of $\frac{2}{7}$?

Multiplication of fractions

We have seen that $\frac{1}{3}$ of 2 is $\frac{2}{3}$. We also know that $\frac{2}{3}$ is $2 \times \frac{1}{3}$. That is

$$\frac{1}{3} \text{ of } 2 = \frac{2}{3} = \frac{1}{3} \times 2$$

Similarly, $\frac{1}{3}$ of $\frac{1}{2}$ can be written $\frac{1}{3} \times \frac{1}{2}$.

That is,

$$\frac{1}{3} \times \frac{1}{2} = \frac{1}{3} \text{ of } \frac{1}{2} = \frac{1}{3 \times 2} = \frac{1}{6}$$

Likewise

$$\frac{1}{5} \times \frac{1}{3} = \ldots\ldots\ldots\ldots = \ldots\ldots\ldots\ldots$$

What about $\frac{1}{2} \times \frac{3}{4}$?

$$\frac{1}{2} \times \frac{3}{4} = \frac{1}{2} \text{ of } \frac{3}{4}$$

$$= \frac{1}{2} \times \frac{1}{4} \times 3$$

$$= \frac{1}{8} \times 3$$

$$= \frac{3}{8}$$

What does $\frac{4}{5} \times \frac{2}{3}$ mean?

$$\frac{4}{5} \times \frac{2}{3} = \frac{4}{5} \text{ of } \frac{2}{3}$$

$$= 4 \times \frac{1}{5} \times \frac{1}{3} \times 2$$

$$= 4 \times \frac{1}{15} \times 2$$

$$= \frac{4}{15} \times 2 = \frac{8}{15}$$

Such operations can be simplified further.

Instead of writing,

$$\frac{2}{3} \times \frac{4}{7} = 2 \times \left(\frac{1}{3} \times \frac{1}{7}\right) \times 4 = 2 \times \frac{1}{3 \times 7} \times 4 = \frac{8}{21}$$

We can very well write

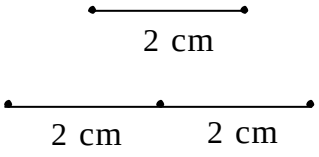
$$\frac{2}{3} \times \frac{4}{7} = \frac{2 \times 4}{3 \times 7} = \frac{8}{21}$$

Now try finding the following products

- $\frac{3}{10} \times \frac{7}{8}$
- $\frac{3}{4} \times \frac{7}{8}$
- $\frac{3}{4} \times \frac{2}{3}$
- $\frac{4}{3} \times \frac{3}{4}$

Fold and part

Suppose a line of length 2 centimeters is drawn and then extended by 2 centimeters.



What is the total length now?

Suppose it is futher extended by 2 centimeters.

The length becomes $3 \times 2 = 6$ centimeters.

The meaning of multiplication

We have given the meaning $\frac{1}{5}$ of $\frac{1}{2}$ to $\frac{1}{2} \times \frac{1}{5}$ Like wise, $\frac{4}{5} \times \frac{2}{3}$ mean $\frac{4}{5}$ of $\frac{2}{3}$.

That is,

$$\frac{4}{5} \times \frac{2}{3} = \frac{4}{5} \text{ of } \frac{2}{3}$$

$$= 4 \times \frac{1}{5} \times \frac{1}{3} \times 2$$

$$= \frac{4 \times 2}{5 \times 3} = \frac{8}{15}$$

Similarly,

$$\frac{5}{6} \times \frac{3}{4} = \frac{5 \times 3}{6 \times 4} = \frac{15}{24}$$

$\frac{15}{24}$ can be further simplified.

$$\frac{15}{24} = \frac{5 \times 3}{8 \times 3} = \frac{5}{8}$$

The 3 could have been disposed of earlier.

That is,

$$\frac{5}{6} \times \frac{3}{4} = \frac{5 \times 3}{6 \times 4} = \frac{5 \times 1}{2 \times 4} = \frac{5}{8}$$

Multiplication and several fold

Four 3's make four times 3. Four 3's and half of 3 make $4\frac{1}{2}$ times 3. That is

$4\frac{1}{2}$ times 3 = 4 times 3 + $\frac{1}{2}$ of 3.

$= (4 \times 3) + (\frac{1}{2} \times 3)$

$= 12 + \frac{3}{2}$

$= 12 + 1 + \frac{1}{2} = 13 + \frac{1}{2} = 13\frac{1}{2}$

Just as we write, $\frac{1}{2}$ of 3 as $\frac{1}{2} \times 3$,

$4\frac{1}{2}$ times 3, can be written $4\frac{1}{2} \times 3$.

That is,

$4\frac{1}{2} \times 3 = (4 \times 3) + \left(\frac{1}{2} \times 3\right) = 13\frac{1}{2}$

$4\frac{1}{2}$ can be written $\frac{9}{2}$.

So, we can write $4\frac{1}{2} \times 3$, as $\frac{9}{2} \times 3$, can't we?

$\frac{9}{2} \times 3 = \frac{9 \times 3}{2} = \frac{27}{2} = 13\frac{1}{2}$

What does this mean?

$4\frac{1}{2}$ times a number is the same as $\frac{1}{2}$ of 9 times a number.

We say, 3×2 is three times (thrice) 2. So, if we extend the line once again, we have 4 times 2.

Suppose we extend it further by 1 centimeter.

Now we have 4 times the original length together with $\frac{1}{2}$ of it.

This may be called $4\frac{1}{2}$ times the length.

So,

$4\frac{1}{2}$ times $2 = (4 \times 2) + (\frac{1}{2} \times 2) = 8 + 1 = 9$

Likewise,

$2\frac{1}{3}$ times $5 = (2 \times 5) + (\frac{1}{3} \times 5) = 10 + \frac{5}{3}$

$= 10 + 1 + \frac{2}{3} = 11\frac{2}{3}$

They could also be written as products.

$4\frac{1}{2} \times 2 = 9$

$2\frac{1}{3} \times 5 = 11\frac{2}{3}$

Another Multiplication

We have seen that $4\frac{1}{2} \times 2 = 9$. We can also change $4\frac{1}{2}$ into a fraction and do this.

$4\frac{1}{2} = 4 + \frac{1}{2}$

In order to turn it into a fraction we must first write $4 = \frac{4}{1}$, as a fraction. Because we have to add $\frac{1}{2}$, it is convenient to express 4 as a fraction with denominator 2.

Suppose we write

$$4 = \frac{4}{1} = \frac{4\times 2}{1\times 2} = \frac{8}{2}?$$

Then,

$$4\frac{1}{2} = \frac{8}{2} + \frac{1}{2} = \frac{9}{2}$$

Now,

$$4\frac{1}{2} \times 2 = \frac{9}{2} \times 2 = 9$$

Likewise, express $2\frac{1}{3}$ as a fraction and calculate $5 \times 2\frac{1}{3}$.

Now try the following problems.

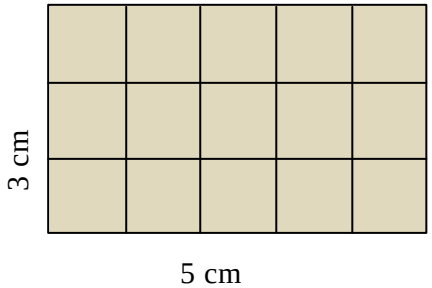
- The price of 1 kilogram of lady's finger is 15 rupees. What is the price of $2\frac{1}{4}$ kilograms?
- A bottle contains 2 liters of milk. 4 such bottles full and then $\frac{3}{4}$ of a bottle of milk were emptied into another vessel. How many liters of milk does the vessel contain now?
- Anu can walk $3\frac{1}{2}$ kilometers in 1 hour. How far can she walk in $1\frac{1}{2}$ hours?

Fractional area

We have learnt about the area of a rectangle in class 5.

What is the area of a rectangle of length 5 centimeters and breadth 3 centimeters?

We find it by filling it with squares of side 1 centimeter.



Area $= 5 \times 3 = 15$ sq.cm.

Fraction and area

If the length and breadth of a rectangle measured in centimeters are natural numbers, then the area (in square centimeters) is their product.

How did we get it?

What if the length and breadth are fractions?

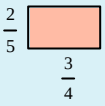
For example, we can't place a square of side 1 centimeter within a rectangle of length $\frac{1}{2}$ centimeter and breadth $\frac{1}{3}$ centimeter.

Here, we should look at it the other way round:

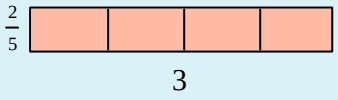
how many such small rectangles make up a square of side 1 centimeter?

Area again

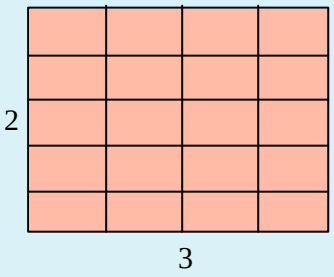
What is the area of a rectangle of length $\frac{3}{4}$ cm and breadth $\frac{2}{5}$ cm?



If 4 such rectangles are placed end to end, we get a rectangle as shown below.



Suppose 5 such rectangles are poled up as given below?



What is the area of this rectangle? How many small rectangles does it contain?

What part of the large rectangle is each small rectangle?

We find from this that the area of a small rectangle is $\frac{1}{20}$ of 6 sq.cm. What is it?

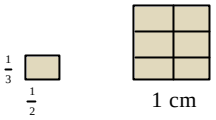
6 sq.cm. $\times \frac{1}{20} = \frac{6}{20}$ sq.cm.

This can be simplified to $\frac{3}{10}$ sq.cm.

Again, the area of the rectangle is the product of its sides.

What is the area of a rectangle of length $\frac{1}{2}$ centimeter and breadth $\frac{1}{3}$ centimeter?

6 such rectangles fill up a square of side 1 centimeter.



So, the area of this small rectangle is $\frac{1}{6}$ of the area of a square of side 1centimeter. So, we can say that the area of this small rectangle is $\frac{1}{6}$ square centimeter.

Similarly, we can see that the area of a rectangle of length $\frac{1}{4}$ centimeter and breadth $\frac{1}{6}$ centimeter. is $\frac{1}{4} \times \frac{1}{6} = \frac{1}{24}$ square centimeter.

What is the area of a rectangle of length $5\frac{1}{2}$ centimeter and breadth $3\frac{1}{3}$ centimeter?

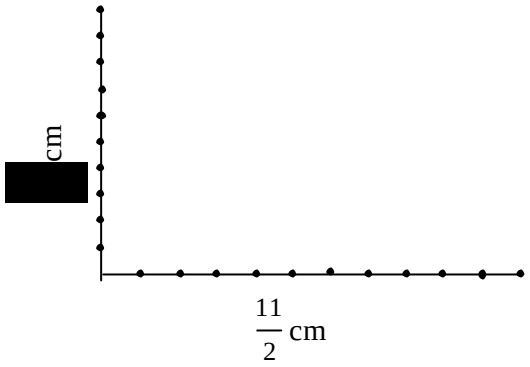
$5\frac{1}{2} = \frac{11}{2}$ right? This means one side of this rectangle is made up of 11 small lengths of $\frac{1}{2}$ centimeter each.

$\frac{11}{2}$ cm

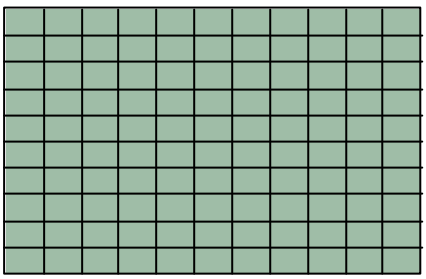
Likewise,

$$3\frac{1}{3} = 3 + \frac{1}{3} = \frac{9}{3} + \frac{1}{3} = \frac{10}{3}$$

and this means that the other side of the rectangle is made up of 10 small lengths of $\frac{1}{3}$ centimeter each.



So we can fill this rectangle with 11×10 small rectangles each of length $\frac{1}{2}$ centimeter and breadth $\frac{1}{3}$ centimeter.



The area of each small rectangle is $\frac{1}{6}$ square centimeter, as seen earlier.

So, area of the large rectangle

$$= 110 \times \frac{1}{6} = \frac{110}{6} = 18\frac{1}{3} \text{ sq.cm.}$$

Is this equal to the product of the length and breadth of this rectangle?

$$5\frac{1}{2} \times 3\frac{1}{3} = \frac{11}{2} \times \frac{10}{3} = \frac{11 \times 10}{2 \times 3} = \frac{110}{6}$$

Thus, we find that the area of a rectangle is the product of the length and the breadth, even if their measures involve fractions.

So, what is the area of a rectangle of length $4\frac{1}{3}$ cm and breadth $2\frac{1}{5}$ cm?

Fraction and decimal

Nowadays, quantities are often expressed in decimals, instead of fractions. If the length of a rectangle is 4.5 meters and the breadth 3.2 meters, how do we find the area?

First we express the lengths in fractions.

$$4.5 \text{ meters} = 4\frac{5}{10} \text{ meters} = 4\frac{1}{2} \text{ meters}$$

$$3.2 \text{ meters} = 3\frac{2}{10} \text{ meters} = 3\frac{1}{5} \text{ meters}$$

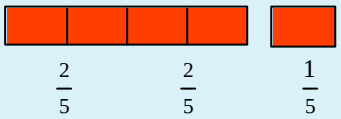
Now, in order to find the area, we need only find $4\frac{1}{2} \times 3\frac{1}{5}$. The product is to be written as a decimal. Go ahead!

Part and Whole

Take a long strip of paper and cut it into 5 equal parts.

Join two of the parts together.

This is $\frac{2}{5}$ of the strip. Again, add 2 more parts to it.



We now have two parts, each $\frac{2}{5}$ of the strip. That is twice $\frac{2}{5}$. What remains is half of $\frac{2}{5}$. Add it also to it. We now have twice of $\frac{2}{5}$ together with $\frac{1}{2}$ of $\frac{2}{5}$ making it $2\frac{1}{2}$ times $\frac{2}{5}$ altogether. That is, $\frac{5}{2}$ times $\frac{2}{5}$. And this is the whole strip. What do we see here?

Now you add attempt these problems.

- The length and breadth of a few rectangles are given below. Find the area of each rectangle.
  - $3\frac{1}{2}$ centimeters, $2\frac{1}{4}$ centimeters
  - $4\frac{1}{2}$ meters, $3\frac{1}{2}$ meters
  - 5.2 meters, 2.5 meters
- The perimeter of a square is 14 meters. What is its area?

Topsy-turvy multiplication

What is $\frac{3}{4} \times \frac{4}{3}$?

$$\frac{3}{4} \times \frac{4}{3} = \frac{3\times 4}{4\times 3} = \frac{12}{12} = 1$$

What about $\frac{2}{5} \times \frac{5}{2}$?

Interchanging of the numerator and denominator of the fraction $\frac{3}{4}$ yields $\frac{4}{3}$. It is called the *reciprocal* of $\frac{3}{4}$.

Similarly, the reciprocal of $\frac{2}{5}$ is $\frac{5}{2}$.

What can you now say of the products we've just found out?

The product of a fraction and its reciprocal is 1.

The natural number 2 has the fractional form $\frac{2}{1}$.

So, the reciprocal of 2 is $\frac{1}{2}$.

What is the product? $2 \times \frac{1}{2} = 1$, again.

Another point can be noted here. We have seen that any division can be expressed as a fraction. For example,

$6 \div 3 = 2$

And we can write this as,

$\frac{6}{3} = 2$

Also

$6 \times \frac{1}{3} = 2$

What do we see here?

$6 \div 3 = 6 \times \frac{1}{3}$

That is, division by a natural number is the same as multiplication by its reciprocal.

Division with fractions

The price of 2 kilograms of beans is 50 rupees. What is the price of 1 kilogram?

The price of 1 kilogram of beans

$= 50 \times \frac{1}{2}$

$= 50 \div 2$

$= 25$ rupees

Look at another problem.

The price of $1\frac{1}{2}$ kilograms of green chilli is 24 rupees. What is the price of 1 kilogram?

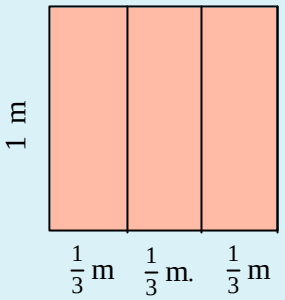
$1\frac{1}{2} = \frac{3}{2}$, isn't it? That is, half of 3 kilograms.

Thus 24 rupees is the price of $\frac{1}{2}$ of 3 kilograms. So, the price of 3 kilograms is 2 times this.

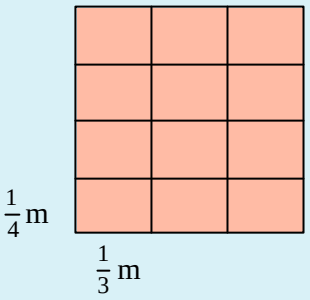
Topsy - turvy

What is the area of a square of side 1 meter?

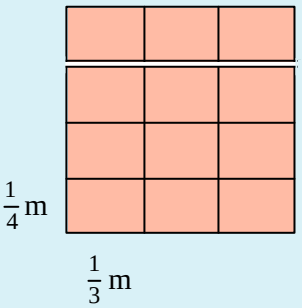
Suppose, we divide the square vertically into 3 equal parts as shown below.

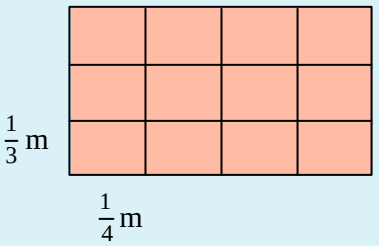


Next let's we divide it horizontally into 4 equal parts.



Suppose we cut off the three pieces on the top and place them, on the left, as shown below.





What is the length of the new rectangle? And the breadth? What is its area?

That is,

the price of 3 kilograms of green chilli

$= 24 \times 2 = 48$ rupees.

Now we can find the price of 1 kilogram.

The price of 1 kilogram $= \frac{48}{3}$

$= 48 \div 3$

$= 16$ rupees

What all operations did we do here? First we multiplied 24 by 2 and then divided the product by 3.

That is,

$$(24 \times 2) \div 3 = 24 \times 2 \times \frac{1}{3}$$

$$= 24 \times \frac{2}{3}$$

That is to say, we have multiplied by the reciprocal, as in the first problem.

The area of a rectangle is 8 square. Its length is 4 meters, What is the breadth?

Breadth of the rectangle $= 8 \div 4 = 2$ meters

What happens if fractions are involved?

The area of a rectangle is $8\frac{3}{4}$ square meters and the length is $3\frac{1}{2}$ meters. What is the breadth?

The area is the product of the length and the breadth.

Using the measurements given,

breadth $\times 3\frac{1}{2} = 8\frac{3}{4}$

Doing it in reverse

Half of three times a number is 15. What is the number?

First, let's find three times the number. Since half of it is 15, it must be 30.

If three times the number is to be 30, the number must be 10.

That is, 10 multiplied by 3 and then divided by 2 gives 15. Going the other way round, 15 multiplied by 2 and divided by 3 gives 10 back!

Now,

$$3\frac{1}{2} = \frac{6}{2} + \frac{1}{2} = \frac{7}{2}$$

$$8\frac{3}{4} = \frac{32}{4} + \frac{3}{4} = \frac{35}{4}$$

So,

$$\frac{\text{breadth} \times 7}{2} = \frac{35}{4}$$

What does this mean?

Half of 7 times the breadth is $\frac{35}{4}$. So, 7 times the breadth must be twice of $\frac{35}{4}$.

$$\text{Breadth} \times 7 = \frac{35}{4} \times 2 = \frac{35 \times 2}{4} = \frac{35 \times 2}{2 \times 2} = \frac{35}{2}$$

Now we have 7 times the breadth. What next? The breadth is $\frac{1}{7}$ of this.

$$\text{Breadth} = \frac{35}{2} \times \frac{1}{7} = \frac{35}{2 \times 7} = \frac{5 \times 7}{2 \times 7} = \frac{5}{2} = 2\frac{1}{2}\text{m}$$

The operations used were multiplication by 2 and $\frac{1}{7}$.

$$8\frac{3}{4} \times 2 \times \frac{1}{7} = 8\frac{3}{4} \times \frac{2}{7}$$

That is, multiplication by the reciprocal of $\frac{7}{2}$.

Since $\frac{7}{2} = 3\frac{1}{2}$, we can say that we have multiplied by the reciprocal of $3\frac{1}{2}$.

Multiplication by the reciprocal can be written as a division, even in the case of fractions. Thus, the operations in the two problems above can be written like this.

Division means...

What does $12 \div 4$ means? We must find out with what number we should multiply 4, to get 12. Since

$$4 \times 3 = 12$$

We write

$$12 \div 4 = 3$$

We have seen that this can be written as multiplication by the reciprocal.

$$12 \div 3 = 12 \times \frac{1}{3}$$
$$= 4$$

Likewise,

$$24 \div \frac{3}{2} = 24 \times \frac{2}{3}$$
$$= 16$$

Here also, suppose we turn back and multiply 16 by $\frac{3}{2}$?

$$16 \times \frac{3}{2} = \frac{16 \times 3}{2}$$
$$= 8 \times 3$$
$$= 24$$

Why is this so?

Since

$$16 = 24 \times \frac{2}{3}$$

We have,

$$16 \times \frac{3}{2} = 24 \times \frac{2}{3} \times \frac{3}{2}$$
$$= 24$$

A poser

How many pieces of length $\frac{1}{2}$ meter can be cut off from a rod of length 36 meters? What is the length of the remaining piece?

Appu did it like this:

$$36 \div 2\frac{1}{2} = 36 \div \frac{5}{2}$$

$$= 36 \times \frac{2}{5}$$

$$= \frac{72}{5}$$

When 72 is divided by 5, the quotient is 14 and the remainder 2. So there are 14 pieces with 2 meters remaining.

Ammu did it another way. 2 pieces each of length $2\frac{1}{2}$ meters make up 5 meters.

$$7 \times 5 = 35$$

So there are $7 \times 2 = 14$ pieces.

The reminder is $36 - 35 = 1$ meter.

Who got it right?

$$24 \div 1\frac{1}{2} = 24 \div \frac{3}{2} = 24 \times \frac{2}{3} = 16$$

$$8\frac{3}{4} \div 3\frac{1}{2} = \frac{35}{4} \div \frac{7}{2} = \frac{35}{4} \times \frac{2}{7} = 2\frac{1}{2}$$

Now, try these problems.

- The length of $\frac{2}{3}$ of a string is $3\frac{1}{2}$ meters. What is the length of the string?
- $1\frac{1}{2}$ liters of water full $\frac{3}{4}$ of a pot. How much water makes the pot full?